Formula Booklet
for the Standardised Competence-Oriented
Written School-Leaving Examination (SRP)
Mathematics (AHS)
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1 Sets

\[ \in \] is an element of...
\[ \notin \] is not an element of...
\[ \cap \] intersection
\[ \cup \] union
\[ \subset \] proper subset
\[ \subseteq \] subset
\[ \setminus \] difference ("without")
\[ \{ \} \] empty set

Sets of numbers

\[ \mathbb{N} = \{0, 1, 2, \ldots\} \] natural numbers
\[ \mathbb{Z} \] integers
\[ \mathbb{Q} \] rational numbers
\[ \mathbb{R} \] real numbers
\[ \mathbb{C} \] complex numbers
\[ \mathbb{R}^+ \] positive real numbers
\[ \mathbb{R}_0^+ \] positive real numbers including zero

2 Prefixes

<table>
<thead>
<tr>
<th>Prefix</th>
<th>Symbol</th>
<th>Base</th>
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<tbody>
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<td>tera-</td>
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<td>pico-</td>
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3 Powers

Powers with integer exponents

\[ a \in \mathbb{R}; n \in \mathbb{N}\setminus\{0\} \]
\[ a \in \mathbb{R}\setminus\{0\}; n \in \mathbb{N}\setminus\{0\} \]

\[ a^n = a \cdot a \cdot \ldots \cdot a \]
\[ a^1 = a \]
\[ a^0 = 1 \]
\[ a^{-1} = \frac{1}{a} \]
\[ a^{-n} = \left(\frac{1}{a}\right)^n \]
\[ n \text{ factors} \]

Powers with rational exponents (roots)

\[ a, b \in \mathbb{R}_0^+; n, k \in \mathbb{N}\setminus\{0\} \text{ where } n \geq 2 \]

\[ a = \sqrt[n]{b} \iff a^n = b \]
\[ a^\frac{k}{n} = \sqrt[n]{a^k} \]
\[ a^{-\frac{k}{n}} = \frac{1}{\sqrt[n]{a^k}} \]
\[ a^{\frac{k}{n}} = \left(\sqrt[n]{a}\right)^k \]
\[ \text{where } a > 0 \]
Calculation rules

\( a, b \in \mathbb{R}\setminus\{0\}; r, s \in \mathbb{Z} \)

or \( a, b \in \mathbb{R}^+; r, s \in \mathbb{Q} \)

\[
\begin{align*}
& a \cdot a = a^{r+s} \\
& \frac{a^r}{a^s} = a^{r-s} \\
& (a^s)^r = a^{rs} \\
& (a \cdot b)^r = a^r \cdot b^r \\
& \sqrt{a} = \frac{a}{\sqrt{a}} \\
& \sqrt[3]{a} = \left(\sqrt[3]{a}\right)^2 \quad (b \neq 0)
\end{align*}
\]

Binomial formulae

\( a, b \in \mathbb{R}; n \in \mathbb{N} \)

\[
\begin{align*}
(a + b)^2 &= a^2 + 2 \cdot a \cdot b + b^2 \\
(a - b)^2 &= a^2 - 2 \cdot a \cdot b + b^2 \\
(a + b) \cdot (a - b) &= a^2 - b^2
\end{align*}
\]

4 Logarithms

\( a, b, c \in \mathbb{R}^+ \) where \( a \neq 1; x, r \in \mathbb{R} \)

\[
\begin{align*}
x &= \log_a(b) \iff a^x = b \\
\log_a(b \cdot c) &= \log_a(b) + \log_a(c) \\
\log_a\left(\frac{b}{c}\right) &= \log_a(b) - \log_a(c) \\
\log_a(a^r) &= x \\
\log_a(1) &= 0 \\
\log_a\left(\frac{1}{a}\right) &= -1
\end{align*}
\]

natural logarithm (logarithm with base \( e \)): \( \ln(b) = \log_e(b) \)

common logarithm (logarithm with base 10): \( \lg(b) = \log_{10}(b) \)

5 Quadratic Equations

\( p, q \in \mathbb{R} \)

\( a, b, c \in \mathbb{R} \) where \( a \neq 0 \)

\[
\begin{align*}
x^2 + p \cdot x + q &= 0 \\
x_{1,2} &= -\frac{p}{2} \pm \sqrt{\left(\frac{p}{2}\right)^2 - q}
\end{align*}
\]

Vieta’s Theorem

\( x_1 \) and \( x_2 \) are the solutions to the equation \( x^2 + p \cdot x + q = 0 \) if and only if:

\[
\begin{align*}
x_1 + x_2 &= -p \\
x_1 \cdot x_2 &= q
\end{align*}
\]

Linear factorisation:

\( x^2 + p \cdot x + q = (x - x_1) \cdot (x - x_2) \)
6 Two-Dimensional Shapes

Triangle

\[ u = a + b + c \]

General triangle

\[ A = \frac{a \cdot h_a}{2} = \frac{b \cdot h_b}{2} = \frac{c \cdot h_c}{2} \]

Heron’s Formula

\[ A = \sqrt{s \cdot (s-a) \cdot (s-b) \cdot (s-c)} \text{ where } s = \frac{a+b+c}{2} \]

Pythagorean theorem

\[ a^2 + b^2 = c^2 \]

Quadrilateral

Square

\[ A = a^2 \]
\[ u = 4 \cdot a \]

Rhombus

\[ A = a \cdot h_a = \frac{e \cdot f}{2} \]
\[ u = 4 \cdot a \]

Parallelogram

\[ A = a \cdot h_a = b \cdot h_b \]
\[ u = 2 \cdot a + 2 \cdot b \]

Trapezium

\[ A = \frac{(a+c) \cdot h}{2} \]
\[ u = a + b + c + d \]

Kite

\[ A = \frac{e \cdot f}{2} \]
\[ u = 2 \cdot a + 2 \cdot b \]

Circle

\[ A = \pi \cdot r^2 = \frac{\pi \cdot d^2}{4} \]
\[ u = 2 \cdot \pi \cdot r = \pi \cdot d \]

Arc length and sector of a circle

\[ \alpha \text{ in degrees (°)} \]
\[ b = \pi \cdot r \cdot \frac{\alpha}{180°} \]
\[ A = \pi \cdot r^2 \cdot \frac{\alpha}{360°} = \frac{b \cdot r}{2} \]
7 Solids

<table>
<thead>
<tr>
<th>Formula</th>
<th>Prism</th>
<th>Cylinder</th>
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<tbody>
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<td>$V = G \cdot h$</td>
<td>$V = G \cdot h$</td>
<td>$V = G \cdot h$</td>
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<tr>
<td>$M = u_G \cdot h$</td>
<td>$M = u_G \cdot h$</td>
<td>$M = u_G \cdot h$</td>
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<tr>
<td>$O = 2 \cdot G + M$</td>
<td>$O = 2 \cdot G + M$</td>
<td>$O = 2 \cdot G + M$</td>
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<td>$V = \frac{G \cdot h}{3}$</td>
<td>$V = \frac{G \cdot h}{3}$</td>
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<th>Sphere</th>
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<td>$V = \frac{4}{3} \cdot \pi \cdot r^3$</td>
<td>$V = \frac{4}{3} \cdot \pi \cdot r^3$</td>
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<tr>
<td>$O = 4 \cdot \pi \cdot r^2$</td>
<td>$O = 4 \cdot \pi \cdot r^2$</td>
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8 Trigonometry

Converting between degrees and radians

Right-angled triangle trigonometry

- **Sine:** $\sin(\alpha) = \frac{\text{side opposite to } \alpha}{\text{hypotenuse}}$
- **Cosine:** $\cos(\alpha) = \frac{\text{side adjacent to } \alpha}{\text{hypotenuse}}$
- **Tangent:** $\tan(\alpha) = \frac{\text{side opposite to } \alpha}{\text{side adjacent to } \alpha}$
Unit circle trigonometry

\[ \sin^2(\alpha) + \cos^2(\alpha) = 1 \]
\[ \tan(\alpha) = \frac{\sin(\alpha)}{\cos(\alpha)} \text{ for } \cos(\alpha) \neq 0 \]

9 Vectors

**Vectors in \( \mathbb{R}^2 \)**

Arrow from \( P \) to \( Q \):

\[ P = (p_1, p_2), \quad Q = (q_1, q_2) \]

\[ \overrightarrow{PQ} = (q_1 - p_1, q_2 - p_2) \]

**Calculation rules in \( \mathbb{R}^2 \)**

\[ \begin{align*}
\vec{a} &= (a_1, a_2), \quad \vec{b} = (b_1, b_2), \\
\vec{a} \pm \vec{b} &= (a_1 \pm b_1, a_2 \pm b_2)
\end{align*} \]

\[ k \cdot \vec{a} = k \cdot (a_1, a_2) \quad \text{where } k \in \mathbb{R} \]

**Scalar product in \( \mathbb{R}^2 \)**

\[ \vec{a} \cdot \vec{b} = a_1 \cdot b_1 + a_2 \cdot b_2 \]

**Absolute value (length) of a vector in \( \mathbb{R}^2 \)**

\[ |\vec{a}| = \sqrt{a_1^2 + a_2^2} \]

Vector perpendicular to \( \vec{a} = (a_2) \) in \( \mathbb{R}^2 \)

\[ \vec{n} = k \cdot \left( -\frac{a_2}{a_1} \right) \text{ where } k \in \mathbb{R}\{0\} \text{ and } |\vec{a}| \neq 0 \]

**Criterion for two vectors to be perpendicular in \( \mathbb{R}^2 \) and \( \mathbb{R}^3 \)**

\[ \vec{a} \cdot \vec{b} = 0 \iff \vec{a} \perp \vec{b} \text{ where } |\vec{a}| \neq 0; |\vec{b}| \neq 0 \]

**Vectors in \( \mathbb{R}^n \)**

Arrow from \( P \) to \( Q \):

\[ P = (p_1, p_2, \ldots, p_n), \quad Q = (q_1, q_2, \ldots, q_n) \]

\[ \overrightarrow{PQ} = (q_1 - p_1, q_2 - p_2, \ldots, q_n - p_n) \]

**Calculation rules in \( \mathbb{R}^n \)**

\[ \begin{align*}
\vec{a} &= (a_1, a_2, \ldots, a_n), \quad \vec{b} = (b_1, b_2, \ldots, b_n), \\
\vec{a} \pm \vec{b} &= (a_1 \pm b_1, a_2 \pm b_2, \ldots, a_n \pm b_n)
\end{align*} \]

\[ k \cdot \vec{a} = k \cdot (a_1, a_2, \ldots, a_n) \quad \text{where } k \in \mathbb{R} \]

**Scalar product in \( \mathbb{R}^n \)**

\[ \vec{a} \cdot \vec{b} = a_1 \cdot b_1 + a_2 \cdot b_2 + \ldots + a_n \cdot b_n \]

**Absolute value (length) of a vector in \( \mathbb{R}^n \)**

\[ |\vec{a}| = \sqrt{a_1^2 + a_2^2 + \ldots + a_n^2} \]

Angle \( \varphi \) between \( \vec{a} \) and \( \vec{b} \) in \( \mathbb{R}^2 \) and \( \mathbb{R}^3 \)

\[ \cos(\varphi) = \frac{\vec{a} \cdot \vec{b}}{|\vec{a}| \cdot |\vec{b}|} \quad \text{where } |\vec{a}| \neq 0; |\vec{b}| \neq 0 \]

**Criterion for two vectors to be parallel in \( \mathbb{R}^2 \) and \( \mathbb{R}^3 \)**

\[ \vec{a} \parallel \vec{b} \iff \vec{a} = k \cdot \vec{b} \text{ where } k \in \mathbb{R}\{0\} \text{ and } |\vec{a}| \neq 0; |\vec{b}| \neq 0 \]
10 Straight Lines

\[ g \ldots \text{line} \]
\[ \vec{g} \ldots \text{a direction vector for the line } g \]
\[ \vec{n} \ldots \text{a vector perpendicular to the line } g \]
\[ X, P \ldots \text{points on the line } g \]
\[ m \ldots \text{gradient of the line } g \]
\[ \alpha \ldots \text{angle of slope of the line } g \]
\[ a, b, c, k \in \mathbb{R} \]

Vector equation of a line \( g \) in \( \mathbb{R}^2 \) and \( \mathbb{R}^3 \)
\[ g: X = P + t \cdot \vec{g} \quad \text{where} \quad t \in \mathbb{R} \]

Equation of a line \( g \) in \( \mathbb{R}^2 \)
the explicit equation of a line:
\[ g: y = m \cdot x + c \quad \text{where} \quad m = \tan(\alpha) \]
a general equation of a line:
\[ g: a \cdot x + b \cdot y = c \]
a normal vector representation:
\[ g: \vec{n} \cdot X = \vec{n} \cdot P \]
\[ \vec{n} \parallel \begin{pmatrix} a \\ b \end{pmatrix} \quad \text{and} \quad \begin{pmatrix} a \\ b \end{pmatrix} \neq \begin{pmatrix} 0 \\ 0 \end{pmatrix} \]

11 Rates of Change

For a real function \( f \) defined over an interval \([a, b]\):

Absolute change of \( f \) in \([a, b]\)
\[ f(b) - f(a) \]

Relative (percentage) change of \( f \) in \([a, b]\)
\[ \frac{f(b) - f(a)}{f(a)} \quad \text{where} \quad f(a) \neq 0 \]

Difference quotient (average rate of change) of \( f \) in \([a, b]\) or \([x, x + \Delta x]\)
\[ \frac{f(b) - f(a)}{b - a} \quad \text{or} \quad \frac{f(x + \Delta x) - f(x)}{\Delta x} \quad \text{where} \quad b = a \quad \text{and} \quad \Delta x \neq 0 \]

Differential quotient (instantaneous rate of change) of \( f \) at the point \( x \)
\[ f'(x) = \lim_{x \to x} \frac{f(x_i) - f(x)}{x_i - x} \quad \text{or} \quad f'(x) = \lim_{x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x} \]
12 Differentiation and Integration

\[ f, g, h \ldots \text{functions that are differentiable over } \mathbb{R} \text{ or over a defined interval} \]

\[ f' \ldots \text{first derivative of } f \]
\[ g' \ldots \text{first derivative of } g \]
\[ h' \ldots \text{first derivative of } h \]
\[ C, k, q \in \mathbb{R}; a \in \mathbb{R} \setminus \{1\} \]

Indefinite integral
\[ \int f(x) \, dx = F(x) + C \quad \text{where } F' = f \]

Definite integral
\[ \int_a^b f(x) \, dx = F(b) - F(a) \]

Function | Derivative | Antiderivative
--- | --- | ---
\( f(x) = k \) | \( f'(x) = 0 \) | \( F(x) = k \cdot x \)
\( f(x) = x^q \) | \( f'(x) = q \cdot x^{q-1} \) | \( F(x) = \frac{x^{q+1}}{q+1} \quad \text{where } q \neq -1 \)
\( f(x) = e^x \) | \( f'(x) = e^x \) | \( F(x) = e^x \)
\( f(x) = a^x \) | \( f'(x) = \ln(a) \cdot a^x \) | \( F(x) = \frac{a^x}{\ln(a)} \)
\( f(x) = \sin(x) \) | \( f'(x) = \cos(x) \) | \( F(x) = -\cos(x) \)
\( f(x) = \cos(x) \) | \( f'(x) = -\sin(x) \) | \( F(x) = \sin(x) \)
\( g(x) = k \cdot f(x) \) | \( g'(x) = k \cdot f'(x) \) | \( G(x) = k \cdot F(x) \)
\( h(x) = f(x) \pm g(x) \) | \( h'(x) = f'(x) \pm g'(x) \) | \( H(x) = F(x) \pm G(x) \)
\( g(x) = f(k \cdot x) \) | \( g'(x) = k \cdot f'(k \cdot x) \) | \( G(x) = \frac{1}{k} \cdot F(k \cdot x) \)

13 Statistics

\( x_1, x_2, \ldots, x_n \ldots \text{a list of } n \text{ real numbers} \)
\( x_{(1)} \leq x_{(2)} \leq \ldots \leq x_{(n)} \ldots \text{ordered list of } n \text{ values} \)

Arithmetic mean
\[ \bar{x} = \frac{x_1 + x_2 + \ldots + x_n}{n} = \frac{1}{n} \cdot \sum_{i=1}^{n} x_i \]

Median
\[ \bar{x} = \begin{cases} x_{\frac{n+1}{2}} & \text{... when } n \text{ is odd} \\ \frac{1}{2} \cdot \left( x_{\left[ \frac{n}{2} \right]} + x_{\left[ \frac{n}{2} + 1 \right]} \right) & \text{... when } n \text{ is even} \end{cases} \]

Measures of spread

\( s^2 \ldots \text{(empirical) variance of a sample} \)
\( s \ldots \text{(empirical) standard deviation of a sample} \)

\[ s^2 = \frac{1}{n} \cdot \sum_{i=1}^{n} (x_i - \bar{x})^2 \]
\[ s = \sqrt{\frac{1}{n} \cdot \sum_{i=1}^{n} (x_i - \bar{x})^2} \]

If the variance of a population should be estimated using a sample of size \( n \):

\[ s_{n-1}^2 = \frac{1}{n-1} \cdot \sum_{i=1}^{n} (x_i - \bar{x})^2 \]
14 Probability

\[ n \in \mathbb{N}\setminus\{0\}; \, k \in \mathbb{N} \text{ where } k \leq n \]

A, B ... events

\( \neg A \) or \( \bar{A} \) ... complementary event of A

A \land B \text{ or } A \cap B \text{ ... A and B (the event A and the event B both occur)}

A \lor B \text{ or } A \cup B \text{ ... A or B (at least one of the two events A or B occurs)}

\( P(A) \) ... probability of event A occurring

\( P(A \mid B) \) ... probability of event A occurring given that B has occurred (conditional probability)

<table>
<thead>
<tr>
<th>Factorial</th>
<th>Binomial coefficient</th>
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<tbody>
<tr>
<td>( n! = n \cdot (n-1) \cdot ... \cdot 1 )</td>
<td>( \binom{n}{k} = \frac{n!}{k! \cdot (n-k)!} )</td>
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Probability for a Laplace experiment

\( P(A) = \frac{\text{number of successful outcomes for } A}{\text{number of possible outcomes}} \)

Elementary rules

\[ P(\neg A) = 1 - P(A) \]
\[ P(A \land B) = P(A) \cdot P(B \mid A) = P(B) \cdot P(A \mid B) \]
\[ P(A \land B) = P(A) \cdot P(B) \text{ ... if A and B are (stochastically) independent of one another} \]
\[ P(A \lor B) = P(A) + P(B) - P(A \land B) \]
\[ P(A \lor B) = P(A) + P(B) \text{ ... if A and B are mutually exclusive} \]

Expectation value \( \mu \) of a discrete random variable \( X \) with values \( x_1, x_2, \ldots, x_n \)

\[ \mu = E(X) = x_1 \cdot P(X = x_1) + x_2 \cdot P(X = x_2) + \ldots + x_n \cdot P(X = x_n) = \sum_{i=1}^{n} x_i \cdot P(X = x_i) \]

Variance \( \sigma^2 \) of a discrete random variable \( X \) with values \( x_1, x_2, \ldots, x_n \)

\[ \sigma^2 = V(X) = \sum_{i=1}^{n} (x_i - \mu)^2 \cdot P(X = x_i) \]

Standard deviation \( \sigma \)

\[ \sigma = \sqrt{V(X)} \]

Binomial distribution

\[ n \in \mathbb{N}\setminus\{0\}; \, k \in \mathbb{N}; \, p \in \mathbb{R} \text{ where } k \leq n \text{ and } 0 \leq p \leq 1 \]

The random variable \( X \) is binomially distributed with parameters \( n \) and \( p \)

\[ P(X = k) = \binom{n}{k} \cdot p^k \cdot (1-p)^{n-k} \]
\[ E(X) = \mu = n \cdot p \]
\[ V(X) = \sigma^2 = n \cdot p \cdot (1-p) \]
Normal distribution

\( \mu, \sigma \in \mathbb{R} \) where \( \sigma > 0 \)

\( f \ldots \) probability density function

\( \varphi \ldots \) probability density function of the standard normal distribution

\( \phi \ldots \) cumulative density function of the standard normal distribution

Normal distribution \( N(\mu; \sigma^2) \): The random variable \( X \) is normally distributed with expectation value \( (\mu) \), standard deviation \( (\sigma) \) and variance \( (\sigma^2) \)

\[
P(X \leq x) = \int_{-\infty}^{x} f(x) \, dx = \int_{-\infty}^{x} \frac{1}{\sigma \sqrt{2 \pi}} \cdot e^{-\frac{(x-\mu)^2}{2 \sigma^2}} \, dx
\]

Probabilities for standard deviation bands

\[
P(\mu - \sigma \leq X \leq \mu + \sigma) \approx 0.683
\]

\[
P(\mu - 2 \cdot \sigma \leq X \leq \mu + 2 \cdot \sigma) \approx 0.954
\]

\[
P(\mu - 3 \cdot \sigma \leq X \leq \mu + 3 \cdot \sigma) \approx 0.997
\]

Standard normal distribution \( N(0, 1) \)

\[
z = \frac{x - \mu}{\sigma}
\]

\[
\phi(z) = P(Z \leq z) = \int_{-\infty}^{z} \varphi(x) \, dx = \frac{1}{\sqrt{2 \pi}} \cdot \int_{-\infty}^{z} e^{-\frac{x^2}{2}} \, dx
\]

\[
\phi(-z) = 1 - \phi(z)
\]

\[
P(-z \leq Z \leq z) = 2 \cdot \phi(z) - 1
\]

<table>
<thead>
<tr>
<th>Probability</th>
<th>z</th>
</tr>
</thead>
<tbody>
<tr>
<td>90 %</td>
<td>1.645</td>
</tr>
<tr>
<td>95 %</td>
<td>1.960</td>
</tr>
<tr>
<td>99 %</td>
<td>2.576</td>
</tr>
</tbody>
</table>

Confidence interval

\( h \ldots \) relative frequency in a sample

\( p \ldots \) unknown relative proportion of the population

\( \gamma \ldots \) confidence level

\( \gamma \)-confidence interval for \( p \) (the values of \( p \) for which the value \( h \) is contained in the given range with probability \( \gamma \)):

\[
\left[ h - z \cdot \sqrt{\frac{h \cdot (1-h)}{n}} ; h + z \cdot \sqrt{\frac{h \cdot (1-h)}{n}} \right], \text{ where for } z: \gamma = 2 \cdot \phi(z) - 1
\]
## 15 Units of Measurement

<table>
<thead>
<tr>
<th>Quantity</th>
<th>Unit</th>
<th>Symbol</th>
<th>Relationship</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Temperature</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>degrees Celsius</td>
<td>°C</td>
<td>$\Delta t = \Delta T$</td>
</tr>
<tr>
<td></td>
<td>or kelvin</td>
<td>K</td>
<td></td>
</tr>
<tr>
<td><strong>Frequency</strong></td>
<td>hertz</td>
<td>Hz</td>
<td>$1 \text{ Hz} = 1 \text{ s}^{-1}$</td>
</tr>
<tr>
<td><strong>Energy, work done,</strong></td>
<td>joules</td>
<td>J</td>
<td>$1 \text{ J} = 1 \text{ kg} \cdot \text{ m}^2 \cdot \text{ s}^{-2}$</td>
</tr>
<tr>
<td><strong>amount of heat</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>newtons</td>
<td>N</td>
<td>$1 \text{ N} = 1 \text{ kg} \cdot \text{ m} \cdot \text{ s}^{-2}$</td>
</tr>
<tr>
<td><strong>Torque</strong></td>
<td>newton metres</td>
<td>N \cdot m</td>
<td>$1 \text{ N} \cdot \text{ m} = 1 \text{ kg} \cdot \text{ m}^2 \cdot \text{ s}^{-2}$</td>
</tr>
<tr>
<td><strong>Electric resistance</strong></td>
<td>ohms</td>
<td>Ω</td>
<td>$1 \Omega = 1 \text{ V} \cdot \text{ A}^{-1}$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>$= 1 \text{ kg} \cdot \text{ m}^2 \cdot \text{ A}^{-2} \cdot \text{ s}^{-3}$</td>
</tr>
<tr>
<td><strong>Pressure</strong></td>
<td>pascals</td>
<td>Pa</td>
<td>$1 \text{ Pa} = 1 \text{ N} \cdot \text{ m}^{-2}$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>$= 1 \text{ kg} \cdot \text{ m}^{-1} \cdot \text{ s}^{-2}$</td>
</tr>
<tr>
<td><strong>Electric current</strong></td>
<td>amperes</td>
<td>A</td>
<td>$1 \text{ A} = 1 \text{ C} \cdot \text{ s}^{-1}$</td>
</tr>
<tr>
<td><strong>Potential difference</strong></td>
<td>volts</td>
<td>V</td>
<td>$1 \text{ V} = 1 \cdot \text{ J} \cdot \text{ C}^{-1}$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>$= 1 \text{ kg} \cdot \text{ m}^2 \cdot \text{ A}^{-1} \cdot \text{ s}^{-3}$</td>
</tr>
<tr>
<td><strong>Power</strong></td>
<td>watts</td>
<td>W</td>
<td>$1 \text{ W} = 1 \text{ J} \cdot \text{ s}^{-1}$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>$= 1 \text{ kg} \cdot \text{ m}^2 \cdot \text{ s}^{-3}$</td>
</tr>
</tbody>
</table>
16 Physical Quantities and Definitions

Density
\[ \varrho = \frac{m}{V} \]

Power
\[ P = \frac{\Delta E}{\Delta t} = \frac{\Delta W}{\Delta t} \quad P = \frac{dW}{dt} \]

Force
\[ F = m \cdot a \]

Work done
\[ W = F \cdot s \]
\[ W = \int F(s) \, ds \quad F = \frac{dW}{ds} \]

Kinetic energy
\[ E_{\text{kin}} = \frac{1}{2} \cdot m \cdot v^2 \]

Potential energy
\[ E_{\text{pot}} = m \cdot g \cdot h \]

Uniform linear motion
\[ v = \frac{s}{t} \quad v = \frac{ds}{dt} \quad v(t) = s'(t) \]

Uniform acceleration
\[ v = a \cdot t + v_0 \quad a = \frac{dv}{dt} = \frac{d^2s}{dt^2} \quad a(t) = v'(t) = s''(t) \]

17 Financial Mathematics

Compound interest calculation
\[ K_0 \quad \text{initial investment} \]
\[ K_n \quad \text{final capital} \]
\[ p \quad \text{annual percentage rate of interest} \]
\[ K_n = K_0 \cdot (1 + i)^n \quad \text{where} \quad i = \frac{p}{100} \]

Cost-of-production theory of value
\[ x \quad \text{amount produced, offered, required or sold} \quad (x \geq 0) \]

| Variable costs | \( K_v(x) \) |
| Fixed costs | \( K_f \) |
| (Total) costs | \( K(x) = K_v(x) + K_f \) |
| Marginal costs | \( K'(x) \) |
| Demand price | \( p(x) \) |
| Revenue/income | \( E(x) = p(x) \cdot x \) |
| Marginal revenue | \( E'(x) \) |
| Profit | \( G(x) = E(x) - K(x) \) |
| Marginal profit | \( G'(x) \) |
| Break-even point | \( E(x) = K(x) \quad \text{at the (first) zero of the profit function} \) |
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