

# Formula Booklet

for the Standardised Competence-Oriented  
Written School-Leaving Examination (SRP)

## Mathematics (AHS)

# Table of Contents

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Chapter	Page
1 Sets	3
2 Prefixes	3
3 Powers	3
4 Logarithms	4
5 Quadratic Equations	4
6 Two-Dimensional Shapes	5
7 Solids	6
8 Trigonometry	6
9 Vectors	7
10 Straight Lines	8
11 Rates of Change	8
12 Differentiation and Integration	9
13 Statistics	9
14 Probability	10
15 Units of Measurement	12
16 Physical Quantities and Definitions	13
17 Financial Mathematics	13
Index	14

# 1 Sets

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$\in$	is an element of...
$\notin$	is not an element of...
$\cap$	intersection
$\cup$	union
$\subset$	proper subset
$\subseteq$	subset
$\setminus$	difference ("without")
$\{\}$	empty set

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## Sets of numbers

$\mathbb{N} = \{0, 1, 2, \dots\}$	natural numbers
$\mathbb{Z}$	integers
$\mathbb{Q}$	rational numbers
$\mathbb{R}$	real numbers
$\mathbb{C}$	complex numbers
$\mathbb{R}^+$	positive real numbers
$\mathbb{R}_0^+$	positive real numbers including zero

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## 2 Prefixes

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tera-	T	$10^{12}$	deci-	d	$10^{-1}$
giga-	G	$10^9$	centi-	c	$10^{-2}$
mega-	M	$10^6$	milli-	m	$10^{-3}$
kilo-	k	$10^3$	micro-	$\mu$	$10^{-6}$
hecto-	h	$10^2$	nano-	n	$10^{-9}$
deca-	da	$10^1$	pico-	p	$10^{-12}$

## 3 Powers

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### Powers with integer exponents

$$a \in \mathbb{R}; n \in \mathbb{N} \setminus \{0\} \qquad a \in \mathbb{R} \setminus \{0\}; n \in \mathbb{N} \setminus \{0\}$$

$$a^n = \underbrace{a \cdot a \cdot \dots \cdot a}_{n \text{ factors}} \qquad a^1 = a \qquad a^0 = 1 \qquad a^{-1} = \frac{1}{a} \qquad a^{-n} = \frac{1}{a^n} = \left(\frac{1}{a}\right)^n$$

### Powers with rational exponents (roots)

$$a, b \in \mathbb{R}_0^+; n, k \in \mathbb{N} \setminus \{0\} \text{ where } n \geq 2$$

$$a = \sqrt[n]{b} \Leftrightarrow a^n = b \qquad a^{\frac{1}{n}} = \sqrt[n]{a} \qquad a^{\frac{k}{n}} = \sqrt[n]{a^k} \qquad a^{-\frac{k}{n}} = \frac{1}{\sqrt[n]{a^k}} \text{ where } a > 0$$

## Calculation rules

$$a, b \in \mathbb{R} \setminus \{0\}; r, s \in \mathbb{Z}$$

$$\text{or } a, b \in \mathbb{R}^+; r, s \in \mathbb{Q}$$

$$a, b \in \mathbb{R}_0^+; m, n, k \in \mathbb{N} \setminus \{0\} \text{ where } m, n \geq 2$$

$$a^r \cdot a^s = a^{r+s}$$

$$\frac{a^r}{a^s} = a^{r-s}$$

$$(a^r)^s = a^{r \cdot s}$$

$$(a \cdot b)^r = a^r \cdot b^r$$

$$\left(\frac{a}{b}\right)^r = \frac{a^r}{b^r}$$

$$\sqrt[n]{a \cdot b} = \sqrt[n]{a} \cdot \sqrt[n]{b}$$

$$\sqrt[n]{a^k} = (\sqrt[n]{a})^k$$

$$\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}} \quad (b \neq 0)$$

$$\sqrt[n]{\sqrt[m]{a}} = \sqrt[n \cdot m]{a}$$

## Binomial formulae

$$a, b \in \mathbb{R}; n \in \mathbb{N}$$

$$(a + b)^2 = a^2 + 2 \cdot a \cdot b + b^2$$

$$(a - b)^2 = a^2 - 2 \cdot a \cdot b + b^2$$

$$(a + b) \cdot (a - b) = a^2 - b^2$$

$$(a + b)^n = \sum_{k=0}^n \binom{n}{k} \cdot a^{n-k} \cdot b^k$$

$$(a - b)^n = \sum_{k=0}^n (-1)^k \cdot \binom{n}{k} \cdot a^{n-k} \cdot b^k$$

## 4 Logarithms

$$a, b, c \in \mathbb{R}^+ \text{ where } a \neq 1; x, r \in \mathbb{R}$$

$$x = \log_a(b) \Leftrightarrow a^x = b$$

$$\log_a(b \cdot c) = \log_a(b) + \log_a(c)$$

$$\log_a\left(\frac{b}{c}\right) = \log_a(b) - \log_a(c)$$

$$\log_a(b^r) = r \cdot \log_a(b)$$

$$\log_a(a^x) = x$$

$$\log_a(a) = 1$$

$$\log_a(1) = 0$$

$$\log_a\left(\frac{1}{a}\right) = -1$$

natural logarithm (logarithm with base  $e$ ):  $\ln(b) = \log_e(b)$

common logarithm (logarithm with base 10):  $\lg(b) = \log_{10}(b)$

## 5 Quadratic Equations

$$p, q \in \mathbb{R}$$

$$a, b, c \in \mathbb{R} \text{ where } a \neq 0$$

$$x^2 + p \cdot x + q = 0$$

$$x_{1,2} = -\frac{p}{2} \pm \sqrt{\left(\frac{p}{2}\right)^2 - q}$$

$$a \cdot x^2 + b \cdot x + c = 0$$

$$x_{1,2} = \frac{-b \pm \sqrt{b^2 - 4 \cdot a \cdot c}}{2 \cdot a}$$

### Vieta's Theorem

$x_1$  and  $x_2$  are the solutions to the equation  $x^2 + p \cdot x + q = 0$  if and only if:

$$x_1 + x_2 = -p$$

$$x_1 \cdot x_2 = q$$

Linear factorisation:

$$x^2 + p \cdot x + q = (x - x_1) \cdot (x - x_2)$$

# 6 Two-Dimensional Shapes

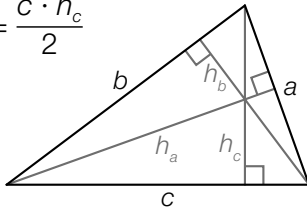
A ... area  
u ... perimeter

## Triangle

$$u = a + b + c$$

General triangle

$$A = \frac{a \cdot h_a}{2} = \frac{b \cdot h_b}{2} = \frac{c \cdot h_c}{2}$$

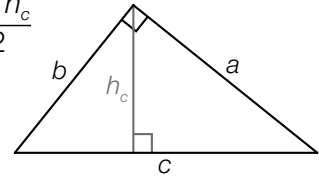


Heron's Formula

$$A = \sqrt{s \cdot (s - a) \cdot (s - b) \cdot (s - c)} \text{ where } s = \frac{a + b + c}{2}$$

Right-angled triangle with hypotenuse c and sides a, b

$$A = \frac{a \cdot b}{2} = \frac{c \cdot h_c}{2}$$



Pythagorean theorem

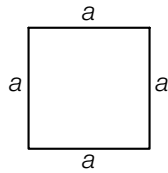
$$a^2 + b^2 = c^2$$

## Quadrilateral

Square

$$A = a^2$$

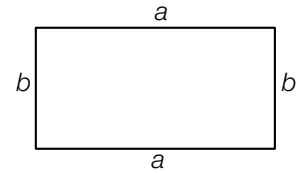
$$u = 4 \cdot a$$



Rectangle

$$A = a \cdot b$$

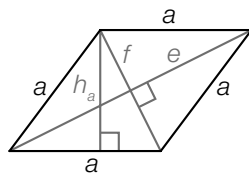
$$u = 2 \cdot a + 2 \cdot b$$



Rhombus

$$A = a \cdot h_a = \frac{e \cdot f}{2}$$

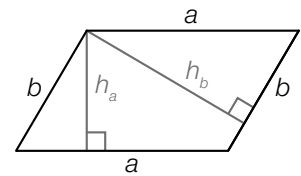
$$u = 4 \cdot a$$



Parallelogram

$$A = a \cdot h_a = b \cdot h_b$$

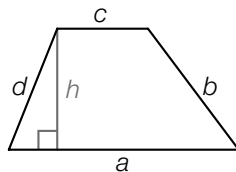
$$u = 2 \cdot a + 2 \cdot b$$



Trapezium

$$A = \frac{(a + c) \cdot h}{2}$$

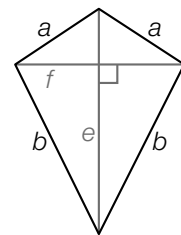
$$u = a + b + c + d$$



Kite

$$A = \frac{e \cdot f}{2}$$

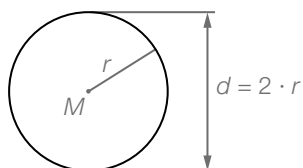
$$u = 2 \cdot a + 2 \cdot b$$



## Circle

$$A = \pi \cdot r^2 = \frac{\pi \cdot d^2}{4}$$

$$u = 2 \cdot \pi \cdot r = \pi \cdot d$$

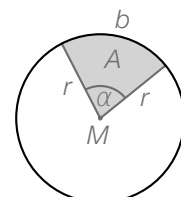


Arc length and sector of a circle

$\alpha$  in degrees ( $^\circ$ )

$$b = \pi \cdot r \cdot \frac{\alpha}{180^\circ}$$

$$A = \pi \cdot r^2 \cdot \frac{\alpha}{360^\circ} = \frac{b \cdot r}{2}$$



# 7 Solids

$V$  ... volume  
 $O$  ... surface area  
 $G$  ... area of the base

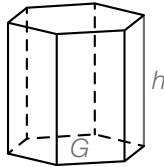
$M$  ... lateral surface area  
 $u_G$  ... perimeter of the base

## Prism

$$V = G \cdot h$$

$$M = u_G \cdot h$$

$$O = 2 \cdot G + M$$



## Cylinder

$$V = G \cdot h$$

$$M = u_G \cdot h$$

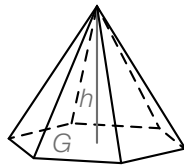
$$O = 2 \cdot G + M$$



## Pyramid

$$V = \frac{G \cdot h}{3}$$

$$O = G + M$$

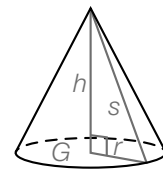


## Cone

$$V = \frac{G \cdot h}{3}$$

$$M = \pi \cdot r \cdot s$$

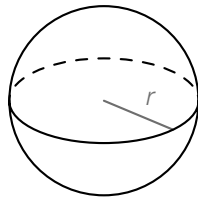
$$O = G + M$$



## Sphere

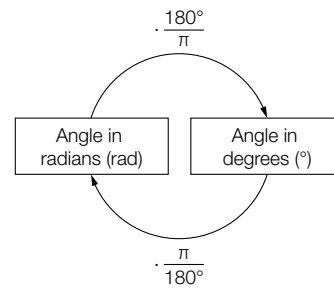
$$V = \frac{4}{3} \cdot \pi \cdot r^3$$

$$O = 4 \cdot \pi \cdot r^2$$



# 8 Trigonometry

Converting between degrees and radians

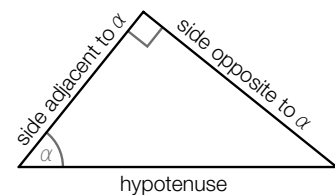


## Right-angled triangle trigonometry

Sine:  $\sin(\alpha) = \frac{\text{side opposite to } \alpha}{\text{hypotenuse}}$

Cosine:  $\cos(\alpha) = \frac{\text{side adjacent to } \alpha}{\text{hypotenuse}}$

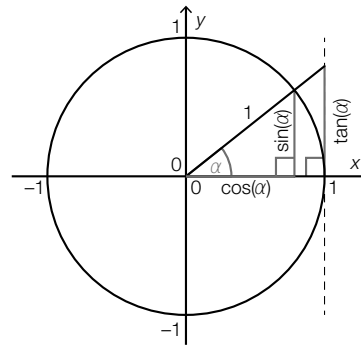
Tangent:  $\tan(\alpha) = \frac{\text{side opposite to } \alpha}{\text{side adjacent to } \alpha}$



## Unit circle trigonometry

$$\sin^2(\alpha) + \cos^2(\alpha) = 1$$

$$\tan(\alpha) = \frac{\sin(\alpha)}{\cos(\alpha)} \text{ for } \cos(\alpha) \neq 0$$



## 9 Vectors

$P, Q \dots$  points

### Vectors in $\mathbb{R}^2$

Arrow from  $P$  to  $Q$ :

$$P = (p_1 | p_2), Q = (q_1 | q_2)$$

$$\overrightarrow{PQ} = \begin{pmatrix} q_1 - p_1 \\ q_2 - p_2 \end{pmatrix}$$

### Calculation rules in $\mathbb{R}^2$

$$\vec{a} = \begin{pmatrix} a_1 \\ a_2 \end{pmatrix}, \vec{b} = \begin{pmatrix} b_1 \\ b_2 \end{pmatrix}, \vec{a} \pm \vec{b} = \begin{pmatrix} a_1 \pm b_1 \\ a_2 \pm b_2 \end{pmatrix}$$

$$k \cdot \vec{a} = k \cdot \begin{pmatrix} a_1 \\ a_2 \end{pmatrix} = \begin{pmatrix} k \cdot a_1 \\ k \cdot a_2 \end{pmatrix} \text{ where } k \in \mathbb{R}$$

### Scalar product in $\mathbb{R}^2$

$$\vec{a} \cdot \vec{b} = a_1 \cdot b_1 + a_2 \cdot b_2$$

### Absolute value (length) of a vector in $\mathbb{R}^2$

$$|\vec{a}| = \sqrt{a_1^2 + a_2^2}$$

### Vector perpendicular to $\vec{a} = \begin{pmatrix} a_1 \\ a_2 \end{pmatrix}$ in $\mathbb{R}^2$

$$\vec{n} = k \cdot \begin{pmatrix} -a_2 \\ a_1 \end{pmatrix} \text{ where } k \in \mathbb{R} \setminus \{0\} \text{ and } |\vec{a}| \neq 0$$

### Criterion for two vectors to be perpendicular in $\mathbb{R}^2$ and $\mathbb{R}^3$

$$\vec{a} \cdot \vec{b} = 0 \Leftrightarrow \vec{a} \perp \vec{b} \text{ where } |\vec{a}| \neq 0; |\vec{b}| \neq 0$$

### Vectors in $\mathbb{R}^n$

Arrow from  $P$  to  $Q$ :

$$P = (p_1 | p_2 | \dots | p_n), Q = (q_1 | q_2 | \dots | q_n)$$

$$\overrightarrow{PQ} = \begin{pmatrix} q_1 - p_1 \\ q_2 - p_2 \\ \vdots \\ q_n - p_n \end{pmatrix}$$

### Calculation rules in $\mathbb{R}^n$

$$\vec{a} = \begin{pmatrix} a_1 \\ a_2 \\ \vdots \\ a_n \end{pmatrix}, \vec{b} = \begin{pmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{pmatrix}, \vec{a} \pm \vec{b} = \begin{pmatrix} a_1 \pm b_1 \\ a_2 \pm b_2 \\ \vdots \\ a_n \pm b_n \end{pmatrix}$$

$$k \cdot \vec{a} = k \cdot \begin{pmatrix} a_1 \\ a_2 \\ \vdots \\ a_n \end{pmatrix} = \begin{pmatrix} k \cdot a_1 \\ k \cdot a_2 \\ \vdots \\ k \cdot a_n \end{pmatrix} \text{ where } k \in \mathbb{R}$$

### Scalar product in $\mathbb{R}^n$

$$\vec{a} \cdot \vec{b} = a_1 \cdot b_1 + a_2 \cdot b_2 + \dots + a_n \cdot b_n$$

### Absolute value (length) of a vector in $\mathbb{R}^n$

$$|\vec{a}| = \sqrt{a_1^2 + a_2^2 + \dots + a_n^2}$$

### Angle $\varphi$ between $\vec{a}$ and $\vec{b}$ in $\mathbb{R}^2$ and $\mathbb{R}^3$

$$\cos(\varphi) = \frac{\vec{a} \cdot \vec{b}}{|\vec{a}| \cdot |\vec{b}|} \text{ where } |\vec{a}| \neq 0; |\vec{b}| \neq 0$$

### Criterion for two vectors to be parallel in $\mathbb{R}^2$ and $\mathbb{R}^3$

$$\vec{a} \parallel \vec{b} \Leftrightarrow \vec{a} = k \cdot \vec{b} \text{ where } k \in \mathbb{R} \setminus \{0\} \text{ and } |\vec{a}| \neq 0; |\vec{b}| \neq 0$$

## 10 Straight Lines

$g$ ... line	$\vec{g}$ ... a direction vector for the line $g$
	$\vec{n}$ ... a vector perpendicular to the line $g$
	$X, P$ ... points on the line $g$
	$m$ ... gradient of the line $g$
	$\alpha$ ... angle of slope of the line $g$
	$a, b, c, k \in \mathbb{R}$

**Vector equation of a line  $g$  in  $\mathbb{R}^2$  and  $\mathbb{R}^3$**

$$g: X = P + t \cdot \vec{g} \text{ where } t \in \mathbb{R}$$

**Equation of a line  $g$  in  $\mathbb{R}^2$**

the explicit equation of a line:

$$g: y = m \cdot x + c \quad \text{where } m = \tan(\alpha)$$

a general equation of a line:

$$g: a \cdot x + b \cdot y = c$$

a normal vector representation:

$$g: \vec{n} \cdot X = \vec{n} \cdot P \quad \left. \begin{array}{l} \\ \\ \end{array} \right\} \text{ where } \vec{n} \parallel \begin{pmatrix} a \\ b \end{pmatrix} \text{ and } \begin{pmatrix} a \\ b \end{pmatrix} \neq \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

## 11 Rates of Change

For a real function  $f$  defined over an interval  $[a, b]$ :

**Absolute change** of  $f$  in  $[a, b]$

$$f(b) - f(a)$$

**Relative (percentage) change** of  $f$  in  $[a, b]$

$$\frac{f(b) - f(a)}{f(a)} \text{ where } f(a) \neq 0$$

**Difference quotient (average rate of change)** of  $f$  in  $[a, b]$  or  $[x, x + \Delta x]$

$$\frac{f(b) - f(a)}{b - a} \text{ or } \frac{f(x + \Delta x) - f(x)}{\Delta x} \text{ where } b \neq a \text{ and } \Delta x \neq 0$$

**Differential quotient (instantaneous rate of change)** of  $f$  at the point  $x$

$$f'(x) = \lim_{x_1 \rightarrow x} \frac{f(x_1) - f(x)}{x_1 - x} \text{ or } f'(x) = \lim_{\Delta x \rightarrow 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$$



## 12 Differentiation and Integration

$f, g, h \dots$  functions that are differentiable over  $\mathbb{R}$  or over a defined interval  
 $f'$  ... first derivative of  $f$   $F$  ... antiderivative of  $f$   
 $g'$  ... first derivative of  $g$   $G$  ... antiderivative of  $g$   
 $h'$  ... first derivative of  $h$   $H$  ... antiderivative of  $h$   
 $C, k, q \in \mathbb{R}; a \in \mathbb{R}^+ \setminus \{1\}$

### Indefinite integral

$$\int f(x) dx = F(x) + C \text{ where } F' = f$$

### Definite integral

$$\int_a^b f(x) dx = F(x) \Big|_a^b = F(b) - F(a)$$

Function	Derivative	Antiderivative
$f(x) = k$	$f'(x) = 0$	$F(x) = k \cdot x$
$f(x) = x^q$	$f'(x) = q \cdot x^{q-1}$	$F(x) = \frac{x^{q+1}}{q+1}$ where $q \neq -1$ $F(x) = \ln( x )$ where $q = -1$
$f(x) = e^x$	$f'(x) = e^x$	$F(x) = e^x$
$f(x) = a^x$	$f'(x) = \ln(a) \cdot a^x$	$F(x) = \frac{a^x}{\ln(a)}$
$f(x) = \sin(x)$	$f'(x) = \cos(x)$	$F(x) = -\cos(x)$
$f(x) = \cos(x)$	$f'(x) = -\sin(x)$	$F(x) = \sin(x)$
$g(x) = k \cdot f(x)$	$g'(x) = k \cdot f'(x)$	$G(x) = k \cdot F(x)$
$h(x) = f(x) \pm g(x)$	$h'(x) = f'(x) \pm g'(x)$	$H(x) = F(x) \pm G(x)$
$g(x) = f(k \cdot x)$	$g'(x) = k \cdot f'(k \cdot x)$	$G(x) = \frac{1}{k} \cdot F(k \cdot x)$

## 13 Statistics

$x_1, x_2, \dots, x_n \dots$  a list of  $n$  real numbers  
 $x_{(1)} \leq x_{(2)} \leq \dots \leq x_{(n)}$  ... ordered list of  $n$  values

### Arithmetic mean

$$\bar{x} = \frac{x_1 + x_2 + \dots + x_n}{n} = \frac{1}{n} \cdot \sum_{i=1}^n x_i$$

### Median

$$\tilde{x} = \begin{cases} x_{(\frac{n+1}{2})} & \dots \text{ when } n \text{ is odd} \\ \frac{1}{2} \cdot (x_{(\frac{n}{2})} + x_{(\frac{n}{2}+1)}) & \dots \text{ when } n \text{ is even} \end{cases}$$

### Measures of spread

$s^2$  ... (empirical) variance of a sample  
 $s$  ... (empirical) standard deviation of a sample

$$s^2 = \frac{1}{n} \cdot \sum_{i=1}^n (x_i - \bar{x})^2$$

$$s = \sqrt{\frac{1}{n} \cdot \sum_{i=1}^n (x_i - \bar{x})^2}$$

If the variance of a population should be estimated using a sample of size  $n$ :

$$s_{n-1}^2 = \frac{1}{n-1} \cdot \sum_{i=1}^n (x_i - \bar{x})^2$$

$$s_{n-1} = \sqrt{\frac{1}{n-1} \cdot \sum_{i=1}^n (x_i - \bar{x})^2}$$

# 14 Probability

$n \in \mathbb{N} \setminus \{0\}; k \in \mathbb{N}$  where  $k \leq n$

$A, B \dots$  events

$\neg A$  or  $\bar{A} \dots$  complementary event of  $A$

$A \wedge B$  or  $A \cap B \dots$   $A$  and  $B$  (the event  $A$  and the event  $B$  both occur)

$A \vee B$  or  $A \cup B \dots$   $A$  or  $B$  (at least one of the two events  $A$  or  $B$  occurs)

$P(A) \dots$  probability of event  $A$  occurring

$P(A|B) \dots$  probability of event  $A$  occurring given that  $B$  has occurred (conditional probability)

Factorial

$$n! = n \cdot (n-1) \cdot \dots \cdot 1$$

$$0! = 1$$

$$1! = 1$$

Binomial coefficient

$$\binom{n}{k} = \frac{n!}{k! \cdot (n-k)!}$$

## Probability for a Laplace experiment

$$P(A) = \frac{\text{number of successful outcomes for } A}{\text{number of possible outcomes}}$$

## Elementary rules

$$P(\neg A) = 1 - P(A)$$

$$P(A \wedge B) = P(A) \cdot P(B|A) = P(B) \cdot P(A|B)$$

$$P(A \wedge B) = P(A) \cdot P(B) \dots \text{ if } A \text{ and } B \text{ are (stochastically) independent of one another}$$

$$P(A \vee B) = P(A) + P(B) - P(A \wedge B)$$

$$P(A \vee B) = P(A) + P(B) \dots \text{ if } A \text{ and } B \text{ are mutually exclusive}$$

## Expectation value $\mu$ of a discrete random variable $X$ with values $x_1, x_2, \dots, x_n$

$$\mu = E(X) = x_1 \cdot P(X = x_1) + x_2 \cdot P(X = x_2) + \dots + x_n \cdot P(X = x_n) = \sum_{i=1}^n x_i \cdot P(X = x_i)$$

## Variance $\sigma^2$ of a discrete random variable $X$ with values $x_1, x_2, \dots, x_n$

$$\sigma^2 = V(X) = \sum_{i=1}^n (x_i - \mu)^2 \cdot P(X = x_i)$$

## Standard deviation $\sigma$

$$\sigma = \sqrt{V(X)}$$

## Binomial distribution

$n \in \mathbb{N} \setminus \{0\}; k \in \mathbb{N}; p \in \mathbb{R}$  where  $k \leq n$  and  $0 \leq p \leq 1$

The random variable  $X$  is binomially distributed with parameters  $n$  and  $p$

$$P(X = k) = \binom{n}{k} \cdot p^k \cdot (1-p)^{n-k}$$

$$E(X) = \mu = n \cdot p$$

$$V(X) = \sigma^2 = n \cdot p \cdot (1-p)$$

## Normal distribution

$\mu, \sigma \in \mathbb{R}$  where  $\sigma > 0$

$f$  ... probability density function

$\varphi$  ... probability density function of the standard normal distribution

$\Phi$  ... cumulative density function of the standard normal distribution

Normal distribution  $N(\mu; \sigma^2)$ : The random variable  $X$  is normally distributed with expectation value ( $\mu$ ), standard deviation ( $\sigma$ ) and variance ( $\sigma^2$ )

$$P(X \leq x_1) = \int_{-\infty}^{x_1} f(x) dx = \int_{-\infty}^{x_1} \frac{1}{\sigma \cdot \sqrt{2 \cdot \pi}} \cdot e^{-\frac{1}{2} \cdot \left(\frac{x-\mu}{\sigma}\right)^2} dx$$

Probabilities for standard deviation bands

$$P(\mu - \sigma \leq X \leq \mu + \sigma) \approx 0.683$$

$$P(\mu - 2 \cdot \sigma \leq X \leq \mu + 2 \cdot \sigma) \approx 0.954$$

$$P(\mu - 3 \cdot \sigma \leq X \leq \mu + 3 \cdot \sigma) \approx 0.997$$

Standard normal distribution  $N(0, 1)$

$$z = \frac{x - \mu}{\sigma}$$

$$\Phi(z) = P(Z \leq z) = \int_{-\infty}^z \varphi(x) dx = \frac{1}{\sqrt{2 \cdot \pi}} \cdot \int_{-\infty}^z e^{-\frac{x^2}{2}} dx$$

$$\Phi(-z) = 1 - \Phi(z)$$

$$P(-z \leq Z \leq z) = 2 \cdot \Phi(z) - 1$$

$P(-z \leq Z \leq z)$	= 90 %	= 95 %	= 99 %
$z$	$\approx 1.645$	$\approx 1.960$	$\approx 2.576$

## Confidence interval

$h$  ... relative frequency in a sample

$p$  ... unknown relative proportion of the population

$\gamma$  ... confidence level

$\gamma$ -confidence interval for  $p$  (the values of  $p$  for which the value  $h$  is contained in the given range with probability  $\gamma$ ):

$$\left[ h - z \cdot \sqrt{\frac{h \cdot (1-h)}{n}}; h + z \cdot \sqrt{\frac{h \cdot (1-h)}{n}} \right], \text{ where for } z: \gamma = 2 \cdot \Phi(z) - 1$$

## 15 Units of Measurement

Quantity	Unit	Symbol	Relationship
Temperature	degrees Celsius or kelvin	°C K	$\Delta t = \Delta T$
Frequency	hertz	Hz	$1 \text{ Hz} = 1 \text{ s}^{-1}$
Energy, work done, amount of heat	joules	J	$1 \text{ J} = 1 \text{ kg} \cdot \text{m}^2 \cdot \text{s}^{-2}$
Force	newtons	N	$1 \text{ N} = 1 \text{ kg} \cdot \text{m} \cdot \text{s}^{-2}$
Torque	newton metres	N · m	$1 \text{ N} \cdot \text{m} = 1 \text{ kg} \cdot \text{m}^2 \cdot \text{s}^{-2}$
Electric resistance	ohms	Ω	$1 \Omega = 1 \text{ V} \cdot \text{A}^{-1}$ $= 1 \text{ kg} \cdot \text{m}^2 \cdot \text{A}^{-2} \cdot \text{s}^{-3}$
Pressure	pascals	Pa	$1 \text{ Pa} = 1 \text{ N} \cdot \text{m}^{-2}$ $= 1 \text{ kg} \cdot \text{m}^{-1} \cdot \text{s}^{-2}$
Electric current	amperes	A	$1 \text{ A} = 1 \text{ C} \cdot \text{s}^{-1}$
Potential difference	volts	V	$1 \text{ V} = 1 \cdot \text{J} \cdot \text{C}^{-1}$ $= 1 \text{ kg} \cdot \text{m}^2 \cdot \text{A}^{-1} \cdot \text{s}^{-3}$
Power	watts	W	$1 \text{ W} = 1 \text{ J} \cdot \text{s}^{-1}$ $= 1 \text{ kg} \cdot \text{m}^2 \cdot \text{s}^{-3}$

## 16 Physical Quantities and Definitions

Density	$\rho = \frac{m}{V}$		
Power	$P = \frac{\Delta E}{\Delta t} = \frac{\Delta W}{\Delta t}$	$P = \frac{dW}{dt}$	
Force	$F = m \cdot a$		
Work done	$W = F \cdot s$		
	$W = \int F(s) ds$	$F = \frac{dW}{ds}$	
Kinetic energy	$E_{\text{kin}} = \frac{1}{2} \cdot m \cdot v^2$		
Potential energy	$E_{\text{pot}} = m \cdot g \cdot h$		
Uniform linear motion	$v = \frac{s}{t}$	$v = \frac{ds}{dt}$	$v(t) = s'(t)$
Uniform acceleration	$v = a \cdot t + v_0$	$a = \frac{dv}{dt} = \frac{d^2s}{dt^2}$	$a(t) = v'(t) = s''(t)$

## 17 Financial Mathematics

### Compound interest calculation

$K_0$  ... initial investment  
 $K_n$  ... final capital  
 $p$  ... annual percentage rate of interest

$$K_n = K_0 \cdot (1 + i)^n \text{ where } i = \frac{p}{100}$$

### Cost-of-production theory of value

$x$  ... amount produced, offered, required or sold ( $x \geq 0$ )

Variable costs	$K_v(x)$
Fixed costs	$K_f$
(Total) costs	$K(x) = K_v(x) + K_f$
Marginal costs	$K'(x)$
Demand price	$p(x)$
Revenue/income	$E(x) = p(x) \cdot x$
Marginal revenue	$E'(x)$
Profit	$G(x) = E(x) - K(x)$
Marginal profit	$G'(x)$
Break-even point	$E(x) = K(x)$ ... at the (first) zero of the profit function

# Index

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## A

absolute change 8  
absolute value of a vector 7  
adjacent side 6  
amperes 12  
angle 6  
angle between vectors 7  
angle of slope of a line 8  
annual percentage rate of interest 13  
antiderivative 9  
arc length 5  
area 5  
area of the base 6  
arithmetic mean 9  
average rate of change 8

## B

binomial coefficient 10  
binomial distribution 10  
binomial formulae 4  
break-even point 13

## C

calculation rules 4  
Celsius 12  
centi- 3  
circle 5  
common logarithm 4  
complementary event 10  
complex numbers 3  
compound interest 13  
conditional probability 10  
cone 6  
confidence interval 11  
confidence level 11  
cosine 6  
cost-of-production theory of value 13  
costs 13  
current 12  
cylinder 6

## D

deca- 3  
deci- 3  
definite integral 9  
degrees (unit of measurement for angles) 6  
degrees Celsius 12  
demand price 13  
density 13  
density function 11  
derivative 9  
difference 3  
difference quotient 8

differential quotient 8  
differentiation 9  
direction vector 8  
discrete random variable 10  
distribution function 11

## E

electric current 12  
electric resistance 12  
element 3  
empirical standard deviation 9  
empirical variance 9  
empty set 3  
energy 12, 13  
equation of a line 8  
events 10  
expectation value 10  
exponent 3

## F

factorial 10  
final capital 13  
financial mathematics 13  
fixed costs 13  
force 12, 13  
frequency 12

## G

general triangle 5  
giga- 3  
gradient 8

## H

heat 12  
hecto- 3  
Heron's formula 5  
hertz 12  
hypotenuse 5

## I

income 13  
indefinite integral 9  
initial investment 13  
instantaneous rate of change 8  
integers 3  
integral 9  
intersection (of sets) 3

## J

joules 12

## K

kelvin 12  
kilo- 3  
kinetic energy 13  
kite 5

## L

Laplace experiment 10  
lateral surface area 6  
length of a vector 7  
linear factorisation 4  
logarithms 4

## M

marginal costs 13  
marginal profit 13  
marginal revenue 13  
mean 9  
measures of spread 9  
median 9  
mega- 3  
micro- 3  
milli- 3

## N

nano- 3  
natural logarithm 4  
natural numbers 3  
newton metres 12  
newtons 12  
normal distribution 8  
normal vector representation of a line 8

## O

ohms 12  
opposite side 6

## P

parallel vectors 7  
parallelogram 5  
pascals 12  
percentage change 8  
perimeter 5, 6  
perpendicular vectors 7, 8  
physical quantities 13  
pico- 3  
potential difference 12  
potential energy 13  
power 12, 13  
powers 3  
prefixes 3  
pressure 12  
prism 6  
probability 10  
probability density function 11  
profit 13  
proper subset 3  
pyramid 6  
Pythagorean theorem 5

**Q**

quadratic equations 4  
quadrilateral 5

**R**

radians 6  
random variable 10  
rates of change 8  
rational numbers 3  
real numbers 3  
rectangle 5  
relative change 8  
relative frequency 11  
resistance 12  
revenue 13  
rhombus 5  
right-angled triangle 5, 6  
roots 3

**S**

sample 9  
scalar product 7  
sector of a circle 5  
sets 3  
sets of numbers 3  
side of a triangle 5  
sine 6  
solids 6  
sphere 6  
square 5  
standard deviation 9, 10  
standard deviation bands 11  
standard normal distribution 11  
statistics 9  
straight lines 8  
subset 3  
surface area 6

**T**

tangent 6  
temperature 12  
tera- 3  
torque 12  
total costs 13  
trapezium 5  
triangle 5  
trigonometry 6, 7  
two-dimensional shapes 5

**U**

uniform acceleration 13  
uniform linear motion 13  
union (of sets) 3  
unit circle 7  
units 12  
units of measurement 12

**V**

variable costs 13  
variance 9, 10  
vector equation of a line 8  
vectors 7  
Vieta's theorem 4  
volts 12  
volume 6

**W**

watts 12  
work done 12, 13