Exemplar für Prüfer/innen

Supplementary Examination for the
Standardised Competence-Oriented
Written School-Leaving Examination

AHS

January 2020

Mathematics

Supplementary Examination 2
Examiner's Version

Bundesministerium
Bildung, Wissenschaft
und Forschung

Instructions for the supplementary examination

The following supplementary examination is comprised of five tasks that can be completed independently of one another.

Each task contains two parts: The statement of the task requires the candidate to demonstrate core competencies, and the guiding question that follows it requires the candidate to show their ability to communicate their ideas.

In the following document, the examiner will find the tasks as well as the expected solutions and the answer key.

The preparation time shall be at least 30 minutes and the examination time shall be at most 25 minutes.

Assessment

Each task can be awarded zero, one or two points. There is one point available for each demonstration of core competencies as well as for each guiding question. A maximum of 10 points can be achieved.

The following scale will be used for the grading of the examination:

Grade	Number of points
Pass	4 points for the core competencies + 0 points for the guiding questions 3 points for the core competencies + 1 point for the guiding questions
Satisfactory	5 points for the core competencies + 0 points for the guiding questions 4 points for the core competencies + 1 point for the guiding questions 3 points for the core competencies + 2 points for the guiding questions
Good	5 points for the core competencies + 1 point for the guiding questions 4 points for the core competencies + 2 points for the guiding questions 3 points for the core competencies + 3 points for the guiding questions
Very good	5 points for the core competencies + 2 (or more) points for the guiding questions 4 points for the core competencies + 3 (or more) points for the guiding questions

The examination board will decide on the final grade based on the candidate’s performance in the supplementary examination as well as the result of the written examination.

Evaluation grid for the supplementary examination

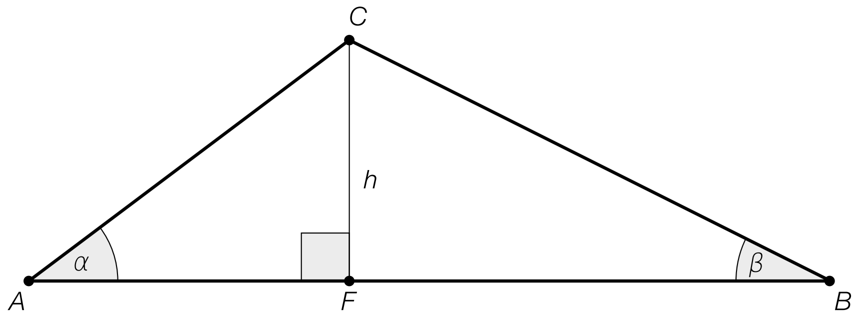
This evaluation grid may be used to assist the examiner in assessing the candidate's performance.

	Point for core competencies reached	Point for the guiding question reached
Task 1		
Task 2		
Task 3		
Task 4		
Task 5		

Task 1

Trigonometry

The triangle *ABC* is shown below. The foot *F* of the altitude *h* is closer to the vertex *A* and divides the line segment *AB* at a ratio of 2 : 5.



Task:

– Determine the size of the angle β when $h = 7$ cm and $\overline{AB} = 21$ cm.

Guiding question:

– Show by calculation that the triangle *ABC* is not a right-angled triangle.

The point *C* is moved so that the foot *F* of the altitude *h* lies to the left of the vertex *A*. The length of the altitude *h* and the length of the line segment *AB* remain the same.

– Write down whether this change causes the value of $\tan(\beta)$ to increase or decrease and justify your answer.

Solution to Task 1

Trigonometry

Expected solution to the statement of the task:

$\overline{FB} = \frac{21}{7} \cdot 5 = 15$

$\tan(\beta) = \frac{h}{\overline{FB}} = \frac{7}{15} \quad \Rightarrow \quad \beta = 25.016...° \approx 25°$

The size of the angle is approximately 25°.

Answer key:

The point for the core competencies shall be awarded if the correct angle is given.

Expected solution to the guiding question:

Possible justification by calculation with an indirect proof:

If it is assumed that *ABC* is a right-angled triangle, then according to the Pythagorean theorem, the following statement must hold:

$225 + h^2 + 36 + h^2 = 21^2 \quad \Rightarrow \quad 261 + 2 \cdot h^2 = 441 \quad \Rightarrow \quad h^2 = 90$

This contradicts $h = 7 \quad \Rightarrow$ the triangle *ABC* is not a right-angled triangle.

The value of $\tan(\beta)$ decreases.

Possible justification:

As the size of β decreases and the value of the tangent of $\beta \in [0°, 90°]$ is strictly monotonically increasing, then the value of $\tan(\beta)$ also decreases.

Answer key:

The point for the guiding question shall be awarded if it has been correctly shown by calculation that the triangle *ABC* is not a right-angled triangle and the correct change in the value of $\tan(\beta)$ has been given along with a correct justification.

Task 2

Powder Dye

If 500 g of powder dye is put into a jug of water, then after one minute 70 g of this powder will have dissolved.

The amount of powder dye that has dissolved is modelled by the function p where
$p(t) = 500 - 500 \cdot e^{k \cdot t}$ in terms of the time t (t in min, $p(t)$ in g).

Task:

– Determine the value of k.

Guiding question:

The function p fulfils the difference equation $p(t + 1) - p(t) = a \cdot (500 - p(t))$ with $a \in \mathbb{R}$.

– Determine the value of a and interpret your result in the context given.

Solution to Task 2

Powder Dye

Expected solution to the statement of the task:

$70 = 500 - 500 \cdot e^{k \cdot 1}$
$k = \ln\left(\frac{43}{50}\right) = -0.150823... \approx -0,15082$

Answer key:

The point for the core competencies shall be awarded if the correct value of k has been given.

Expected solution to the guiding question:

$p(1) - p(0) = a \cdot (500 - p(0))$
$70 = a \cdot (500 - 0) \quad \Rightarrow \quad a = \frac{70}{500} = 0.14$

Every minute, 14 % of the powder that has not yet dissolved dissolves in the water.

Answer key:

The point for the guiding question shall be awarded if the correct value of a has been determined and a correct interpretation has been given.

Task 3

Maxima and Minima of a Fourth Degree Polynomial Function

The equation of a fourth degree polynomial function f is $f(x) = a \cdot x^4 + b \cdot x^3 + c \cdot x^2 + d \cdot x + e$ with $a, b, c, d, e \in \mathbb{R}$ and $a > 0$.

Task:

– Justify why f can have at most 3 maxima or minima.

Guiding question:

The following applies: $g(x) = p \cdot x^4 + q \cdot x^2 + r$ with $p, q, r \in \mathbb{R}$ and $p > 0$.

– Write down every number of local maxima or minima that g can have.
– Demonstrate by calculation how the sign of q influences the number of maxima or minima and for each case, sketch a typical graph of the function.

Solution to Task 3

Maxima and Minima of a Fourth Degree Polynomial Function

Expected solution to the statement of the task:

Possible justification:

$f'(x) = 4 \cdot a \cdot x^3 + 3 \cdot b \cdot x^2 + 2 \cdot c \cdot x + d$
The equation $4 \cdot a \cdot x^3 + 3 \cdot b \cdot x^2 + 2 \cdot c \cdot x + d = 0$ is a third degree equation and has a maximum of three solutions $\Rightarrow$ there are at most three maxima or minima.

Answer key:

The point for the core competencies shall be awarded if it has been correctly explained why at most three maxima or minima are possible.

Expected solution to the guiding question:

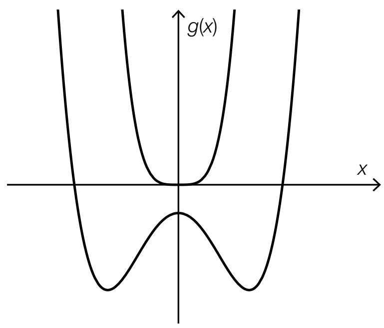
The number of local maxima or minima can only be 1 or 3.

Demonstration by calculation:

$g(x) = p \cdot x^4 + q \cdot x^2 + r$
$g'(x) = 4 \cdot p \cdot x^3 + 2 \cdot q \cdot x = 0$
$x \cdot (4 \cdot p \cdot x^2 + 2 \cdot q) = 0 \quad \Rightarrow \quad x_1 = 0 \quad x_{2,3} = \pm\sqrt{-\frac{q}{2 \cdot p}}$

For $q \geq 0$, there is no second solution and therefore only one maximum or minimum.
For $q < 0$, there are two further solutions and therefore three maxima or minima.

Possible graphs:



Answer key:

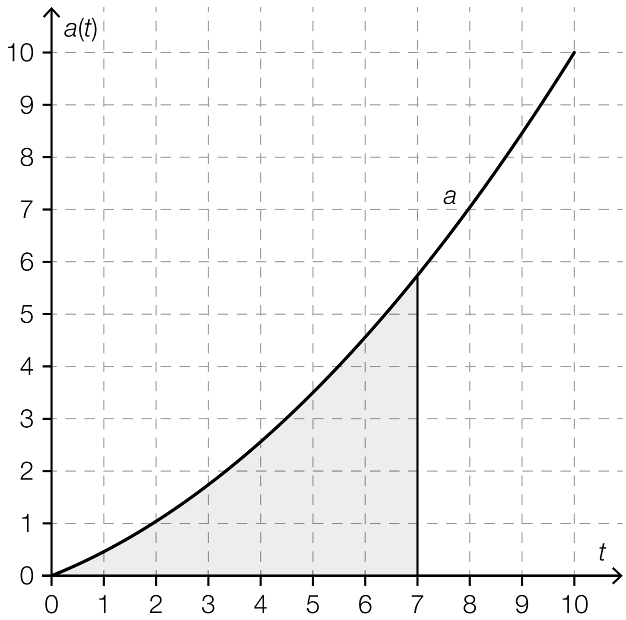
The point for the guiding question shall be awarded if the correct number of possible maxima or minima has been given and the influence of the sign of q has been correctly demonstrated. A correct sketch also must have been shown in which the number of maxima or minima and the symmetry of the graph must be recognisable.

Task 4

Acceleration

An object with an initial speed of 5 m/s accelerates for 10 s.
The acceleration of the object is modelled by the function a in terms of time t.
The equation of the function a is: $a(t) = 0.06 \cdot t^2 + 0.4 \cdot t$ with t in s and $a(t)$ in m/s^2.

The diagram below shows the graph of the function a with a shaded area.



Task:

– Determine the size of the shaded area and explain what this value means in terms of the velocity of the object.

Guiding question:

– Determine the length of the distance covered during this 10 s long acceleration and explain your method.

Solution to Task 4

Acceleration

Expected solution to the statement of the task:

Size of the shaded area:
$\int_0^7 (0.06 \cdot t^2 + 0.4 \cdot t)\,dt = 16.66$

This value gives the increase in the velocity of the object in the first 7 s.

Answer key:

The point for the core competencies shall be awarded if the correct size of the shaded area has been determined and this value has been correctly interpreted.

Expected solution to the guiding question:

Possible method:
By integrating the acceleration function, the velocity function is obtained.
By integrating again, the length of the distance covered can be calculated.

$\int a(t)\,dt = \int (0.06 \cdot t^2 + 0.4 \cdot t)\,dt = 0.02 \cdot t^3 + 0.2 \cdot t^2 + c = v(t)$
$v(0) = 5 \quad \Rightarrow \quad v(t) = 0.02 \cdot t^3 + 0.2 \cdot t^2 + 5$
$s(10) = \int_0^{10} (0.02 \cdot t^3 + 0.2 \cdot t^2 + 5)\,dt = 166.\dot{6}$

Length of the distance covered: around 167 m

Answer key:

The point for the guiding question shall be awarded if the correct length of the distance covered has been calculated and a correct method has been given.

Task 5

Fitness Training

The list of data shown below gives the number of hours per week that eight young people spend training at a gym.

3, 3, 5, 6, 7, 8, 9, x

The median and the mean of the training times have the same value.

Task:

– Assuming that x is the largest value of the list of data, write down the training time x.

Guiding question:

– Assuming that x is any integer value of the list of data, write down a second possible value for the training time x.

Three out of the eight young people are chosen at random.

– For each of the possible values of x, determine the probability that exactly two out of the three young people train at least five hours per week.

Solution to Task 5

Fitness Training

Expected solution to the statement of the task:

$$\frac{3+3+5+6+7+8+9+x}{8} = 6.5 \quad \Rightarrow \quad x = 11$$

Answer key:

The point for the core competencies shall be awarded if the correct training time x has been given.

Expected solution to the guiding question:

In order to obtain a different median value, then a value of $x \leq 6$ must be chosen.
For $x = 6 \quad \Rightarrow \quad$ median = 6, this does not correspond to the mean (5.875).
Therefore a value of $x \leq 5$ must be chosen $\quad \Rightarrow \quad$ median = 5.5.

$$\frac{x+3+3+5+6+7+8+9}{8} = 5.5 \quad \Rightarrow \quad x = 3$$

$$\frac{6}{8} \cdot \frac{5}{7} \cdot \frac{2}{6} \cdot 3 = \frac{15}{28}$$

$$\frac{5}{8} \cdot \frac{4}{7} \cdot \frac{3}{6} \cdot 3 = \frac{15}{28}$$

The probability for both possible values of x is 53.6 %.

Answer key:

The point for the guiding question shall be awarded if the correct second value of x and the correct probability for both possible values of x have been given.