Name:	
Class:	

Standardised Competence-Oriented
Written School-Leaving Examination

AHS

14th January 2020

Mathematics

Part 1 and Part 2 Tasks

Bundesministerium
Bildung, Wissenschaft
und Forschung

Advice for Completing the Tasks

Dear candidate!

The following booklet contains Part 1 tasks and Part 2 tasks (divided into sub-tasks). The tasks can be completed independently of one another. You have a total of *270 minutes* available in which to work through this booklet.

Please do all of your working out solely in this booklet and the paper provided to you. Write your name and that of your class on the cover page of the booklet in the spaces provided. Also, write your name and consecutive page numbers on each sheet of paper used. When answering each sub-task, indicate its name/number on your sheet.

In the assessment of your work, everything that is not crossed out will be considered. Your solutions must be marked clearly. If a solution is not clearly marked or if more than one solution is given, the task will be considered to be unsolved.

You may use the official formula booklet for this examination session as well as approved electronic device(s), provided there is no possibility to communicate via internet, Bluetooth, mobile networks, etc. and there is no access to your own data stored on the device.

An explanation of the task types is available in the examination room and can be viewed on request.

Please hand in the task booklet and all used sheets at the end of the examination.

Changing an answer for a task that requires a cross:

1. Fill in the box that contains the cross.
2. Put a cross in the box next to your new answer.

   In this instance, the answer "5 + 5 = 9" was originally chosen. The answer was later changed to be "2 + 2 = 4".

1 + 1 = 3	☐
2 + 2 = 4	☒
3 + 3 = 5	☐
4 + 4 = 4	☐
5 + 5 = 9	■

Selecting an item that has been filled in:

1. Fill in the box that contains the cross for the answer you do not wish to give.
2. Put a circle around the filled-in box you would like to select.

   In this instance, the answer "2 + 2 = 4" was filled in and then selected again.

1 + 1 = 3	☐
2 + 2 = 4	◉
3 + 3 = 5	☐
4 + 4 = 4	■
5 + 5 = 9	☐

Assessment

The tasks in Part 1 will be awarded either 0 points or 1 point or 0, ½ or 1 point, respectively. The points that can be reached in each task are listed in the booklet for all Part 1 tasks. Every sub-task in Part 2 will be awarded 0, 1 or 2 points. The tasks marked with an [A] will be awarded either 0 points or 1 point.

Two assessment options

1) If you have reached **at least 16** of the 28 points (24 Part 1 points + 4 [A] points from Part 2), a grade will be awarded as follows:

Pass	16–23.5 points
Satisfactory	24–32.5 points
Good	33–40.5 points
Very Good	41–48 points

2) If you have reached **fewer than 16** of the 28 points (24 Part 1 points + 4 [A] points from Part 2), but have reached a **total of 24 points or more** (from Part 1 and Part 2 tasks), then a "Pass" or "Satisfactory" grade is possible as follows:

Pass	24–28.5 points
Satisfactory	29–35.5 points

From 36 points upward, the assessment key specified in 1) applies.

If you have reached fewer than 16 points in Part 1 (including the compensation tasks marked with an [A] from Part 2) and if the total is less than 24 points, you will not pass the examination.

Good luck!

Task 1

Equivalent Equations

The following equation is given: $\frac{x}{2} - 4 = 3$ in $x \in \mathbb{R}$.

Task:

Put a cross next to each of the two equations in $x \in \mathbb{R}$, that are equivalent to the given equation.

$x - 4 = 6$	☐
$\frac{x}{2} = -1$	☐
$\frac{x}{2} - 3 = 4$	☐
$\frac{x-8}{2} = 3$	☐
$\left(\frac{x}{2} - 4\right)^2 = 9$	☐

[0 / 1 point]

Task 2

Statistics on Traffic Accidents

The following data refers to traffic accidents in the time range from 2014 to 2016.

A ... number of traffic accidents in the year 2014, of which *a* % include human injuries
B ... number of traffic accidents in the year 2015, of which *b* % include human injuries
C ... number of traffic accidents in the year 2016, of which *c* % include human injuries

Task:

Write down an expression that describes the total number *N* of traffic accidents which include human injuries in the time range from 2014 to 2016.

N = _____

[0 / 1 point]

Task 3

Lion Pack

A pack of lions consists of lionesses and male lions. The number of male lions in this specific pack is modelled by m, the number of lionesses as w.

The following two equations contain information regarding this pack.

$m + w = 21$

$4 \cdot m + 1 = w$

Task:

Put a cross next to each of the two correct statements regarding this pack.

There are more male lions than lionesses in this pack.	☐
The number of lionesses is more than four times the number of male lions.	☐
The number of male lions is 1 smaller than the number of lionesses.	☐
The total number of lions (male lions and lionesses) is greater than 20 lions.	☐
The quadruple number of male lions is 1 greater than the number of lionesses.	☐

[0 / 1 point]

Task 4

Quadratic Equation

The quadratic equation $x^2 + r \cdot x + s = 0$ in which $x \in \mathbb{R}$ applies with $r, s \in \mathbb{R}$ is given.

Task:

Match each of the four solution possibilities to its corresponding statement about the parameters r and s (from A to F), so that it always contains the specific solution case.

The quadratic equation doesn‘t have a real solution.	
The quadratic equation only holds one real solution $x = -\frac{r}{2}$.	
The quadratic equation has the real solutions $x_1 = 0$ and $x_2 = -r$.	
The quadratic equation has the real solutions $x_1 = -\sqrt{-s}$ and $x_2 = \sqrt{-s}$.	

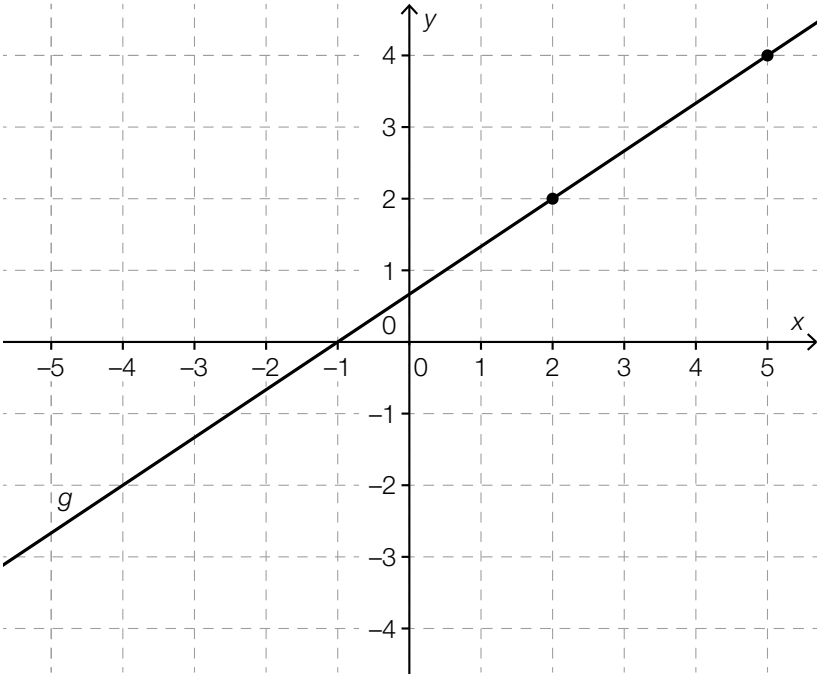
A	$\frac{r^2}{4} = s$
B	$\frac{r^2}{4} - s > 0$ with $r, s \neq 0$
C	$r \in \mathbb{R},\ s > 0$
D	$r = 0,\ s < 0$
E	$r \neq 0,\ s = 0$
F	$r = 0,\ s > 0$

[0/½/1 point]

Task 5

A Parallel Line through a Point

The following coordinate system shows the graph of the line *g*. The marked points on the graph *g* have integer coordinates.



Task:

Give a vector equation of the line *h* that goes through the point (3,–1) and is parallel to *g*.

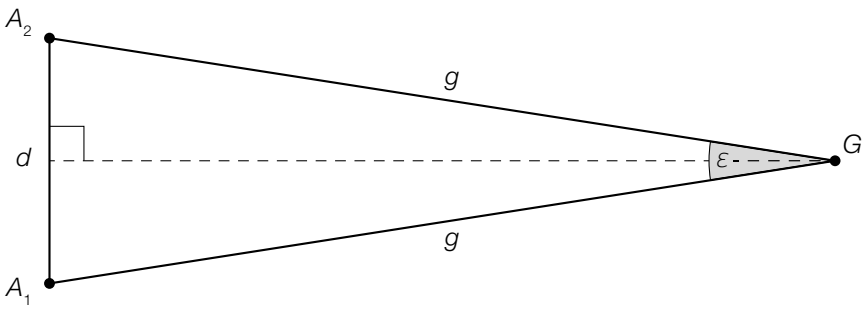
h: *X* = _____

[0 / 1 point]

Task 6

Depth Perception

When visualizing an object the direction of sight from both eyes enclose an angle ε. In the following scenario the object G is the same distance g from both eyes A_1 and A_2. The distance between the eyes is described by d.



Task:

Write down an expression for the distance g in terms of the distance between the eyes d and the angle ε.

$g =$ _____

[0 / 1 point]

Task 7

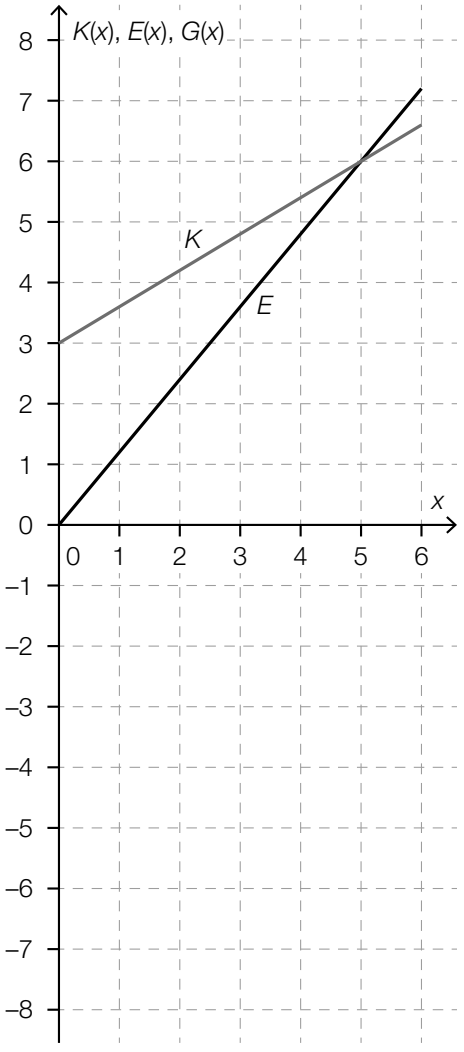
Function of Profit

The following diagram shows a linear cost function $K: x \mapsto K(x)$ as well as a linear revenue function $E: x \mapsto E(x)$ for which $x \in [0, 6]$ applies.

The profit function $G: x \mapsto G(x)$ holds for all $x \in [0, 6]$: $G(x) = E(x) - K(x)$.

Task:

Draw the graph of G into the following diagram.



[0 / 1 point]

Task 8

Functional Coherencies

The following equation $w = \frac{y \cdot z^2}{2 \cdot x}$ with $w, x, y, z \in \mathbb{R}^+$ is given.

The given equations show the functional coherencies between two variables when the other two are assumed to be constants.

Task:

Put a cross next to each of the two correct answers.

Assuming z is dependent on x, then $z\colon \mathbb{R}^+ \to \mathbb{R}^+, x \mapsto z(x)$ is an exponential function.	☐
Assuming w is dependent on z, then $w\colon \mathbb{R}^+ \to \mathbb{R}^+, z \mapsto w(z)$ is a quadratic function.	☐
Assuming w is dependent on x, then $w\colon \mathbb{R}^+ \to \mathbb{R}^+, x \mapsto w(x)$ is a linear function.	☐
Assuming y is dependent on z, then $y\colon \mathbb{R}^+ \to \mathbb{R}^+, z \mapsto y(z)$ is a polynomial function with degree 2.	☐
Assuming x is dependent on y, then $x\colon \mathbb{R}^+ \to \mathbb{R}^+, y \mapsto x(y)$ is a linear function.	☐

[0 / 1 point]

Task 9

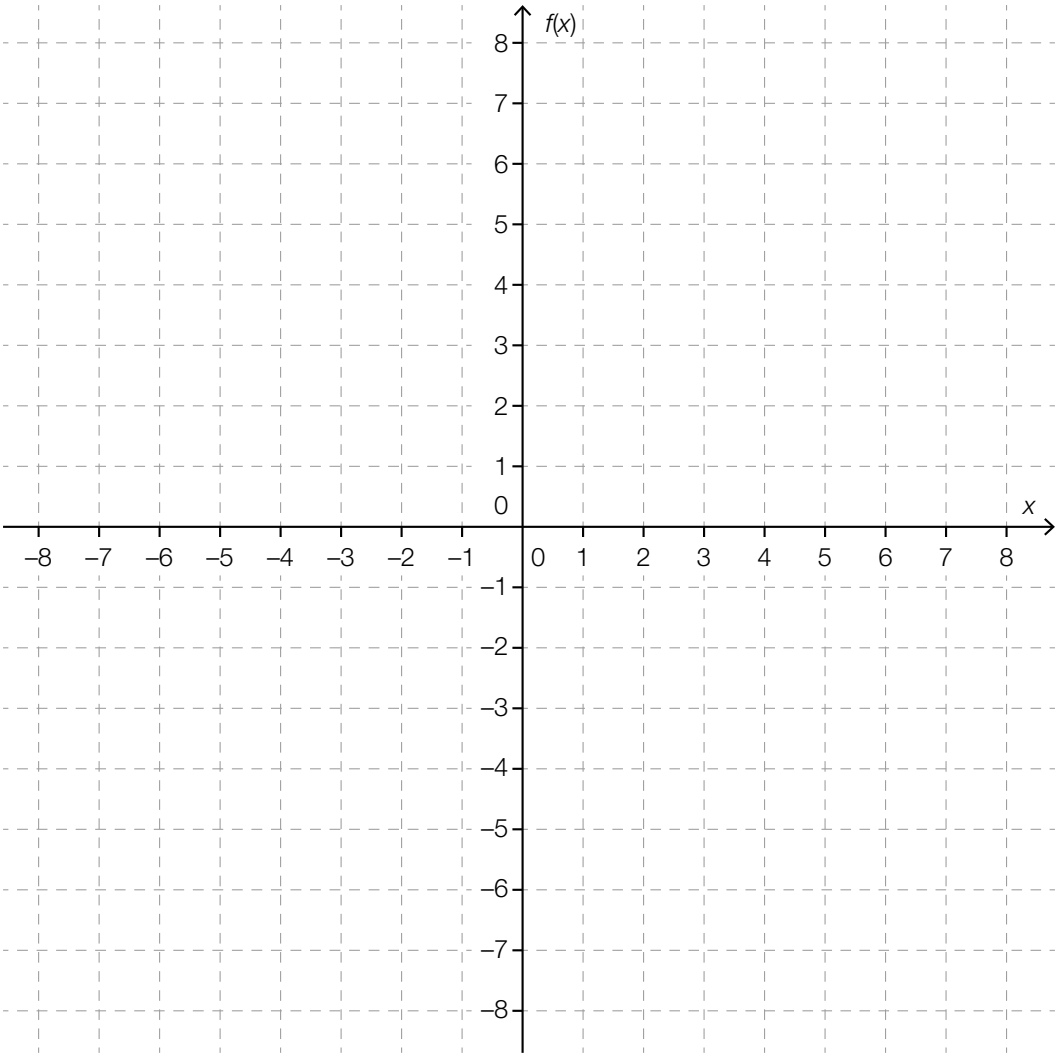
Drawing a Graph

The following characteristics of a linear function f are known:

- The gradient of f is –0.4.
- The function value of f at 2 is 1.

Task:

Draw the graph of f into the coordinate system below for the interval [–7, 7].



[0 / 1 point]

Task 10

Gross Income and Net Income

On the website of the ministry for finance one can find a gross-net-calculator which calculates the respective net income based on the monthly gross income.

The following table shows some incomes:

gross income in €	1 500	2 000	2 500
net income in €	1 199	1 483	1 749

Task:

By using data from the table, show that there is no linear coherency between the gross income and the net income.

[0 / 1 point]

Task 11

Interest

A capital investment K_0 is placed in a savings account which bears an interest rate of 1 % (per year).

For the following task it may be assumed that all taxes and fees need not be considered and that no further deposits or payments are made.

Task:

Calculate the number of years until the capital K_0, at a constant interest rate, is doubled.

[0 / 1 point]

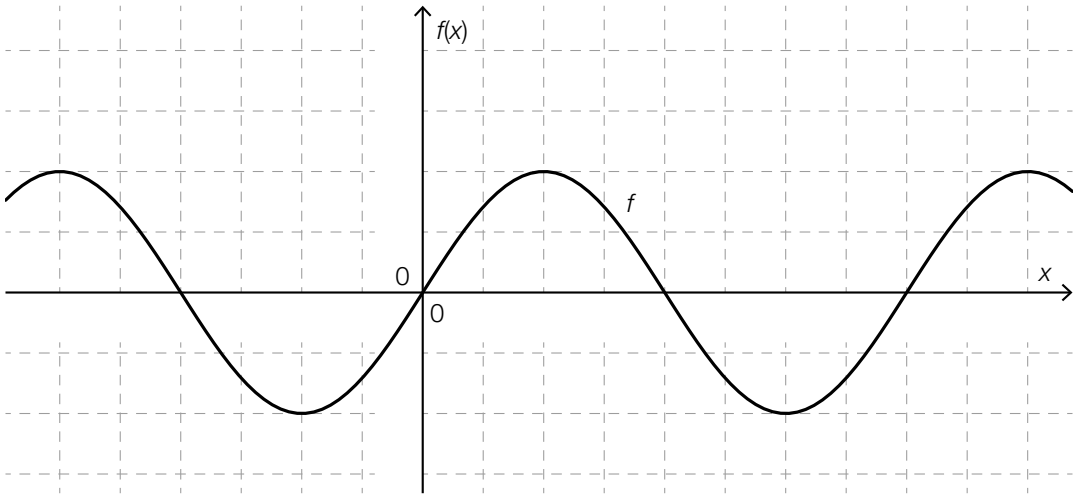
Task 12

Sine Function

The function $f\colon \mathbb{R} \to \mathbb{R}$ with $f(x) = a \cdot \sin\left(\frac{\pi \cdot x}{b}\right)$ with $a, b \in \mathbb{R}^+$.

Task:

Include the parameters *a* and *b* in the illustration by marking on the appropriate axis, so that the graph pictured is equivalent to the graph of the function *f*.

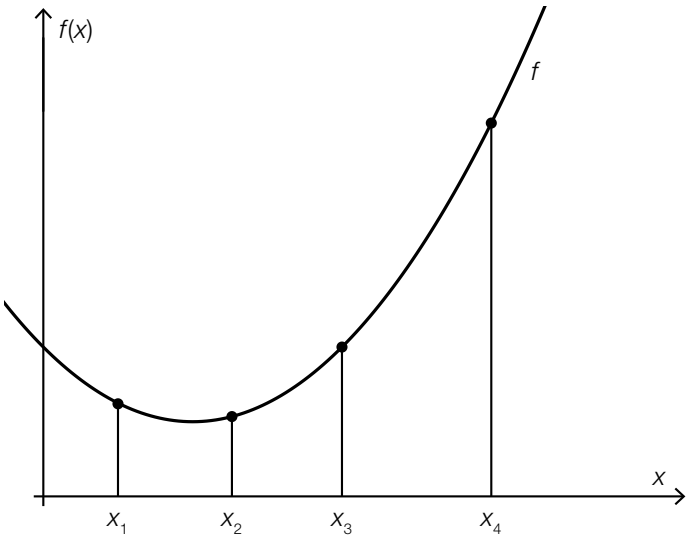


[0 / 1 point]

Task 13

Difference Quotient and Differential Quotient

The following diagram shows the graph of a second degree polynomial function f. Furthermore, there are four points marked along the graph with the x-coordinates x_1, x_2, x_3 and x_4.



Task:

Put a cross next to each of the two statements which are correct for the function f.

The difference quotient for the interval $[x_1, x_2]$ is smaller than the differential quotient at the position x_1.	☐
The difference quotient for the interval $[x_1, x_3]$ is smaller than the differential quotient at the position x_3.	☐
The difference quotient for the interval $[x_1, x_4]$ is smaller than the differential quotient at the position x_2.	☐
The difference quotient for the interval $[x_2, x_4]$ is greater than the differential quotient at the position x_2.	☐
The difference quotient for the interval $[x_3, x_4]$ is greater than the differential quotient at the position x_4.	☐

[0 / 1 point]

Task 14

Motion

An object starts its linear motion at the time $t = 0$.
The function v assigns every time t to a velocity $v(t)$ of the object at the time t (t in s, $v(t)$ in m/s).

Task:

Interpret the equation $v'(3) = 1$ in the given context with use of the appropriate unit.

[0 / 1 point]

Task 15

Concentration of a Medicinal Substance

A patient is given a medicinal substance intravenously every day at 8 a. m. The concentration of the substance in the patient's blood on day t directly before the next dose of the substance is described as c_t (c_t in milligrams per litre).

For $t \in \mathbb{N}$: $c_{t+1} = 0.3 \cdot (c_t + 4)$ applies.

Task:

Interpret the value 4 in the equation in the given context with use of the appropriate unit.

[0 / 1 point]

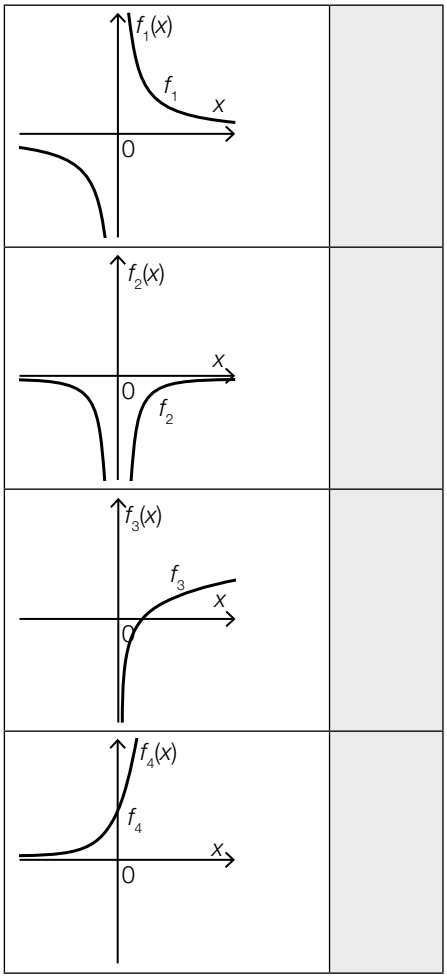
Task 16

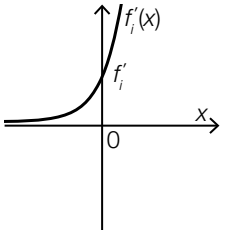
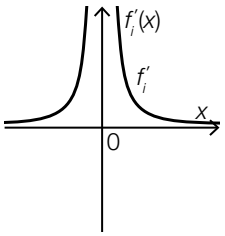
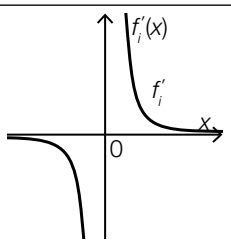
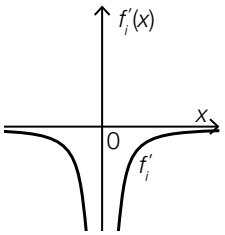
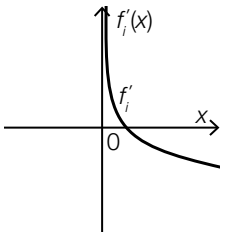
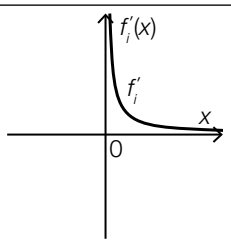
Graphs of Derivatives

Below, there are four graphs of the functions f_1 to f_4 as well as the graphs of six functions (A through F).

Task:

Match each of the four graphs of the functions f_1 to f_4 to the corresponding graph (from A to F) that is the derivative of this function.



A	
B	
C	
D	
E	
F	

[0/½/1 point] 18

Task 17

Characteristics of Polynomial Functions

Let $f\colon \mathbb{R} \to \mathbb{R}$ be a polynomial function and $a, b \in \mathbb{R}$ with $a < b$.

Task:

Complete the following sentence by putting a cross next to one of the given possibilities for each gap so that the sentence is a certainly correct statement.

If for all $x \in (a, b)$ _____①_____ holds, then the function f is _____②_____ in the interval (a, b).

①	
$f(x) > 0$	☐
$f'(x) < 0$	☐
$f''(x) > 0$	☐

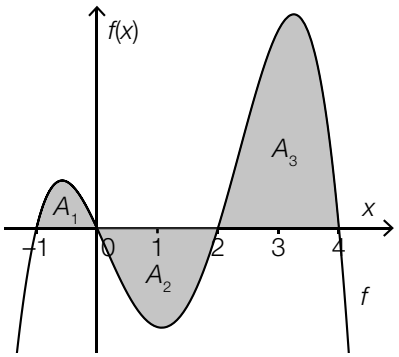
②	
strictly monotonic decreasing	☐
concave down	☐
strictly monotonic increasing	☐

[0 / 1 point]

Task 18

Definite Integral

Below, a graph of a polynomial function f with zeros at $x_1 = -1$, $x_2 = 0$, $x_3 = 2$ and $x_4 = 4$ is pictured.
For the areas labelled as A_1, A_2 and A_3 the following applies: $A_1 = 0.4$, $A_2 = 1.5$ and $A_3 = 3.2$.



Task:

Put a cross next to each of the two equations that are true.

$\int_{-1}^{2} f(x)\,dx = 1.9$	☐
$\int_{0}^{4} f(x)\,dx = 1.7$	☐
$\int_{-1}^{4} f(x)\,dx = 5.1$	☐
$\int_{0}^{2} f(x)\,dx = 1.5$	☐
$\int_{2}^{4} f(x)\,dx = 3.2$	☐

[0 / 1 point]

Task 19

Histogram

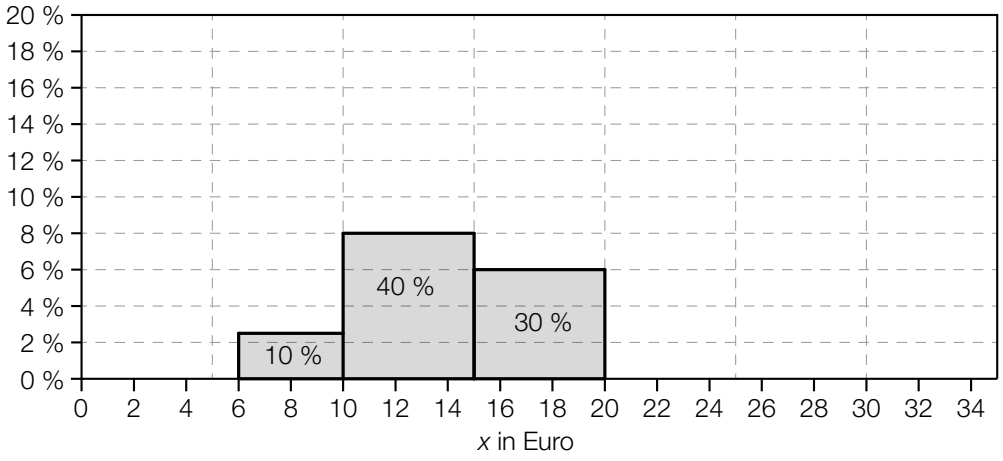
A company has a total of 200 employees. The following table depicts the hourly wages of the employees in groups.

hourly wages x in Euro	number of employees
$6 \leq x < 10$	20
$10 \leq x < 15$	80
$15 \leq x < 20$	60
$20 \leq x \leq 30$	40

The area of the rectangle in the histogram below represents the relative proportion of the employees in each group.

Task:

Add the missing column to the given histogram so that the table is fully represented by the histogram.



[0 / 1 point]

Task 20

Statistical Indicators

A data set is expanded by a value which is greater than all the values in the original set. Two of the following statistical indicators are certainly greater in the new set.

Task:

Put across next to each of the two appropriate statistical indicators.

span	☐
modus	☐
median	☐
3rd quartile	☐
arithmetic mean	☐

[0 / 1 point]

Task 21

Flu in Austria

The Medical University of Vienna has published the data on flu strain infections for a specific week. To compile this data, the blood of people, who were sick with the flu in this specific week was analysed. Of the 1 954 analysed samples, 547 blood samples were infected with the virus *A(H1N1)*, 117 blood samples were infected with the virus *A(H3N2)* and the rest of the blood samples were infected with the virus *Influenza B*.

Task:

Use the given frequencies as probabilities and given these criteria, determine the probability that a randomly selected infected person was infected with the Virus *Influenza B*.

[0 / 1 point]

Task 22

Basketball

Martin and Sebastian each once and one after another throw a ball in the direction of the hoop. Martin dunks (basketball falls through hoop) with a probability of 0.7 and Sebastian dunks with a probability of 0.8 (independent of Martin's success).

Task:

Calculate the probability that exactly one of the two players dunks.

[0 / 1 point]

Task 23

Three Tosses of a Cone

When tossing a cone, it can either land on its curved surface or on its base.

Independent from other tosses, the probability that a cone lands on its base is 30 %.

A cone gets tossed three times during a random experiment. The random variable *X* describes how often the cone lands on its base.

The table below should show the probability distribution of the random variable *X*.

Task:

Complete the missing values in the table.

X	probability (approximated)
0	0.343
1	0.441
2	
3	

[0 / 1 point]

Task 24

Breakfast

In a recent poll, 252 out of 450 questioned youths of a federal state stated that they always eat breakfast before going to school. The proportion of these youths is described by h.

The proportion of all youths in this federal state which always eat breakfast before heading to school is described by p.

Task:

Based on the results of the poll, write down the to h symmetrical 95-%-confidence interval for p.

[0 / 1 point]

Task 25 (Part 2)

Use of Antibiotics

The development of a bacteria culture can be influenced by the addition of antibiotics which should eventually lead to the death of the culture due to the toxic effect of the antibiotic.

In certain cases this development can be approximated by the function B: $\mathbb{R}_0^+ \to \mathbb{R}$:

$B(t) = b \cdot e^{k \cdot t - \frac{c}{2} \cdot t^2}$ with $b, c, k \in \mathbb{R}^+$

t ... time in hours
$B(t)$ ... number of bacteria in millions at the time t
b ... number of bacteria in millions at the time $t = 0$
k ... constant
c ... parameter of the toxic effect

Task:

a) The function B has exactly one positive maximum or minimum t_1.

1) Determine t_1 in dependency of k and c.

2) Describe what effect enlarging c with a given k has on the position of the maximum or minimum t_1 of the function B.

b) The function B_1: $\mathbb{R}_0^+ \to \mathbb{R}$ with $B_1(t) = 20 \cdot e^{2 \cdot t - 0.45 \cdot t^2}$ describes the number of bacteria of a certain bacteria culture in millions dependent on the time t.

At a certain time $t_2 \neq 0$ the bacterial culture reaches its original population of 20 million.

1) [A] Determine t_2.

2) Interpret the meaning of $B_1'(t_2)$ in the given context with use of the appropriate unit.

c) The function B_2: $\mathbb{R}_0^+ \to \mathbb{R}$ with $B_2(t) = 5 \cdot e^{4 \cdot t - \frac{t^2}{2}}$ describes the number of bacteria in a culture in millions. This culture shows its maximum at the time $t = 4$.

1) Determine the time t_3 at which the biggest decrease in bacteria occurs.

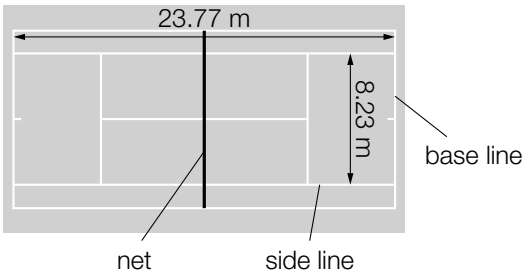
2) Write down what percentage of the maximum number of bacteria is left at the time t_3.

Task 26 (Part 2)

Tennis

Tennis is a recoil game played by two or four people during which a tennis ball is tossed over a net. The playing field is a rectangle and is split into two equal rectangles by the net (see Illustration 1). For a game between two people the playing field is 23.77 m long and 8.23 m wide. The playing field is limited by the lines on the ground along the base line and side lines. The net has a maximum height of 1.07 m.

Illustration 1:



Task:

a) The function $f: \mathbb{R}_0^+ \rightarrow \mathbb{R}$ with $f(x) = -0.0007 \cdot x^3 + 0.005 \cdot x^2 + 0.2 \cdot x + 0.4$ describes the trajectory of a tennis ball until it hits the ground for the first time. In this scenario, x describes the horizontal distance from the position at the time of the serve and $f(x)$ the height of flight of the tennis ball above the ground (x and $f(x)$ in m). The flight pattern of the tennis ball starts between the two side lines at the base line and the plane, in which the flight pattern lies, runs parallel to the side line of the tennis court.

1) [A] Determine in which horizontal distance the tennis ball reaches its maximum height measuring from the point of the serve.

   horizontal distance from point of the serve: _____ m

2) Determine through calculation if the tennis ball lands in the opponents field or behind the base line.

b) If a tennis ball falls onto the ground perpendicularly (without rotation), then it recoils again perpendicularly. The restitution coefficient r is a gage for the ability of a tennis ball to jump.

The equation $r = \frac{v_2}{v_1}$ applies for the jump of the tennis ball, in which v_1 describes the value of the velocity before impact and v_2 describes the value of the velocity after the impact.

The difference of the vertical velocities directly before and after the impact is described by the equation $\Delta v = v_2 - (-v_1)$ due to the different orientation.

1) Find a term that describes Δv, so that Δv is dependent on v_1 and r.

$\Delta v =$ _____

A tennis ball impacts perpendicularly at a speed of $v_1 = 4.4$ m/s. The restitution coefficient for this tennis ball is approximately $r = 0.6$. The contact time with the ground is 0.01 s.

2) Calculate the average acceleration a (in m/s^2) of a tennis ball in vertical direction during the time of the impact.

$a =$ _____ m/s^2

c) A player wins a five-set match as soon as he has won three sets. To win a set one usually must win six games, whereby each game is worth one point.

For different probabilities p for having won a game the resulting probabilities m to win a five-set match have been determined and can be seen in the table below.

p	m
0.5	0.5
0.51	0.6302
0.55	0.9512
0.6	0.9995
0.7	1.000

The probability that player A wins a game is 2 percent points greater that the probability that the opponent B will win a game.

1) Determine by how many percent points the probability that player A wins a five-set match is greater than that of the opponent B winning.

When playing against a different, weaker player C, player A has an advantage of 10 percent points to win a game.

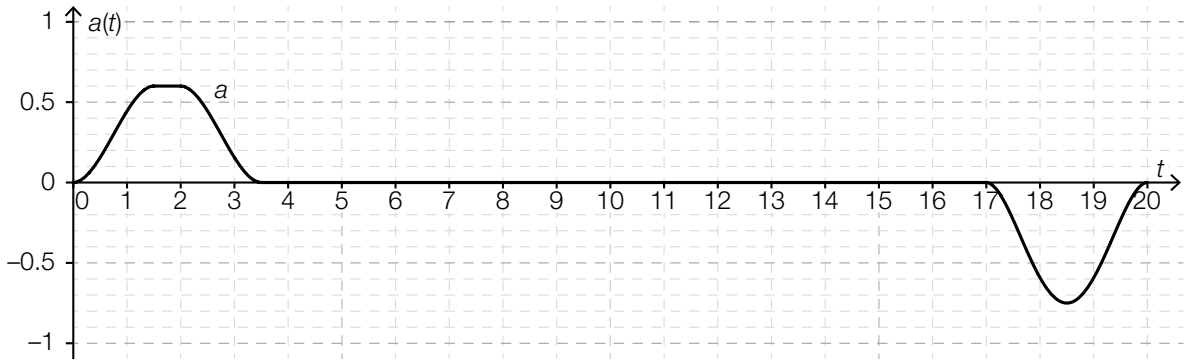
2) Show that the probability that player A wins a five-set match against opponent C is 50.94 percent greater than during a five set match against opponent B.

Task 27 (Part 2)

Elevator Ride

The velocity of elevators for transporting people can vary greatly depending on construction and building height.

The following illustration shows the time-acceleration diagram for an elevator ride of 20 seconds. At the beginning and end of the ride the elevator is at a standstill. The time t is measured in seconds, the acceleration $a(t)$ in m/s^2. The acceleration values were determined by sensors and the course of the acceleration is modelled by a derivable function a.



Task:

a) 1) A Write down the appropriate interval for each later mentioned section of the elevator ride.

elevator is decelerating: _____

elevator is travelling at a constant speed: _____

Kim claims that the velocity of the elevator during the time interval [1.5 s, 2s] remains constant.

2) Argue if Kim's statement is correct and justify your decision.

b) 1) Using the illustration, determine the approximate maximal velocity v_{max} during the graphed elevator ride.

The graph of the function a encloses two areas together with the t-axis during the time intervals [0, 3.5] and [17, 20].

2) Based on the given context, justify why these two areas must be the same size.

c) An elevator producer is planning to construct a new elevator. The acceleration of this elevator during the first three seconds is described by the derivable function a_1: [0, 3] → $\mathbb{R}$ with

$$a_1(t) = \begin{cases} 0.6 \cdot t^2 \cdot (3 - 2 \cdot t) & \text{for } 0 \le t < 1 \\ 0.6 & \text{for } 1 \le t < 2 \\ 0.6 \cdot (t - 3)^2 \cdot (2 \cdot t - 3) & \text{for } 2 \le t \le 3 \end{cases}$$

(t in s, $a_1(t)$ in m/s^2).

1) Calculate the increase in speed of this elevator during the time interval [0, 3].

For the path of an elevator ride, certain criteria regarding the acceleration must be fulfilled. The *jolt*, the instantaneous change in acceleration, should lie between –1 m/s^3 and 1 m/s^3 for the entire elevator ride.

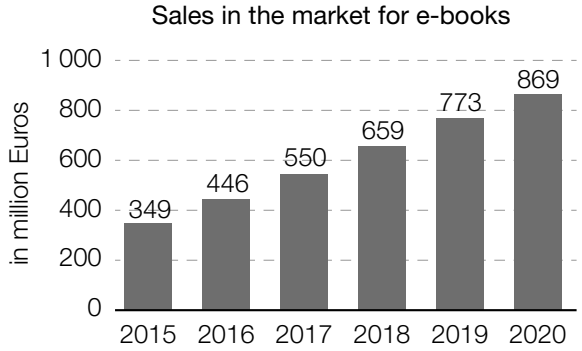
2) Verify if this elevator fulfils the criteria for the jolt at the time $t = 1$.

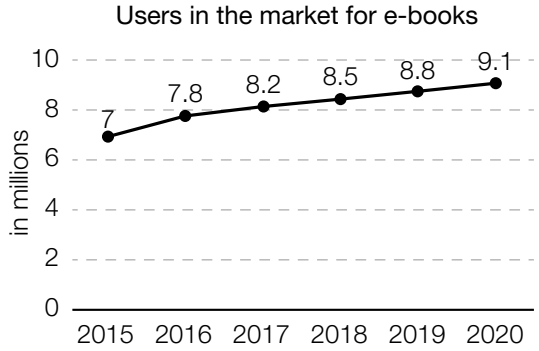
Task 28 (Part 2)

E-Book

The term *e-book* (abr. for electronic book) describes books in a digital form.

The two diagrams regarding Germany show the estimated values for the growth of the e-book market.





Source: http://www.e-book-news.de/20-prozent-wachstum-pro-jahr-statista-sieht-deutschen-e-book-markt-im-aufwind/ [19.06.2019] (adapted).

Task:

a) 1) Calculate the absolute and relative change for the estimated sales per user in Germany in the time range from 2015 to 2020.

absolute change: € _____

relative change: _____

2) Calculate the difference quotient for the estimated sales per user in Germany for the time range from 2015 to 2020.

b) The estimated increase in sales in the market for e-books from 349 million Euros in 2015 to 869 million Euros in 2020 is described by the source as follows:

"20 percent increase per year"

1) [A] Determine how high the estimated sales $U(2017)$ for the year 2017 would have had to be, if the sales had increased by 20 % annually starting with the estimated sales value in 2015.

$U(2017) =$ _____ million Euro

Someone describes the estimated increase in sales in the market for e-books from 349 million Euros in the year 2015 to 869 million Euros in the year 2020 as follows:

"*a* million Euro increase per year"

2) Calculate a.

c) The population of Germany in the year 2015 was approximately 82.18 million. The population of Austria was approximately 8.58 million. Someone asks themselves the following question: "How many people were using e-books in Austria in 2015?"

1) Answer the question assuming that the proportion of e-book users in Austria is the same as the (estimated) proportion of the e-book users in Germany in 2015.

Number: _____ people

500 people are randomly selected in Austria in the year 2020. The binomially distributed random variable X describes the number of people from this selection that are e-book users. The probability that a person is an e-book user is assumed at 12 %.

2) Calculate the probability that at least 50 e-book users were randomly selected.