Exemplar für Prüfer/innen

Supplementary Examination for the
Standardised Competence-Oriented
Written School-Leaving Examination

AHS

May 2020

Mathematics

Supplementary Examination 6
Examiner's Version

Bundesministerium
Bildung, Wissenschaft
und Forschung

Instructions for the supplementary examination

The following supplementary examination is comprised of five tasks that can be completed independently of one another.

Each task contains two parts: The statement of the task requires the candidate to demonstrate core competencies, and the guiding question that follows it requires the candidate to show their ability to communicate their ideas.

In the following document, the examiner will find the tasks as well as the expected solutions and the answer key.

The preparation time shall be at least 30 minutes and the examination time shall be at most 25 minutes.

Assessment

Each task can be awarded zero, one or two points. There is one point available for each demonstration of core competencies as well as for each guiding question. A maximum of 10 points can be achieved.

The following scale will be used for the grading of the examination:

Grade	Number of points
Pass	4 points for the core competencies + 0 points for the guiding questions 3 points for the core competencies + 1 point for the guiding questions
Satisfactory	5 points for the core competencies + 0 points for the guiding questions 4 points for the core competencies + 1 point for the guiding questions 3 points for the core competencies + 2 points for the guiding questions
Good	5 points for the core competencies + 1 point for the guiding questions 4 points for the core competencies + 2 points for the guiding questions 3 points for the core competencies + 3 points for the guiding questions
Very good	5 points for the core competencies + 2 (or more) points for the guiding questions 4 points for the core competencies + 3 (or more) points for the guiding questions

The examination board will decide on the final grade based on the candidate's performance in the supplementary examination as well as the result of the written examination.

Evaluation grid for the supplementary examination

This evaluation grid may be used to assist the examiner in assessing the candidate's performance.

	Point for core competencies reached	Point for the guiding question reached
Task 1		
Task 2		
Task 3		
Task 4		
Task 5		

Task 1

Angle of a Slope

In order to determine the danger of avalanches, it is important to know the angle of a slope.

Task:

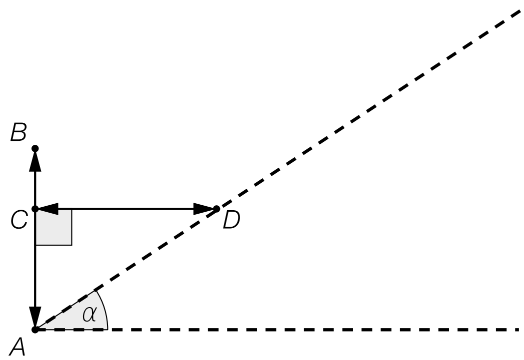
A particular slope has an angle of 30° to the horizontal.

– Determine the gradient of the slope as a percentage.

Guiding question:

The diagram below shows a method used to estimate the angle of a slope using ski poles. The angle of the slope, α, is determined using two ski poles of equal length, *AB* and *CD*.

The ski pole *CD* is held horizontally to the slope; the ski pole *AB* is held vertically at the end of the pole *CD* (as in the diagram).



– Write down the angle of the slope if using this method it is found that the points *B* and *C* have the same position as each other.

– Determine the angle of the slope, α, when the length of the line segment $\overline{BC}$ is one third of the length of the ski pole $\overline{AB}$.

Solution to Task 1

Angle of a Slope

Expected solution to the statement of the task:

$\tan(30°) = 0.57735... \approx 57.74\ \%$

Answer key:

The point for the core competencies is to be awarded if the gradient of the slope is given correctly as a percentage.

Expected solution to the guiding question:

If $B = C$, then the angle of the slope is 45°.

$\tan(\alpha) = \frac{\overline{AC}}{\overline{CD}} = \frac{2}{3} \quad \Rightarrow \quad \alpha \approx 33.7°$

Answer key:

The point for the guiding question is to be awarded if the angle of the slope is given correctly in both cases.

Task 2

Ideal Gas Equation

The equation $p \cdot V = n \cdot R \cdot T$ models the relationship between the pressure p, the volume V, the amount of the substance n, and the absolute temperature T of an ideal gas. In the equation, R is a constant.

Task:

– Justify why the relationship between how the pressure p changes with respect to the temperature T can be modelled by a linear function of the form $p(T) = k \cdot T + d$ (where $k, d \in \mathbb{R}$) if the other values are constant.

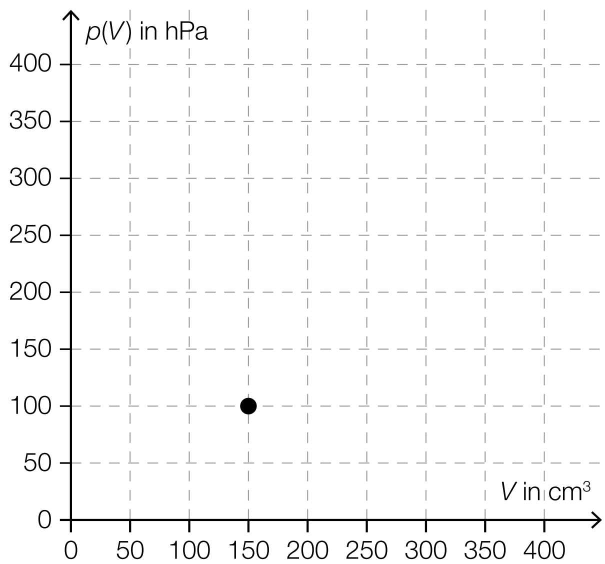
– Write down the parameters k and d of this linear function (in terms of the values given above).

Guiding question:

The pressure p of an ideal gas can be given as a function of the volume V if the values of n, R and T are constant.

– Complete the table of values shown below, sketch the graph of the function p in the coordinate system and write down which type of function p is.

V in cm^3	50	100	150	200	300
$p(V)$ in hPa			100		



Solution to Task 2

Ideal Gas Equation

Expected solution to the statement of the task:

If n, R and V are constant, then $p(T) = \frac{n \cdot R}{V} \cdot T$. This equation corresponds to a linear function with parameters $k = \frac{n \cdot R}{V}$ and $d = 0$.

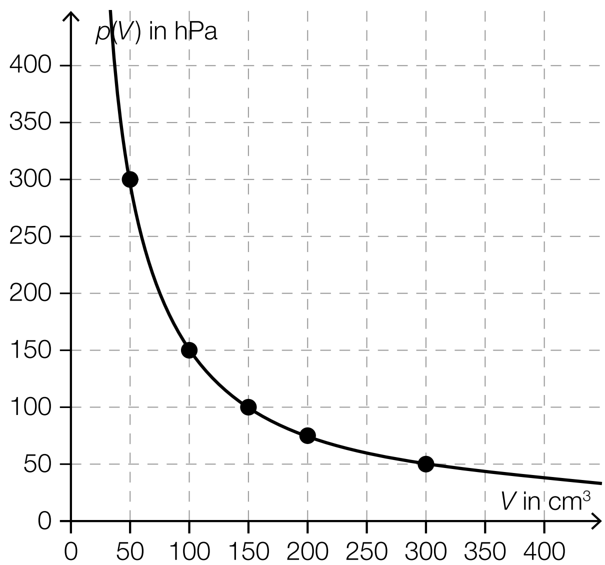
Answer key:

The point for the statement of the task is to be awarded if a correct justification has been given and the parameters of the corresponding linear function k and d have been given correctly.

Expected solution to the guiding question:

$$p(V) = \frac{n \cdot R \cdot T}{V} \quad \Rightarrow \quad p(V) \cdot V = \text{constant} \quad \Rightarrow \quad p(V) \cdot V = 15\,000$$

V in cm^3	50	100	150	200	300
$p(V)$ in hPa	300	150	100	75	50



The function p is a power function (or reciprocal function or indirectly proportional function).

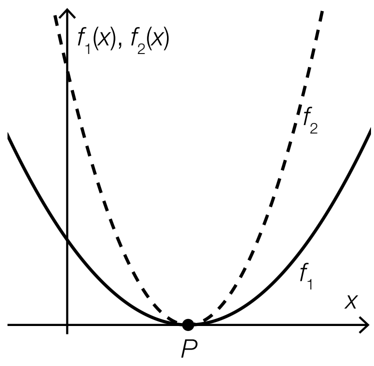
Answer key:

The point for the guiding question is to be awarded if the table of values has been completed correctly, a correct graph has been sketched and a correct function type has been given.

Task 3

Two Parabolas

The graphs of two functions f_1 and f_2 where $f_1(x) = a_1 \cdot x^2 + b_1 \cdot x + c_1$ and $f_2(x) = a_2 \cdot x^2 + b_2 \cdot x + c_2$ are shown in the diagram below.
The graphs of the two functions only have the point P on the positive x-axis in common.



Task:

– Complete each of the gaps shown below with a correct symbol "<"," >" or "=" so that the statements are true and justify your answer.

a_1 ______ a_2

c_1 ______ c_2

Guiding question:

– Given that $a_1 = 0.25$ and $P = (2,0)$, determine the values of the parameters b_1 and c_1 and explain your method.

Solution to Task 3

Two Parabolas

Expected solution to the statement of the task:

$a_1 < a_2$ because the graph corresponding to f_1 is "flatter" than the graph corresponding to f_2.
$c_1 < c_2$ because $f_1(0) < f_2(0)$.

Answer key:

The point for the core competencies is to be awarded if the symbols have been inserted correctly and correct justifications have been given.

Expected solution to the guiding question:

$b_1 = -1, \ c_1 = 1$

Possible method:

$f_1(x) = 0.25 \cdot x^2 + b_1 \cdot x + c_1$

$f_1'(x) = 0.5 \cdot x + b_1$

$f_1'(2) = 0 \quad \Rightarrow \quad 1 + b_1 = 0 \quad \Rightarrow \quad b_1 = -1$

$f_1(2) = 0 \quad \Rightarrow \quad 1 - 2 + c_1 = 0 \quad \Rightarrow \quad c_1 = 1$

Answer key:

The point for the guiding question is to be awarded if the values of both parameters have been given correctly and a correct method has been shown.

Task 4

Velocity of a Vehicle

The velocity of a vehicle between two sets of traffic lights in the time interval $[0, t_1]$ is modelled by the function v with $v(t) = -\frac{4}{15} \cdot t^2 + 4 \cdot t$ where t is given in s and $v(t)$ is given in m/s.
At the time $t = 0$, the vehicle is at the first set of traffic lights.

Task:

At time t_1 the vehicle comes to a stop at the second set of traffic lights.

– Write down the time t_1 and determine the distance covered by the vehicle in this time interval.

Guiding question:

– Determine the time $t_0 \in [0, t_1]$ at which the vehicle reaches its maximum velocity and write down this maximal velocity.

– The time t_2 is the time at which the vehicle has covered 80 % of the distance between the two sets of traffic lights. Using v, write down an equation with which the time t_2 can be found and determine the value of t_2.

Solution to Task 4

Velocity of a Vehicle

Expected solution to the statement of the task:

$v(t) = -\frac{4}{15} \cdot t^2 + 4 \cdot t = 0 \quad \Rightarrow \quad t_1 = 15\text{ s}$

$s(t) = \int v(t)\,\mathrm{d}t = -\frac{4}{45} \cdot t^3 + 2 \cdot t^2 + c$

$s(0) = 0 \quad \Rightarrow \quad c = 0$

$s(15) = 150\text{ m}$

Answer key:

The point for the core competencies is to be awarded if both the time as well as the distance covered have been given correctly.

Expected solution to the guiding question:

$v'(t_0) = 0 \quad \Rightarrow \quad -\frac{8}{15} \cdot t_0 + 4 = 0 \quad \Rightarrow \quad t_0 = 7.5\text{ s}$

$v(7.5) = 15\text{ m/s}$

Possible equation:

$\int_0^{t_2} v(t)\,\mathrm{d}t = 0.8 \cdot 150 \quad \Rightarrow \quad t_2 \approx 10.7\text{ s}$

Answer key:

The point for the guiding question is to be awarded if both the correct time t_0 and the correct velocity $v(t_0)$ have been given along with a correct equation and the correct time t_2.

Task 5

Expanding a Data Set

A set of data that consists of six numbers is shown below:

$x_1 = 4,\ x_2 = 8,\ x_3 = 2,\ x_4 = 7,\ x_5 = 4,\ x_6$

The mean of the set of data is $\bar{x} = 5$.

Task:

– Determine the value of x_6 as well as the median of the set of data.

Guiding question:

– Expand the data set by writing down two whole numbers such that both of the following conditions are fulfilled and justify your answer:
  - The mean of the new data set is the same as the mean of the original data set.
  - The median of the new data set is greater than the original median.

Solution to Task 5

Expanding a Data Set

Expected solution to the statement of the task:

$$\frac{4 + 8 + 2 + 7 + 4 + x_6}{6} = 5 \quad \Rightarrow \quad x_6 = 5$$

Median: $\frac{4 + 5}{2} = 4.5$

Answer key:

The point for the core competencies is to be awarded if both the value of x_6 and the median have been given correctly.

Expected solution to the guiding question:

Numbers to be added to the data set: 5 and 5

Possible justification:

So that the mean $\bar{x}$ stays the same, the extra numbers must be of the form $\bar{x} - c$ and $\bar{x} + c$ with $c \in \mathbb{N}$.

Only when $c = 0$ and therefore only when the data set is expanded with the values 5 and 5 does the value of the median increase. With these values, the median of the data set 2, 4, 4, 5, 5, 5, 7, 8 is 5 > 4.5. For all values $c \in \mathbb{N}\backslash\{0\}$ the value of $5 - c$ lies below the original median and the value of $5 + c$ lies above the original median, which means that the expanded data set also has a median of 4.5.

Answer key:

The point for the guiding question is to be awarded if the correct values of both numbers have been given and the choice of the values has been justified correctly.