Name:	
Class:	

Standardised Competence-Oriented
Written School-Leaving Examination

AHS

5th May 2020

Mathematics

Bundesministerium
Bildung, Wissenschaft
und Forschung

Advice for Completing the Tasks

Dear candidate!

The following booklet contains Part 1 tasks and Part 2 tasks (divided into sub-tasks). The tasks can be completed independently of one another. You have a total of *270 minutes* available in which to work through this booklet.

Please do all of your working out solely in this booklet and the paper provided to you. Write your name and that of your class on the cover page of the booklet in the spaces provided. Also, write your name and consecutive page numbers on each sheet of paper used. When answering each sub-task, indicate its name/number on your sheet.

In the assessment of your work, everything that is not crossed out will be considered. Your solutions must be marked clearly. If a solution is not clearly marked or if more than one solution is given, the task will be considered to be unsolved.

The use of the official formula booklet that has been approved by the relevant government authority is allowed. Furthermore, the use of electronic device(s) (e.g. graphic display calculators or other appropriate technology) is allowed provided there is no possibility to communicate via internet, Bluetooth, mobile networks, etc. and there is no access to your own data stored on the device.

An explanation of the task types is available in the examination room and can be viewed on request.

Please hand in the task booklet and all used sheets at the end of the examination.

Changing an answer for a task that requires a cross:

1. Fill in the box that contains the cross.
2. Put a cross in the box next to your new answer.

In this instance, the answer "5 + 5 = 9" was originally chosen. The answer was later changed to be "2 + 2 = 4".

1 + 1 = 3	☐
2 + 2 = 4	☒
3 + 3 = 5	☐
4 + 4 = 4	☐
5 + 5 = 9	■

Selecting an item that has been filled in:

1. Fill in the box that contains the cross for the answer you do not wish to give.
2. Put a circle around the filled-in box you would like to select.

In this instance, the answer "2 + 2 = 4" was filled in and then selected again.

1 + 1 = 3	☐
2 + 2 = 4	(■)
3 + 3 = 5	☐
4 + 4 = 4	■
5 + 5 = 9	☐

Assessment

The tasks in Part 1 will be awarded either 0 points or 1 point or 0, ½ or 1 point, respectively. The points that can be reached in each task are listed in the booklet for all Part 1 tasks. Every sub-task in Part 2 will be awarded 0, 1 or 2 points. The tasks marked with an [A] will be awarded either 0 points or 1 point.

Two assessment options

1) If you have reached **at least 16** of the 28 points (24 Part 1 points + 4 [A] points from Part 2), a grade will be awarded as follows:

Pass	16–23.5 points
Satisfactory	24–32.5 points
Good	33–40.5 points
Very Good	41–48 points

2) If you have reached **fewer than 16** of the 28 points (24 Part 1 points + 4 [A] points from Part 2), but have reached a **total of 24 points or more** (from Part 1 and Part 2 tasks), then a "Pass" or "Satisfactory" grade is possible as follows:

Pass	24–28.5 points
Satisfactory	29–35.5 points

If you have reached fewer than 16 points in Part 1 (including the compensation tasks marked with an [A] from Part 2) and if the total is less than 24 points, you will not pass the examination.

Good luck!

Task 1

Numbers and Sets of Numbers

Five statements about numbers and sets of numbers are given.

Task:

Put a cross next to each of the two correct statements.

$\sqrt{\frac{9}{2}}$ is a rational number.	☐
$-\sqrt{100}$ is an integer.	☐
$\sqrt{15}$ has a terminating decimal representation.	☐
$\sqrt{2}$ is a rational number.	☐
-4 is not the square of a real number.	☐

[0/1 point]

Task 2

Prize Distribution

A team consisting of three players wins € 10,000. The prize is divided up as follows: player *B* receives 50 % more than player *A*, player *C* receives 20 % less than player *B*.

The variable x represents the amount of money that player *A* receives (x in €).

Task:

Write down an equation that can be used to calculate x.

[0/1 point]

Task 3

Delegation

A delegation is to be formed from a large group of youths and adults.

The following three rules should apply:

1. The delegation should consist of at least 8 members.
2. The delegation should consist of no more than 12 members.
3. The delegation should consist of at least twice as many youths as adults.

Two of the three rules have been described by inequalities below. The number of youths in the delegation is described by J and the number of adults in the delegation is described by E.

Task:

Put a cross next to each of the two correct inequalities.

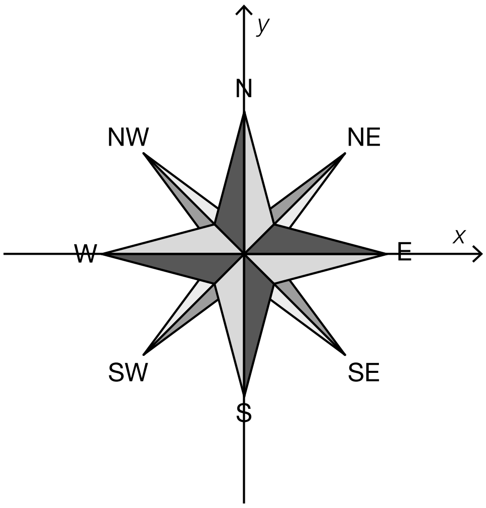
$J + E \leq 12$	☐
$J \geq 2 \cdot E$	☐
$J + E \leq 8$	☐
$J - 2 \cdot E < 0$	☐
$E \geq 2 \cdot J$	☐

[0/1 point]

Task 4

Directions on a Compass

The illustration below shows a symmetric wind rose showing the compass points.



The velocity of a ship travelling in a north-west (NW) direction is described by the vector $\vec{u} = \begin{pmatrix} -a \\ a \end{pmatrix}$ with $a \in \mathbb{R}^+$.

Task:

Write down a vector $\vec{v}$ that describes the velocity of a ship travelling in a north-east (NE) direction.

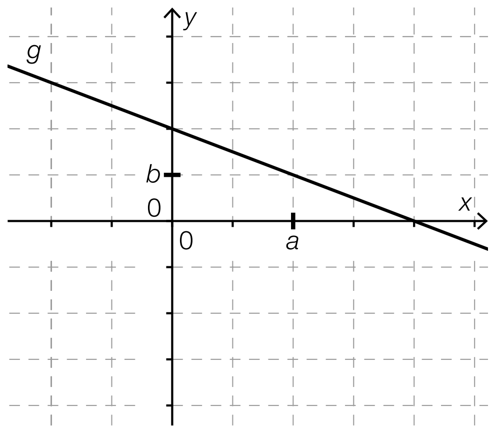
$\vec{v}$ = ____

[0/1 point]

Task 5

Scaling Coordinate Axes

The following coordinate system with axes in different scales shows a straight line g.
a has been marked on the x-axis and b has been marked on the y-axis. Both a and b are integers.
The line g is described by $y = -2 \cdot x + 4$.



Task:

Determine a and b.

a = ____

b = ____

[0/½/1 point]

Task 6

Train Track

The slope of a straight train track is measured in per mille (‰). For example an altitude change of 1 m per 1 000 m distance travelled horizontally translates to a slope of 1 ‰.

Task:

Write down an equation with which one can exactly calculate the angle of the slope α ($\alpha > 0$) of a straight train track with a slope of 30 ‰.

[0/1 point]

Task 7

Cost Function

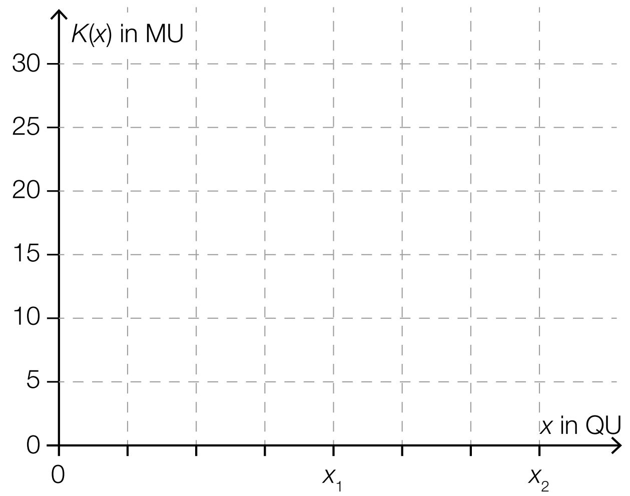
The total costs that accumulate while producing a product can be modelled using the differentiable cost function K. In this function, K assigns the cost $K(x)$ (x in quantity units (QU), $K(x)$ in monetary units (MU)) to the production volume x.

The following conditions apply for the cost function $K: [0, x_2] \to \mathbb{R}$ and x_1 with $0 < x_1 < x_2$:

- K is strictly monotonically increasing in the interval $[0, x_2]$.
- The fixed costs are 10 MU.
- The cost function is digressive in the interval $[0, x_1)$, which means the costs rise more slowly as the production volume increases.
- The point of cost reversal lies at a production volume of x_1. The point of cost reversal of K is the point from which the costs rise more and more.

Task:

Sketch the shape of the graph of one such cost function K into the coordinate system below.



[0/1 point]

Task 8

Train

A train moves forward at a constant speed until the time $t = 0$. After the time $t = 0$, the train increases its speed.

The function v assigns the point in time t with $0 \leq t \leq 60$ to the speed $v(t) = a \cdot t + b$ (t in s, $v(t)$ in m/s, $a, b \in \mathbb{R}$).

Task:

Complete the following sentence by putting a cross next to one of the given possibilities for each gap so that the sentence becomes a correct statement.

For parameter a ______①______ holds and for parameter b ______②______ holds.

①	
$a < 0$	☐
$a = 0$	☐
$a > 0$	☐

②	
$b < 0$	☐
$b = 0$	☐
$b > 0$	☐

[0/½/1 point]

Task 9

Linear Function

A linear function $f\colon \mathbb{R} \to \mathbb{R}$ with $f(x) = k \cdot x + d$ with $k, d \in \mathbb{R}$ and $k \neq 0$ is given.

$\frac{f(5) - f(a)}{2} = k$ for $a \in \mathbb{R}$ holds.

Task:

Determine a.

a = ____

[0/1 point]

Task 10

Grape Harvest

The grape harvest in a vineyard progresses faster the more people are involved. The function f models the indirectly proportional relationship between the time needed to harvest the grapes and the number of people involved. $f(n)$ describes the time needed to harvest the grapes when n people are involved ($n \in \mathbb{N}\setminus\{0\}$, $f(n)$ in hours).

Task:

Write down $f(n)$, if it is known that the time needed to harvest the grapes when 8 people are working is 6 hours.

$f(n) =$ ____ with $n \in \mathbb{N}\setminus\{0\}$

[0/1 point]

Task 11

Number of Animals

It can be assumed that the number of animals of a specific species of animals on earth increases by 1.8 % each year.

Task:

Determine the time period in years that it takes for the number of animals of this species to double on earth.

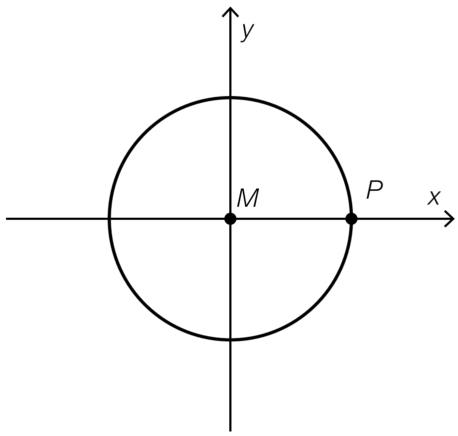
Duration: approximately ___________ years

[0/1 point]

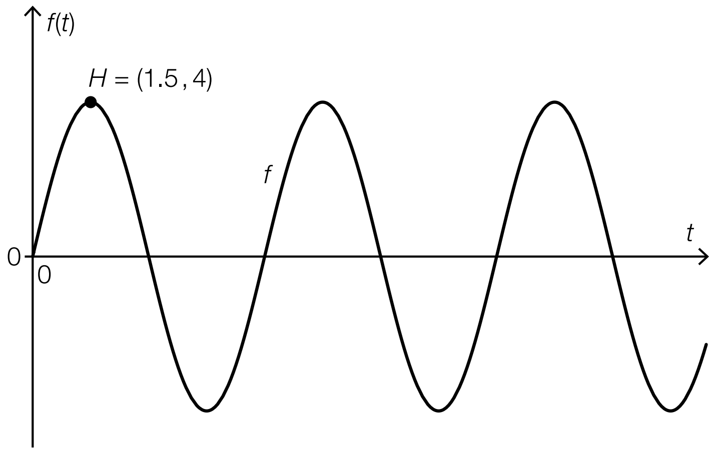
Task 12

Movement on a Circle

A point P moves counter-clockwise along a circle with centre $M = (0,0)$ at constant speed.
At the beginning of the motion (at time $t = 0$), the point P lies on the positive x-axis, as shown in the following illustration.



The function f assigns the time t to the second coordinate $f(t) = a \cdot \sin(b \cdot t)$ of the point P at the time t (t in s, $f(t)$ in dm, $a, b \in \mathbb{R}^+$). The following illustration shows the graph of f, which goes through the point H, whereby $f'(1.5) = 0$ holds.



Task:

Determine the radius of the circle and the period of the point P (the time required for one revolution).

Radius of the circle: ____________ dm

Period: ____________ s

[0 / ½ / 1 point]

Task 13

Absolute and Relative Change of a Function

The absolute change of a function $f\colon \mathbb{R} \to \mathbb{R}$ in an interval $[a, b]$, is described by A. The relative change of f in an interval $[a, b]$ is described by R, whereby $f(a) \neq 0$ and $a < b$ hold.

Task:

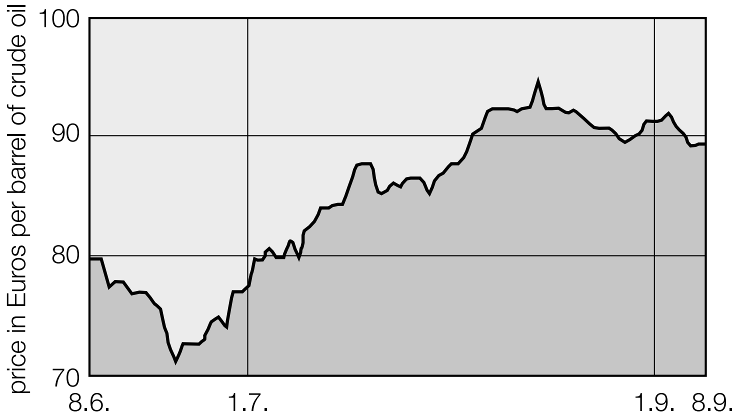
Write down an equation that shows the relationship between A and R.

[0/1 point]

Task 14

Price of Oil

The following diagram shows the price trend for crude oil in the period from 8.6.2012 until 8.9.2012.



Source: http://www.heizoel24.at/charts/rohoel [14.12.2012] (adapted).

Task:

Determine the average rate of change of the price per barrel of crude oil per month during the period from 1.7.2012 until 1.9.2012.

average rate of change: ____________________________ Euros per barrel of crude oil per month

[0/1 point]

Task 15

Population

The number of deer in a forest at the end of the year i ($i = 1, 2, 3$) is described by R_i. By the end of the first year, there are 60 deer in this forest.

The following equation describes the development of the population of the deer.

$R_{i+1} = 1.2 \cdot R_i - 2$ for $i = 1, 2$

Task:

Determine the number of deer in this forest at the end of the third year.

The number of deer at the end of the third year is __________ .

[0/1 point]

Task 16

Growth of a Plant

At the beginning of a three week long observation process, a certain plant is 15 cm tall. The instantaneous rate of change of the height of this plant is modelled by the function v in terms of the time t.
The following holds:
$v(t) = 3 - 0.3 \cdot t^2$ with $t \in [0, 3]$ in weeks and $v(t)$ in cm/week

The function h assigns each time $t \in [0, 3]$ to the height $h(t)$ of the plant (t in weeks, $h(t)$ in cm).

Task:

Write down $h(t)$.

$h(t) =$ ____

[0/1 point]

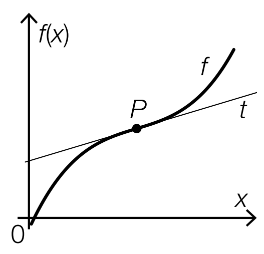
Task 17

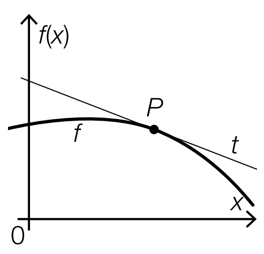
Graphs of Curves

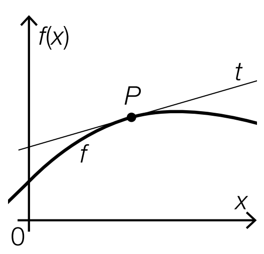
The four graphs given below on the left each show a tangent t to a polynomial function f at a point $P = (x_P, f(x_P))$. The point P is the only common point between the graph of f and the tangent t. In the table given below on the right, there are six statements about $f'(x_P)$ and $f''(x_P)$.

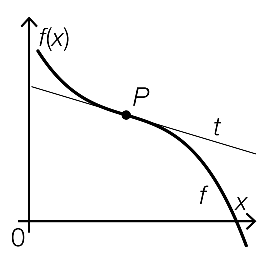
Task:

Match each of the four graphs to the corresponding statement (from A to F).









A	$f'(x_P) > 0$ and $f''(x_P) > 0$
B	$f'(x_P) > 0$ and $f''(x_P) < 0$
C	$f'(x_P) < 0$ and $f''(x_P) > 0$
D	$f'(x_P) < 0$ and $f''(x_P) < 0$
E	$f'(x_P) > 0$ and $f''(x_P) = 0$
F	$f'(x_P) < 0$ and $f''(x_P) = 0$

[0 / ½ / 1 point]

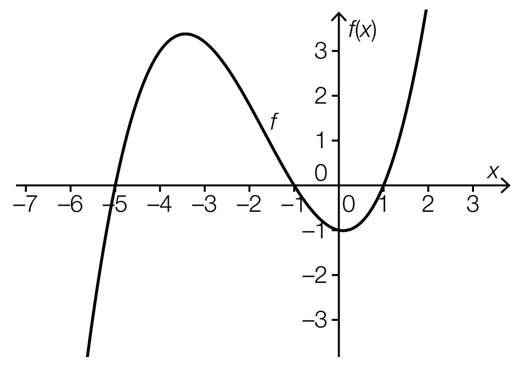
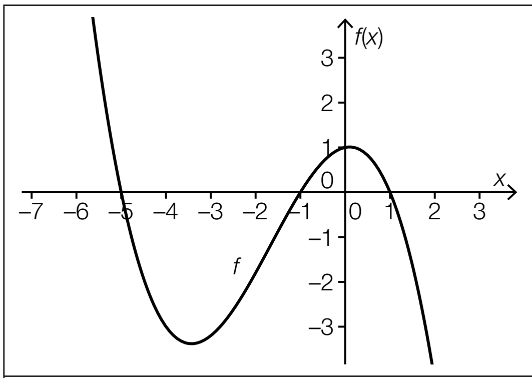
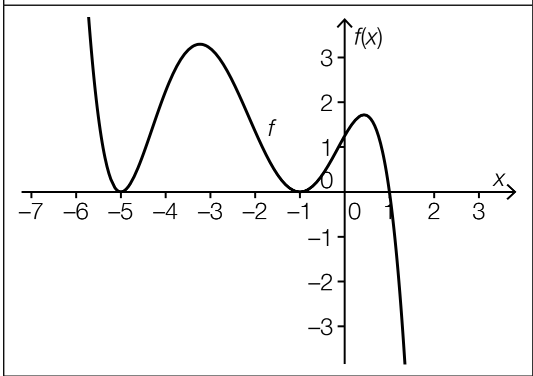
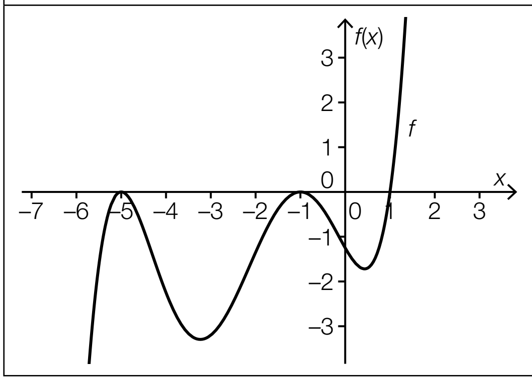
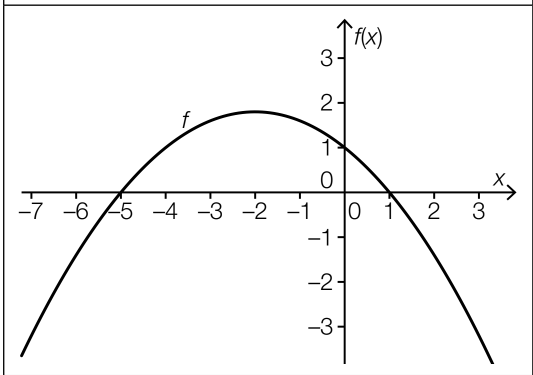
Task 18

Comparison of Definite Integrals

Five graphs of polynomial functions are illustrated below.

Task:

Put a cross next to the two graphs for which $\int_{-5}^{-1} f(x)\,\mathrm{d}x > \int_{-5}^{+1} f(x)\,\mathrm{d}x$ holds.

	☐		☐
	☐		☐
	☐		

[0/1 point]

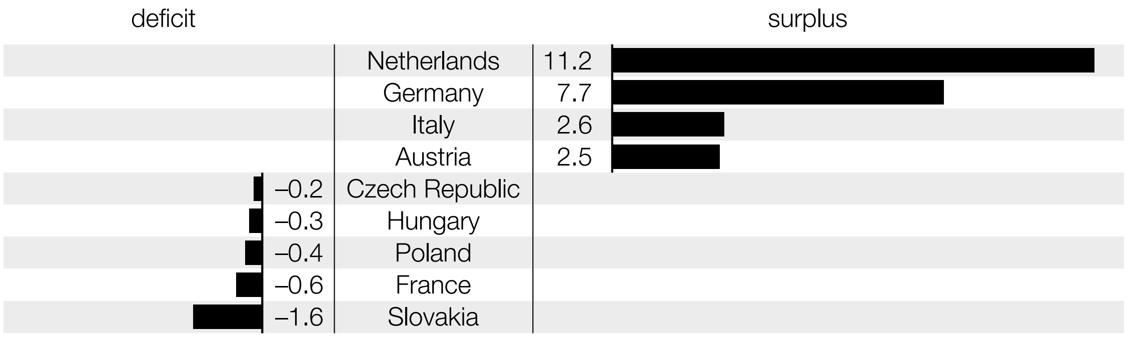
Task 19

GDP 2018

The gross domestic product (GDP) of Austria in the year 2018 amounted to 385.71 billion Euros.
Source: https://de.statista.com/statistik/daten/studie/14390/umfrage/bruttoinlandsprodukt-in-oesterreich/ [21.11.2019].

If the revenue generated from exports exceeds the expenditures from imports then there is an account surplus. Otherwise there is an account deficit. The following illustration shows the account surpluses and deficits, respectively, for certain countries as account balances given as a percentage of each GDP for the year 2018.

account balances 2018 given as a % of the GDP



Source: https://www.oenb.at/isaweb/report.do?report=10.18 [21.11.2019].

Task:

Calculate the account surplus (in billions of Euros) for Austria in the year 2018.

account surplus: ____ billion Euros

[0/1 point]

Task 20

List of Numbers

A list of numbers $x_1, x_2, x_3, \ldots, x_{40}$, for which $x_1 < x_2 < \ldots < x_{40}$ holds, is given.

Task:

Put a cross next to the number that can be added to the list of numbers above so that the median of the given list does not change.

$\frac{x_1 + x_{20}}{2}$	☐
$\frac{x_1 + x_{40}}{2}$	☐
$\frac{x_{20} + x_{21}}{2}$	☐
$\frac{x_{20} + x_{40}}{2}$	☐
x_{20}	☐
x_{21}	☐

[0/1 point]

Task 21

Favourite Subject

All children attending the first or second grade of a school were asked what their favourite subject was. Each child was allowed to choose exactly one subject. The following table shows the gathered data.

	favourite subject math	other favourite subject
first graders	47	241
second graders	33	287
total	80	528

A child attending the first grade is randomly selected. (The probability of selecting any child from the first grade is the same for all first graders.)

Task:

Calculate the probability that this child chose math as his or her favourite subject.

[0/1 point]

Task 22

Probability Distribution

An urn holds only white and black balls. Three balls are selected without replacement. The random variable X describes the number of white balls drawn from the urn.

The following table shows the probability distribution of the random variable X.

x	1	2	3
$P(X = x)$	0.3	0.6	0.1

Task:

Put a cross next to each of the two correct statements.

The probability of selecting no more than two white balls is 0.9.	☐
The probability of selecting at least one white ball is 0.3.	☐
The probability of selecting more than one white ball is 0.6.	☐
The probability of selecting exactly two black balls and one white ball is 0.1.	☐
The probability of selecting at least one black ball is 0.9.	☐

[0/1 point]

Task 23

Room Booking

A hotel manager assumes, due to years of experience, that every room booking made independently from another room booking is cancelled with a probability of 10 %. For a specific date, he accepts 40 independent room bookings.

Task:

Calculate the probability that no more than 5 % of the 40 room bookings on that specific date are cancelled.

[0/1 point]

Task 24

Conditioning Experiment

During a conditioning experiment, German shepherds learn to operate a mechanism to receive feed. After a training phase in which 50 German shepherds participate, 40 of them can operate the mechanism.

The relative proportion of these German shepherds that can operate the mechanism after the training phase is described by h.

Out of this data, a confidence interval $[a, 0.91]$ symmetrical to h with $a \in \mathbb{R}$ for the unknown proportion p of all German shepherds that can operate the mechanism after such a training phase is determined.

Task:

Determine the lower boundary a of the confidence interval.

[0/1 point]

Task 25 (Part 2)

Parachute Jump

During a parachute jump from a height of 4000 m above the ground, the parachute is opened 30 s after jumping off.

The function v_1 with $v_1(t) = 56 - 56 \cdot e^{-\frac{t}{4}}$ for $t \in [0, 30]$ (whilst considering air resistance) describes the speed of the fall of the parachutist at the time t (t in s after jumping off, $v_1(t)$ in m/s).

The function v_2 with $v_2(t) = \frac{51}{(t-29)^2} + 5 - 56 \cdot e^{-7.5}$ for $t \geq 30$ describes the speed of the fall of the parachutist at the time t until the time of landing (t in s after jumping off, $v_2(t)$ in m/s).

It can be assumed that the parachute jump is vertical.

Task:

a) 1) [A] Interpret $w = \frac{v_1(10) - v_1(5)}{10 - 5}$ in the given context.

For a $t_1 \in [0, 30]$ $v_1'(t_1) = w$ holds.

2) Interpret t_1 in the given context.

b) 1) Using the function v_1, calculate at which height the parachute is opened.

2) Calculate the amount of time taken for the entire parachute jump, from jumping off to landing.

c) Without considering air resistance, the parachutist would have an initial speed of 0 m/s and a constant acceleration of 9.81 m/s² during the time interval [0, 30]. The speed of the fall 9 s after jumping off would be v^*.

1) Calculate how much smaller $v_1(9)$ is than v^*.

2) Calculate by how many percent the acceleration of the parachutist is lower than during a jump which does not consider the air resistance 9 s after jumping off.

Task 26 (Part 2)

Growth Processes

Below, models of growth are considered.

The following difference equation describes a growth process.

$N_{t+1} - N_t = r \cdot (S - N_t)$
N_t ... population at time t
r ... growth constant, $r \in \mathbb{R}^+$
S ... (upper) capacity limit

Task:

a) On a cruise ship with 2000 passengers, of which none are sick at the time $t = 0$ days, 5 % of the healthy people get sick every day. N_t gives the number of sick passengers at the time t (with t in days).

1) Write down a difference equation for N_{t+1}.

2) Determine after how many days more than 25 % of the passengers are sick for the first time.

b) The difference equation $N_{t+1} - N_t = r \cdot (S - N_t)$ can also be written as $N_{t+1} = a \cdot N_t + b$ with $a, b \in \mathbb{R}$.

1) Write r and S in terms of a and b.

r = ____

S = ____

The growth of a bacterial culture in a petri dish is observed in order to develop a new vaccine.

The following table shows the area N_t (in cm^2) that is covered by the bacteria culture at the time t (in h).

t in h	N_t in cm^2
0	5.00
1	9.80
2	14.41

2) Determine a and b using the values from the table above.

c) A pharmaceutical company is launching a new vaccine on the market. In the first week after the launch, 15000 people have already purchased the vaccine.

The number $f(t)$ of people who have purchased the vaccine within t weeks after the launch can be modelled by the function f with $f(t) = 1\,000\,000 \cdot (1 - e^{-k \cdot t})$ ($k \in \mathbb{R}^+$).

1) [A] Calculate k.

2) Determine the earliest point in time t_0 at which more than 500000 people have purchased this vaccine.

Task 27 (Part 2)

Quiz with a Game Board

During a quiz, a series of questions, which can each be answered with “yes” or “no”, are asked. On a game board, a token is placed on the field with the number 0 at the beginning of each game. The token moves one field to the right for every correct answer given and one field to the left for every incorrect answer given. Each field is labelled with an integer value in ascending order (see the illustration below). The game board can be extended freely in any direction.

game board

---	–6	–5	–4	–3	–2	–1	0	1	2	3	4	5	6	---

Maria and Tom are playing this quiz. Tom asks Maria the questions.

Task:

a) For one game, the quiz ends when a token lands on the field with the number 2.

The variable A describes the event that a token lands on field number 2 after no more than 4 questions.

Maria answers each question correctly independently from the other ones with the same probability p.

1) Write down the probability $P(A)$ in terms of p.

$P(A) =$ ______________________________

If p is increased, the probability $P(A)$ increases as well.

2) Write down the value of $p \in [0, 1]$ for which the probability $P(A)$ increases the fastest (i. e. the instantaneous rate of change of $P(A)$ is the greatest).

b) Maria is asked exactly 100 questions during a different game. She answers each question correctly independently from the other ones with a probability of 0.8. The random variable Y represents the number on the field on which the token lands after answering these 100 questions.

1) Calculate the expectation value $E(Y)$.

$E(Y) =$ ______________________________

The random variable Y can be approximated by the normally distributed random variable Z. For this random variable, $E(Y) = E(Z)$ holds and the standard deviation σ of Z is 8.

2) Determine the interval $[z_1, z_2]$ that is symmetrical about the expectation value $E(Z)$ for which $P(z_1 \leq Z \leq z_2) = 95.4\ \%$ holds.

c) During another game, Maria answers all the questions by guessing. She therefore answers each question correctly independently from the other ones with a probability of 0.5. For every even number n of questions with $n \geq 2$ the following holds:

$$M(n) = \binom{n}{\frac{n}{2}} \cdot 0.5^n$$

1) [A] Interpret $M(n)$ in the given context.

For every even number of questions with $n \geq 10$, $M(n)$ can be approximated by

$$\tilde{M}(n) = \sqrt{\frac{2}{\pi \cdot n}}.$$

For every even $n \geq 10$, there exists an n^* so that $\tilde{M}(n^*) = \frac{1}{2} \cdot \tilde{M}(n)$ holds.

2) Determine n^* in terms of n.

$n^* =$ ______________________________

Task 28 (Part 2)

Ozone Measurements

The gas ozone affects our health. Due to this, the ozone concentrations in different layers of the atmosphere are measured at measuring stations and using weather balloons.

Task:

a) There is a weather station on the Hohe Warte in Vienna at 220 m above sea level. For a measurement series a weather balloon with an ozone reader is launched from there. The ozone reader starts its measurements as soon as the balloon has reached an altitude of 2 km above sea level.

Assume that the weather balloon (with a starting speed of 0 m/s) rises vertically and accelerates evenly with 0.125 m/s^2 until the balloon reaches a speed of 6 m/s at time t_1. The time is measured in seconds and the altitude above sea level is measured in meters.

1) Determine the height of the weather balloon above the weather station at the point in time t_1.

After the point in time t_1 the balloon continues to rise vertically at a constant speed of 6 m/s.

2) Determine how many seconds after the launch the ozone reader starts measuring.

b) A weather balloon has a volume of 6.3 m^3 at an air pressure of 1 013.25 hPa. Due to the decrease in air pressure while the balloon rises, the weather balloon expands further and further and becomes approximately spherical. It bursts at a diameter of *d* meters.

The air pressure, in terms of the altitude above sea level *h*, can be modelled by the function *p*, which assigns the air pressure $p(h)$ to the altitude above sea level *h*.

The following holds: $p(h) = 1\,013.25 \cdot \left(1 - \frac{0.0065 \cdot h}{288.15}\right)^{5.255}$ with *h* in m, $p(h)$ in hPa

Assume the pressure $p(h)$ and the volume $V(h)$ of the weather balloon are indirectly proportional to each other. $V(h)$ is the volume of the weather balloon at the altitude above sea level *h*.

1) Express the volume $V(h)$ in terms of the altitude above sea level *h*.

$V(h) =$ ____ with *h* in m, $V(h)$ in m^3

The weather balloon bursts at an altitude above sea level of $h = 27\,873.6$ m.

2) Calculate the diameter *d* of the weather balloon in m at which the balloon bursts.

c) The so-called *total ozone* is a measure for the thickness of the ozone layer and is measured in *Dobson-units* (DU).

The data collected by a weather balloon can be modelled by a quadratic function f, whereby f assigns the total ozone density $f(h)$ to the height h (h in km, $f(h)$ in DU/km).

The highest value of 36 DU/km is measured at an altitude of 22 km above sea level. At an altitude of 37 km above sea level, the value measured is 1 DU/km.

1) [A] Determine $f(h)$.

$f(h) =$ ____

In the earth's atmosphere 1 DU corresponds to a 0.01 mm thick layer of pure ozone on the earth's surface. The thickness of that layer of pure ozone on the earth's surface, which is the same as the total ozone between 7 km and 37 km altitude above sea level, is equal to $\int_7^{37} f(h)\,\mathrm{d}h$.

2) Calculate the thickness of this layer.

Thickness of the layer: ____ mm