Exemplar für Prüfer/innen

Supplementary Examination for the
Standardised Competence-Oriented
Written School-Leaving Examination

AHS

October 2020

Mathematics

Supplementary Examination 2
Examiner's Version

Bundesministerium
Bildung, Wissenschaft
und Forschung

Instructions for the supplementary examination

The following supplementary examination is comprised of five tasks that can be completed independently of one another.

Each task contains two parts: The statement of the task requires the candidate to demonstrate core competencies, and the guiding question that follows it requires the candidate to show their ability to communicate their ideas.

In the following document, the examiner will find the tasks as well as the expected solutions and the answer key.

The preparation time shall be at least 30 minutes and the examination time shall be at most 25 minutes.

Assessment

Each task can be awarded zero, one or two points. There is one point available for each demonstration of core competencies as well as for each guiding question. A maximum of 10 points can be achieved.

The following scale will be used for the grading of the examination:

Grade	Number of points
Pass	4 points for the core competencies + 0 points for the guiding questions 3 points for the core competencies + 1 point for the guiding questions
Satisfactory	5 points for the core competencies + 0 points for the guiding questions 4 points for the core competencies + 1 point for the guiding questions 3 points for the core competencies + 2 points for the guiding questions
Good	5 points for the core competencies + 1 point for the guiding questions 4 points for the core competencies + 2 points for the guiding questions 3 points for the core competencies + 3 points for the guiding questions
Very good	5 points for the core competencies + 2 (or more) points for the guiding questions 4 points for the core competencies + 3 (or more) points for the guiding questions

The examination board will decide on the final grade based on the candidate's performance in the supplementary examination as well as the result of the written examination.

Evaluation grid for the supplementary examination

This evaluation grid may be used to assist the examiner in assessing the candidate's performance.

	Point for core competencies reached	Point for the guiding question reached
Task 1		
Task 2		
Task 3		
Task 4		
Task 5		

Task 1

Flight Paths

The three aeroplanes F_1, F_2 and F_3 fly at the same height for a particular period of time. Their flight paths over this period of time can be modelled by the three lines f_1, f_2 and f_3.

The following statements hold: f_1: $X = A + r \cdot \vec{v}_1$ with $r \in \mathbb{R}^+$
f_2: $X = B + s \cdot \vec{v}_2$ with $s \in \mathbb{R}^+$
f_3: $X = C + u \cdot \vec{v}_3$ with $u \in \mathbb{R}^+$

At the time $t = 0$ the aeroplane F_1 is at point $A = (a, 40)$ with $a \in \mathbb{R}$ and the aeroplane F_2 is at point $B = (–30, 20)$. The velocity vectors of the aeroplanes are given by $\vec{v}_1 = \vec{v}_3 = \begin{pmatrix} 10 \\ -10 \end{pmatrix}$ and $\vec{v}_2 = \begin{pmatrix} 5 \\ b \end{pmatrix}$ with $b \in \mathbb{R}$.
The parameters r, s and u give the flight times in minutes from the time $t = 0$. The velocities of the aeroplanes are given in km/min.

Task:

– Determine the values of a and b for which the flight paths of F_1 and F_2 are identical.

Guiding question:

At the time $t = 0$ the aeroplane F_3 is at point $C = (–20, 40)$.

– For this case, determine the value of b such that the flight paths of F_2 and F_3 cross each other at right angles.
– Determine the point S of intersection of these flight paths and justify why there is no collision between the two aeroplanes.

Solution to Task 1

Flight Paths

Expected solution to the statement of the task:

possible method:

$f_1\colon X = \begin{pmatrix} a \\ 40 \end{pmatrix} + r \cdot \begin{pmatrix} 10 \\ -10 \end{pmatrix}$

$f_2\colon X = \begin{pmatrix} -30 \\ 20 \end{pmatrix} + s \cdot \begin{pmatrix} 5 \\ b \end{pmatrix}$

$\vec{v}_1 = \begin{pmatrix} 10 \\ -10 \end{pmatrix}$ is parallel to $\vec{v}_2 = \begin{pmatrix} 5 \\ b \end{pmatrix} \quad \Rightarrow \quad b = -5$

$\begin{pmatrix} -30 \\ 20 \end{pmatrix} = \begin{pmatrix} a \\ 40 \end{pmatrix} + r \cdot \begin{pmatrix} 10 \\ -10 \end{pmatrix} \quad \Rightarrow \quad r = 2,\ a = -50$

$a = -50$
$b = -5$

Answer key:

The point for the core competencies is to be awarded if the values of *a* and *b* have been correctly determined.

Expected solution to the guiding question:

possible method:

$\vec{v}_2 \cdot \vec{v}_3 = 0 \quad \Rightarrow \quad \begin{pmatrix} 5 \\ b \end{pmatrix} \cdot \begin{pmatrix} 10 \\ -10 \end{pmatrix} = 0 \quad \Rightarrow \quad b = 5$

$\begin{pmatrix} -20 \\ 40 \end{pmatrix} + u \cdot \begin{pmatrix} 10 \\ -10 \end{pmatrix} = \begin{pmatrix} -30 \\ 20 \end{pmatrix} + s \cdot \begin{pmatrix} 5 \\ 5 \end{pmatrix} \quad \Rightarrow \quad u = 0.5;\ s = 3 \quad \Rightarrow \quad S = (-15, 35)$

There is no collision because the flight times until point *S* (with $u = 0.5$ min and $s = 3$ min) are not the same.

Answer key:

The point for the guiding question is to be awarded if the value of the parameter *b* and the point *S* of intersection have been correctly determined and a correct justification has been given.

Task 2

Triangle

For an $a \in \mathbb{R}^+$, the linear function g with $g(x) = -2 \cdot a \cdot x + 2$ is given. A triangle is bounded by the graph of g and the two coordinate axes.

Task:

– Write down the area A of this triangle in terms of a.

$A(a) =$ ____________________

Guiding question:

A change in the value of a results in a change of the area A of the triangle.

– Write down how A changes when a is doubled.

– Write down by which percentage A changes if a is reduced by 20 %.

Solution to Task 2

Triangle

Expected solution to the statement of the task:

possible method:
Points of intersection with the axes: $\left(\frac{1}{a}, 0\right)$ and $(0, 2)$
$A(a) = \frac{1}{a}$

Answer key:

The point for the core competencies is to be awarded if a correct function has been given.

Expected solution to the guiding question:

possible method:
$A(2 \cdot a) = \frac{1}{2 \cdot a} = \frac{1}{2} \cdot \frac{1}{a}$
If a is doubled, then A is halved.
$A(0.8 \cdot a) = \frac{1}{0.8 \cdot a} = 1.25 \cdot \frac{1}{a}$
If a is reduced by 20 %, then A becomes 25 % larger.

Answer key:

The point for the guiding question is to be awarded if both of the changes in area have been given correctly.

Task 3

Examination

In a school, class 8a has 27 pupils and class 8b has 24 pupils.

The last examination was held simultaneously in all 8th classes and all students from 8a and 8b were present. In 8a, <u>one</u> more “very good” grade was awarded than in 8b. The relative proportion of “very good” grades was the same in both classes.

Task:

– Determine the number of examinations that were awarded the grade “very good” in 8a.

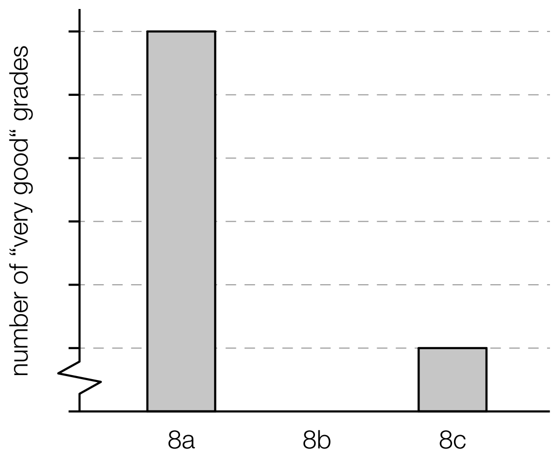
Guiding question:

A group of 9 pupils from 8c also completed this examination.

In total, 35 % of all pupils that took this examination were awarded a “very good” grade.

– Write down the number of pupils in 8c who were awarded a “very good” grade for this examination.

The bar chart shown below shows a graphical representation of the results of this examination.



– In the diagram above, label the vertical axis with a scale so that the situation described is represented correctly and complete the diagram by drawing the bar for the results in 8b.

– Write down a reason why this diagram could be regarded as biased.

Solution to Task 3

Examination

Expected solution to the statement of the task:

possible method:

n ... number of “very good” grades in class 8a
$n - 1$ ... number of “very good” grades in class 8b

$$\frac{n}{27} = \frac{n-1}{24} \quad \Rightarrow \quad n = 9$$

Nine examinations from 8a were awarded a “very good” grade.

Answer key:

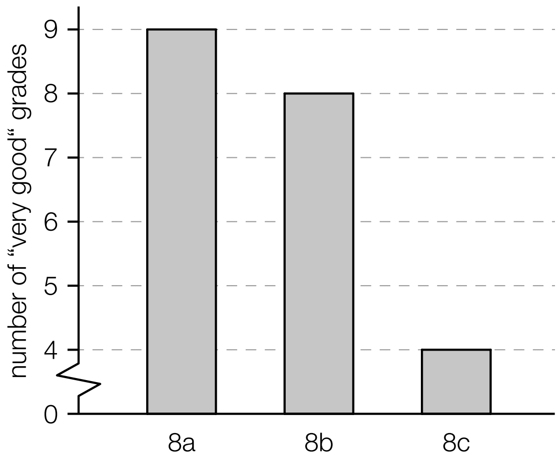
The point for the core competencies is to be awarded if the number of “very good” grades has been correctly determined.

Expected solution to the guiding question:

possible method:

$0.35 \cdot (27 + 24 + 9) = 21$
$21 - 9 - 8 = 4$

Four pupils in 8c were awarded a ”very good” grade.



possible reasons:

As only 9 pupils from class 8c participated in the examination, the relative proportions would be more informative than the absolute frequencies.

or:

The contraction of the vertical axis gives the impression that a lot fewer “very good” grades were awarded in 8c than in 8a.

Answer key:

The point for the guiding question is to be awarded if the correct number of pupils has been given, the diagram has been completed correctly, and a correct reason has been given.

Task 4

Integral

Task:

Let f be a linear (non-constant) function for which $\int_0^3 f(x)\,\mathrm{d}x = 0$ holds.

– Write down the zero of f and justify your answer.

Guiding question:

Let g be a quadratic function with $g(x) = a \cdot x^2 + b$ $(a, b \in \mathbb{R}, a \neq 0)$ for which $\int_0^3 g(x)\,\mathrm{d}x = 0$ holds.

– Based on the behaviour of the graph of g, justify why a and b must have different signs.

– Write down the relationship between a and b by using an equation.

Solution to Task 4

Integral

Expected solution to the statement of the task:

The zero of *f* is 1.5.

The graph of *f* must enclose two equally large areas with the *x*-axis (one above the *x*-axis and the other below the *x*-axis). Therefore the zero must lie exactly in the middle of the interval [0, 3].

Answer key:

The point for the core competencies is to be awarded if the correct zero and a correct justification have been given.

Expected solution to the guiding question:

The parabola must cross the *x*-axis (between 0 and 3). Either the parabola is concave up and the vertex is below the *x*-axis ($a > 0$, $b < 0$), or the parabola is concave down and the vertex is above the *x*-axis ($a < 0$, $b > 0$).

$$\int_0^3 g(x)\,\mathrm{d}x = 0$$

$$\int_0^3 (a \cdot x^2 + b)\,\mathrm{d}x = \frac{a \cdot x^3}{3} + b \cdot x \Big|_0^3$$

$$\Rightarrow \quad 3 \cdot a + b = 0$$

Answer key:

The point for the guiding question is to be awarded if a correct justification and a correct equation have been given.

Task 5

Driving Test

The theoretical part of the driving test comprises solely multiple-choice questions. Each question has four possible answers, and at least one of these possible answers is correct. A question is considered to be answered if at least one possible answer has been selected. A question has been answered correctly if the correct possible answer(s) has/have been selected.

Task:

– Determine the number of possible ways that a multiple choice question of this type could be answered.

Guiding question:

Assume that Elias has to guess for every question of the theoretical part of his driving test and for each question chooses one way of answering the question at random from all the possible ways that a question can be answered. The probability that he chooses one particular way of answering a question is the same for all possible ways.

– Write down the probability that Elias answers a multiple choice question correctly.

For the theoretical part of the driving test, 20 multiple choice questions about *basic knowledge* are asked first.

– Determine the probability that Elias answers less than 20 % of the multiple choice questions about *basic knowledge* correctly and therefore has to attend the theory course again.

Solution to Task 5

Driving Test

Expected solution to the statement of the task:

$\binom{4}{1} + \binom{4}{2} + \binom{4}{3} + \binom{4}{4} = 4 + 6 + 4 + 1 = 15$

There are 15 possible ways of answering a multiple choice question of this type.

Answer key:

The point for the core competencies is to be awarded if the number has been calculated correctly.

Expected solution to the guiding question:

$p = \frac{1}{15}$

The number X of multiple choice questions that have been answered correctly is binomially distributed with parameters $n = 20$ and $p = \frac{1}{15}$.

$P(X < 4) = P(X \leq 3) = 0.959... \approx 96\ \%$

Answer key:

The point for the guiding question is to be awarded if the probabilities have been determined correctly.