Name:	
Class:	

Standardised Competence-Oriented
Written School-Leaving Examination

AHS

16th September 2020

Mathematics

Bundesministerium
Bildung, Wissenschaft
und Forschung

Advice for Completing the Tasks

Dear candidate!

The following booklet contains Part 1 tasks and Part 2 tasks (divided into sub-tasks). The tasks can be completed independently of one another. You have a total of *270 minutes* available in which to work through this booklet.

Please do all of your working out solely in this booklet and the paper provided to you. Write your name and that of your class on the cover page of the booklet in the spaces provided. Also, write your name and consecutive page numbers on each sheet of paper used. When answering each sub-task, indicate its name/number on your sheet.

In the assessment of your work, everything that is not crossed out will be considered. Your solutions must be marked clearly. If a solution is not clearly marked or if more than one solution is given, the task will be considered to be unsolved.

The use of the official formula booklet that has been approved by the relevant government authority is allowed. Furthermore, the use of electronic device(s) (e.g. graphic display calculators or other appropriate technology) is allowed provided there is no possibility to communicate via internet, Bluetooth, mobile networks, etc. and there is no access to your own data stored on the device.

An explanation of the task types is available in the examination room and can be viewed on request.

Please hand in the task booklet and all used sheets at the end of the examination.

Changing an answer for a task that requires a cross:

1. Fill in the box that contains the cross.
2. Put a cross in the box next to your new answer.

In this instance, the answer "5 + 5 = 9" was originally chosen. The answer was later changed to be "2 + 2 = 4".

1 + 1 = 3	☐
2 + 2 = 4	☒
3 + 3 = 5	☐
4 + 4 = 4	☐
5 + 5 = 9	■

Selecting an item that has been filled in:

1. Fill in the box that contains the cross for the answer you do not wish to give.
2. Put a circle around the filled-in box you would like to select.

In this instance, the answer "2 + 2 = 4" was filled in and then selected again.

1 + 1 = 3	☐
2 + 2 = 4	(■)
3 + 3 = 5	☐
4 + 4 = 4	■
5 + 5 = 9	☐

Assessment

The tasks in Part 1 will be awarded either 0 points or 1 point or 0, ½ or 1 point, respectively. The points that can be reached in each task are listed in the booklet for all Part 1 tasks. Every sub-task in Part 2 will be awarded 0, 1 or 2 points. The tasks marked with an [A] will be awarded either 0 points or 1 point.

Two assessment options

1) If you have reached **at least 16** of the 28 points (24 Part 1 points + 4 [A] points from Part 2), a grade will be awarded as follows:

Pass	16–23.5 points
Satisfactory	24–32.5 points
Good	33–40.5 points
Very Good	41–48 points

2) If you have reached **fewer than 16** of the 28 points (24 Part 1 points + 4 [A] points from Part 2), but have reached a **total of 24 points or more** (from Part 1 and Part 2 tasks), then a "Pass" or "Satisfactory" grade is possible as follows:

Pass	24–28.5 points
Satisfactory	29–35.5 points

If you have reached fewer than 16 points in Part 1 (including the compensation tasks marked with an [A] from Part 2) and if the total is less than 24 points, you will not pass the examination.

Good luck!

Task 1

Calculation Operations

Let a and b be two natural numbers for which $b \neq 0$ holds.

Task:

Put a cross next to each of the two expressions that always result in a natural number.

$a + b$	☐
$a - b$	☐
$\frac{a}{b}$	☐
$a \cdot b$	☐
$\sqrt[a]{b}$	☐

[0/1 point]

Task 2

Active Ingredient

A particular medication is consumed in liquid form. Per millilitre of liquid, there are 30 milligrams of the active ingredient. Martin consumes 85 millilitres of this medication. 10 % of the active ingredient reaches his bloodstream.

Task:

Write down how many milligrams of this active ingredient reach Martin’s bloodstream.

____________ milligrams of the active ingredient reach Martin’s bloodstream.

[0/1 point]

Task 3

Movement of a Body

A body moves in a straight line with a constant velocity of 8 m/s and covers 100 m.

Task:

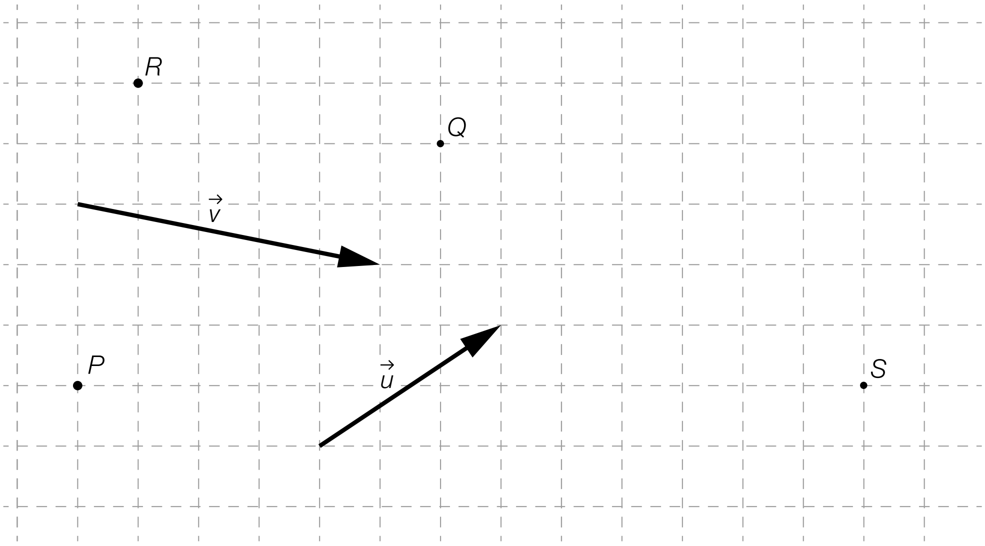
Interpret the solution to the equation $8 \cdot x - 100 = 0$ in the given context.

[0/1 point]

Task 4

Vectors

The diagram below shows four points P, Q, R and S as well as two vectors $\vec{u}$ and $\vec{v}$.



Task:

Match each of the four vectors to the corresponding expression (from A to F).

$\overrightarrow{PQ}$	
$\overrightarrow{PR}$	
$\overrightarrow{QR}$	
$\overrightarrow{PS}$	

A	$2 \cdot \vec{u} - \vec{v}$
B	$2 \cdot \vec{v} - \vec{u}$
C	$-\vec{v}$
D	$2 \cdot \vec{v} + \vec{u}$
E	$2 \cdot \vec{u}$
F	$2 \cdot \vec{u} + 2 \cdot \vec{v}$

[0/½/1 point]

Task 5

Lines in $\mathbb{R}^2$

For two lines g and h in $\mathbb{R}^2$, the following statements hold:

- The line g with direction vector $\vec{g}$ has the normal vector $\overrightarrow{n_g}$.
- The line h with direction vector $\vec{h}$ has the normal vector $\overrightarrow{n_h}$.
- The lines g and h are perpendicular to each other.

Task:

Put a cross next to each of the two statements that are always true.

$\overrightarrow{n_g} \cdot \vec{h} = 0$	☐
$\overrightarrow{n_g} \cdot \overrightarrow{n_h} = 0$	☐
$\vec{g} = r \cdot \vec{h}$ with $r \in \mathbb{R}\backslash\{0\}$	☐
$\vec{g} = r \cdot \overrightarrow{n_h}$ with $r \in \mathbb{R}\backslash\{0\}$	☐
$\vec{g} \cdot \overrightarrow{n_h} = 0$	☐

[0/1 point]

Task 6

Ladder

A 4 m long straight ladder is placed on a horizontal surface and is leaned against the vertical wall of a house.

The ladder must make an angle of between 65° and 75° with the floor in order to avoid either toppling over or slipping.

Task:

Determine the minimum distance and the maximum distance between the lower end of the ladder and the wall of the house.

minimum distance from the wall of the house: __________ m

maximum distance from the wall of the house: __________ m

[0/½/1 point]

Task 7

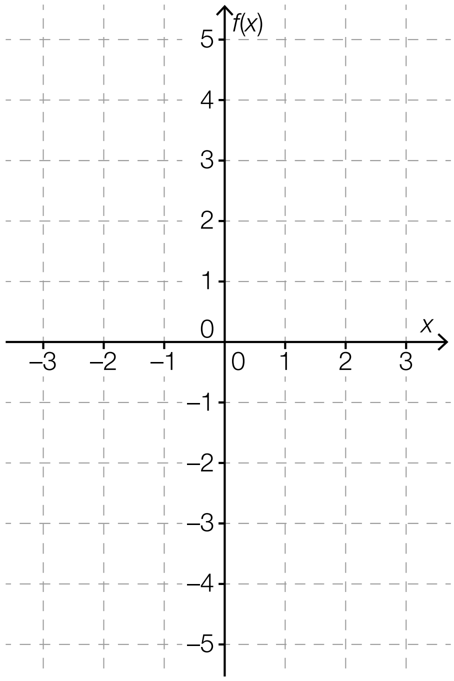
Graph of a Polynomial Function

A polynomial function $f: [-3, 3] \to \mathbb{R}, x \mapsto f(x)$ has the following properties:

- The graph of f is symmetrical about the vertical axis.
- The function f has a local minimum at the point (2,1).
- The graph of f crosses the vertical axis at the point (0,3).

Task:

Sketch the graph of one such function f in the coordinate system shown below over the interval [–3, 3].



[0/1 point]

Task 8

Feed Requirement

Horses are kept in a stable for t days. The daily feed requirement for each of these horses is assumed to be constant and is represented by c.

The function f describes the total feed requirement $f(p)$ for t days in terms of the number p of horses in this stable.

Task:

Put a cross next to the correct equation.

$f(p) = p + t + c$	☐
$f(p) = c + p \cdot t$	☐
$f(p) = c \cdot \frac{t}{p}$	☐
$f(p) = \frac{c}{p \cdot t}$	☐
$f(p) = c \cdot p \cdot t$	☐
$f(p) = \frac{p \cdot t}{c}$	☐

[0/1 point]

Task 9

Power Function

Let $f\colon \mathbb{R}\setminus\{0\} \to \mathbb{R}$ with $f(x) = \frac{a}{x^2}$ with $a \in \mathbb{R}\setminus\{0\}$ be a power function.

Task:

Put a cross next to each of the two statements that are always true about the function *f*.

$f\left(\frac{1}{a}\right) = 1$	☐
$f(x + 1) = \frac{a}{x^2 - 2 \cdot x + 1}$	☐
$f(2 \cdot x) = \frac{a}{4 \cdot x^2}$	☐
$f(2 \cdot a) = \frac{1}{2 \cdot a}$	☐
$f(-x) = f(x)$	☐

[0/1 point]

Task 10

Pressure and Volume of an Ideal Gas

When the temperature remains constant, the pressure and the volume of an ideal gas are indirectly proportional to each other. The function p assigns the pressure $p(V)$ to the volume V (V in m^3, $p(V)$ in pascals).

Task:

Write down $p(V)$ where $V \in \mathbb{R}^+$ if the volume is 4 m^3 at a pressure of 50 000 pascals.

$p(V) =$ ____

[0/1 point]

Task 11

Half-Life

The function f with $f(t) = 80 \cdot b^t$ with $b \in \mathbb{R}^+$ describes the mass $f(t)$ of a radioactive substance in terms of the time t (t in h, $f(t)$ in mg). The half-life of the radioactive substance is 4 h.
The substance is observed from time $t = 0$.

Task:

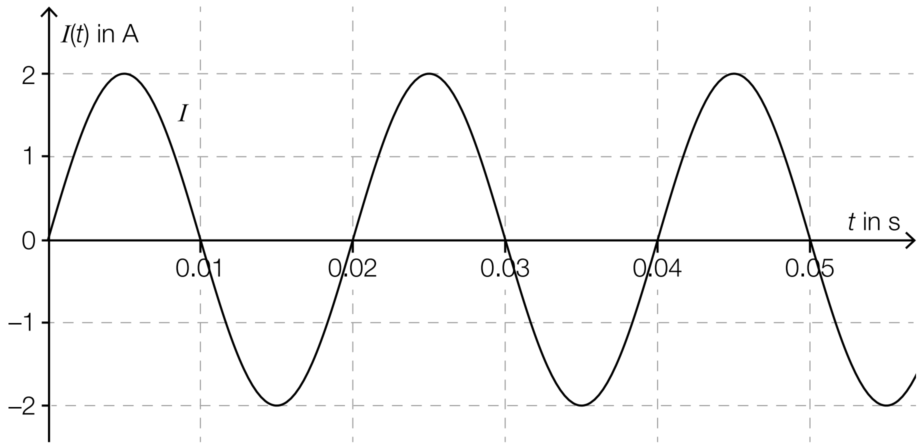
Determine the mass (in mg) of the radioactive substance that remains after the first 3 half-lives.

[0/1 point]

Task 12

Alternating Current

For sinusoidal alternating current, the value of the current changes periodically.
The diagram below shows the current $I(t)$ in terms of the time t for a sinusoidal alternating current (t in s, $I(t)$ in A).



Task:

Write down the maximum value of the current and the length of one period.

maximum value: _______________ A

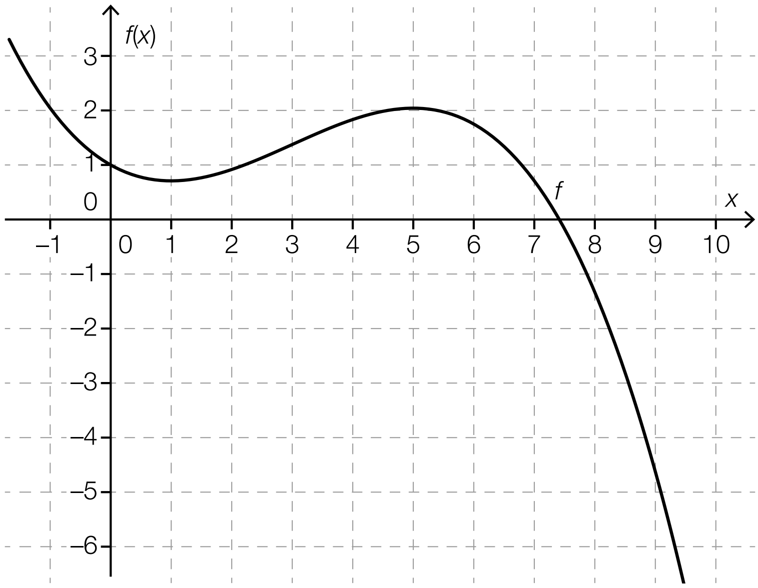
length of one period: _______________ s

[0/½/1 point]

Task 13

Difference Quotient and Differential Quotient

The diagram below shows the graph of a third degree polynomial function f.



Task:

Put a cross next to each of the two correct statements.

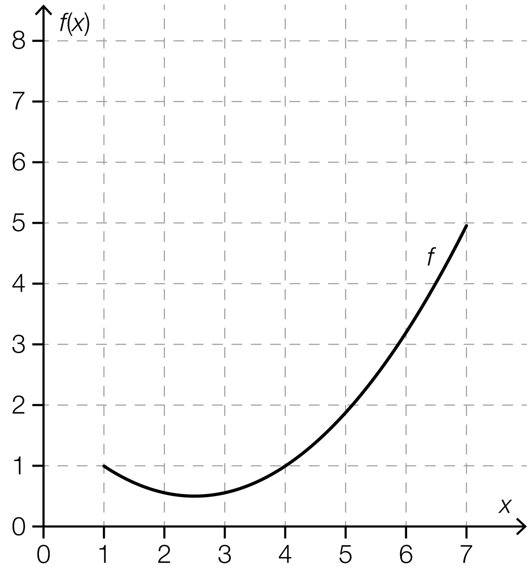
In the interval (0, 2), there is a point a such that the following holds: $\frac{f(a) - f(0)}{a - 0} = f'(0)$	☐
In the interval (4, 6), there is a point a such that the following holds: $\frac{f(a) - f(0)}{a - 0} = f'(0)$	☐
For all $a \in (0, 1)$ the following statement holds: The smaller a is, the less $\frac{f(a) - f(0)}{a - 0}$ differs from $f'(0)$.	☐
For all $a \in (2, 5)$ the following statement holds: The larger a is, the less $\frac{f(a) - f(0)}{a - 0}$ differs from $f'(0)$.	☐
For all $a \in (2, 3)$ the following holds: $\frac{f(a) - f(0)}{a - 0} > f'(0)$	☐

[0/1 point]

Task 14

Rates of Change

The diagram below shows the graph of a function *f* over the interval [1, 7].



Task:

On the diagram above, draw the point *P* on the graph of *f* for which the differential quotient of the function *f* corresponds to the difference quotient over the interval [1, 7].

[0/1 point]

Task 15

Bacterial Culture

The number of bacteria in a bacterial culture in terms of the time t is investigated. The number of bacteria in this bacterial culture increases each minute by the same percentage.
In the equations shown below, $N(t)$ is the number of bacteria in this bacterial culture at time t (in minutes) and $k \in (0, 1)$ is a real number.

Task:

Put a cross next to each of the two correct equations.

$N(t + 1) - N(t) = -k \cdot N(t)$	☐
$N(t + 1) - N(t) = k$	☐
$N(t + 1) - N(t) = k \cdot N(t)$	☐
$N(t + 1) = k \cdot N(t)$	☐
$N(t + 1) = N(t) \cdot (1 + k)$	☐

[0/1 point]

Task 16

Antiderivative

Let f: $\mathbb{R} \to \mathbb{R}$, $x \mapsto f(x)$ be a function.
The function g: $\mathbb{R} \to \mathbb{R}$, $x \mapsto g(x)$ is an antiderivative of f.
For a function h: $\mathbb{R} \to \mathbb{R}$, $x \mapsto h(x)$ and $c \in \mathbb{R}\backslash\{0\}$ the statement $h(x) = g(x) + c$ holds.

Task:

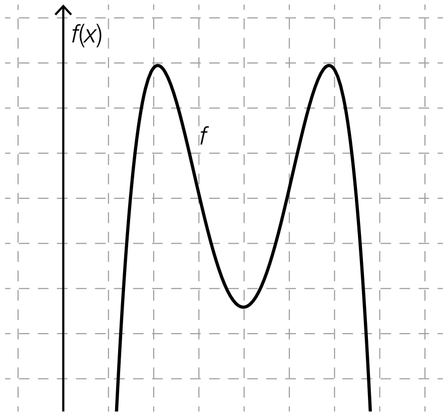
Write down whether h is also an antiderivative of f and justify your answer.

[0/1 point]

Task 17

Polynomial Function

The diagram below shows the graph of a fourth degree polynomial function $f: x \mapsto f(x)$. The x-axis has not been included in the diagram.



Task:

Put a cross next to each of the two statements that are true for the polynomial function f shown above regardless of the position of the x-axis.

There are exactly two x-values x_1 and x_2 for which $f(x_1) = 0$ and $f(x_2) = 0$.	☐
There are exactly two x-values x_1 and x_2 for which $f'(x_1) = 0$ and $f'(x_2) = 0$.	☐
There is exactly one x-value x_1 for which $f''(x_1) = 0$.	☐
There is exactly one x-value x_1 for which $f'(x_1) = 0$ and $f''(x_1) > 0$.	☐
There is exactly one x-value x_1 for which $f'(x_1) > 0$ and $f''(x_1) = 0$.	☐

[0/1 point]

Task 18

Velocity Function

The function v with $v(t) = 0.5 \cdot t + 2$ assigns each point in time t to the velocity $v(t)$ of a body (t in s, $v(t)$ in m/s).

The following calculation is carried out:

$$\int_1^5 (0.5 \cdot t + 2)\,\mathrm{d}t = 14$$

Task:

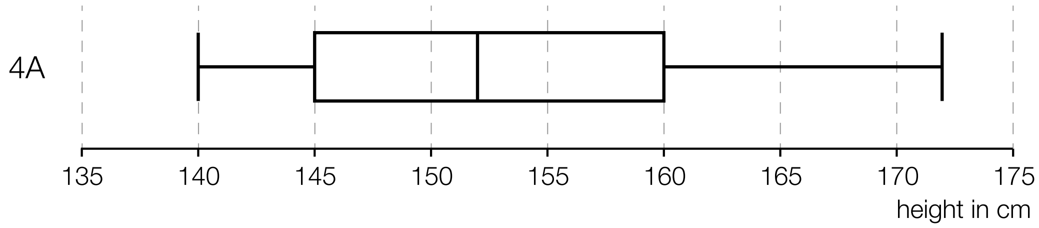
With regard to the movement of the body, write down a question that can be answered with the calculation shown above.

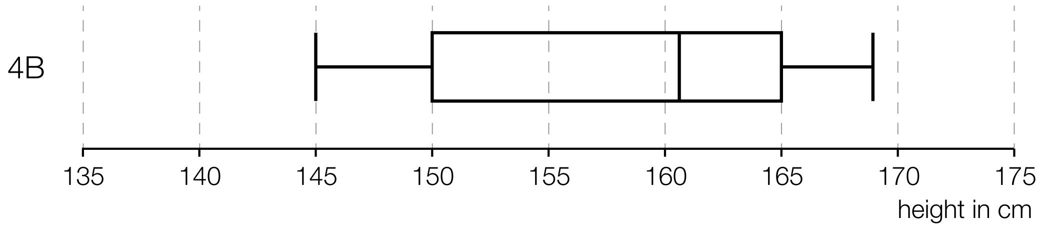
[0/1 point]

Task 19

Boxplots of Heights

The boxplots below show the distributions of the heights of the pupils in two classes (4A and 4B).
There is the same number of pupils in both classes.





Task:

Put a cross next to each of the two statements that are definitely true.

In 4A, more than half of the pupils are shorter than 150 cm.	☐
In 4B, more pupils are taller than 160 cm than in 4A.	☐
The range of the heights in 4A is larger than in 4B.	☐
The tallest pupil across both classes is in 4B.	☐
In 4A, the most common height is 160 cm.	☐

[0/1 point]

Task 20

Estimate of a Probability

A dice with faces that show the numbers 1, 2, 3, 4, 5, and 6 has one corner that is damaged. Therefore, it is assumed that the probability of rolling a particular number is not the same for every number.
A person conducted two separate series of throws in which they rolled the dice 50 times each. The absolute frequencies of the numbers that were rolled were recorded. The table below shows these recordings.

number shown on the dice	1	2	3	4	5	6
frequency in series 1	7	8	7	10	8	10
frequency in series 2	6	9	7	9	10	9

Task:

Based on the results of both series of throws, write down an estimate for the probability p (as a %) of rolling a 6 with this dice.

p = ____ %

[0/1 point]

Task 21

Test Tasks

For an international comparative study, a large number of test tasks have been created. Experience has shown that 20 % of the tasks are discarded during a preliminary evaluation process due to the design of the tasks. The rest of the tasks undergo a second evaluation process. Experience has shown that 10 % of these tasks are discarded during this process due to the task content.

Task:

Determine the probability that a task will be discarded.

[0/1 point]

Task 22

Binomial Coefficient

A group contains 12 school pupils.

Task:

Complete the gaps in the following sentence by putting a cross next to one of the given possibilities for each gap so that the sentence becomes a correct statement.

The binomial coefficient $\binom{12}{2}$ has the value ___①___; it can be used to determine the number of different possibilities of ___②___.

①	
24	☐
66	☐
144	☐

②	
selecting 2 pupils from this group who should hold a presentation together	☐
awarding 2 pupils from this group 2 different prizes	☐
dividing the pupils into 2 groups of 6 pupils each	☐

[0/½/1 point]

Task 23

Tossing a Coin

After being tossed, a coin shows either *heads* or *tails*. For each toss, the probability of the coin showing *heads* is exactly the same as the probability of the coin showing *tails*. The results of the tosses are independent of each other. The coin is tossed 20 times.

Task:

Determine the probability that the coin shows *heads* exactly 12 times during these 20 tosses.

[0/1 point]

Task 24

Confidence Interval

Based on the relative sample frequency h from a representative survey of 500 people, the 95 % confidence interval $[h - 0.04, h + 0.04]$ for the unknown relative proportion of the supporters of a bypass is determined.

A second representative survey of 2000 people gives the same relative sample frequency h.

Task:

For this second survey, write down the 95 % confidence interval that is symmetrical about h for the unknown relative proportion of the supporters of the bypass.

[0/1 point]

Task 25 (Part 2)

Solar Thermal Power Station

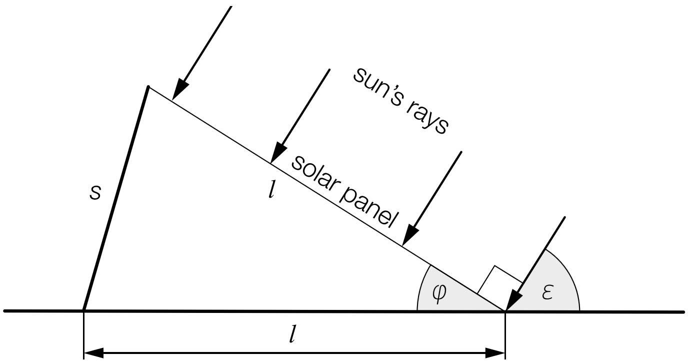
At a solar thermal power station, so-called *solar panels* convert sunlight into heat. This heat can be used to generate hot water or to heat buildings, for example.

Task:

a) One solar panel in a solar thermal power station with length l creates an angle φ with the horizontal ground. This angle φ can be changed by a support of variable length s so that the solar panel is perpendicular to the sun's rays.

The sun's rays make an angle ε with the ground.

A model of the situation is shown in the diagram below.



1) [A] Write down a formula that can be used to calculate s by using l and ε.

s = ____

The solar panel shown above has a length l = 1666 mm. Over the course of a particular day, the angle ε takes values between 14° and 65° for this solar panel.

2) Write down the maximum value of s.

maximum value of s: ____ mm

b) The power generated by a particular solar thermal power station on a cloudless day is modelled by the function P, for which:

$P(t) = 0.0136 \cdot a^3 \cdot t^4 - 0.272 \cdot a^2 \cdot t^3 + 1.36 \cdot a \cdot t^2$

t ... time in h elapsed since sunrise ($t = 0$)
$P(t)$ ... power in kW at time t
a ... parameter

At sunrise and sunset, the power generated by the solar thermal power station is 0 kW. Between sunrise and sunset, the values of the function P are positive.

1) For this solar thermal power station, determine the value of the parameter a for a particular cloudless day on which the sun rises at 7:08 and sets at 18:38.

The work done by the solar thermal power station between two points in time t_1 and t_2 is given by $\int_{t_1}^{t_2} P(t)\,dt$.

2) Determine the work done (in kWh) by the solar thermal power station on this day.

Task 26 (Part 2)

Petrol Consumption

The petrol consumption of a particular small car in terms of its velocity can be modelled by the function B.

$B(v) = 0.000483 \cdot v^2 - 0.0326 \cdot v + 2.1714 + \frac{66}{v}$ with $20 < v < 150$
v ... velocity in km/h
$B(v)$ ... petrol consumption in litres per 100 km (L/100 km) for the velocity v

Task:

a) 1) Determine the percentage by which the petrol consumption increases when the velocity increases from 70 km/h to 90 km/h.

____ %

The petrol consumption for a velocity of 40 km/h is 25 % lower than the petrol consumption for a velocity v_1 where $20 < v_1 < 40$.

2) Determine the velocity v_1.

v_1 = ____ km/h

b) For higher velocities, the function B is to be approximated by a linear function f where $f(v) = k \cdot v + d$ with $k, d \in \mathbb{R}$ such that the following conditions hold:
$f(100) = B(100)$
$f(130) = B(130)$

1) [A] Determine an equation of the function f.

$f(v)$ = ____

This approximation can be used if the difference between the values of the functions f and B is at most 0.3 L/100 km.

2) Write down the largest possible interval for the velocity for which the function f can be used as an approximation.

c) 1) Using the function B, determine the velocity v_{min} at which the petrol consumption is lowest as well as the corresponding petrol consumption B_{min}.

v_{min} = ____________________ km/h

B_{min} = ____________________ L/100 km

The petrol consumption also depends on the tyre pressure.
The function g describes the petrol consumption in terms of the velocity v for a tyre pressure that is a little too low.
The following statement holds: $g(v) = 1.02 \cdot B(v)$

2) Using the function g, determine the two velocities at which the petrol consumption is 2 L/100 km higher than B_{min} when the tyre pressure is a little too low.

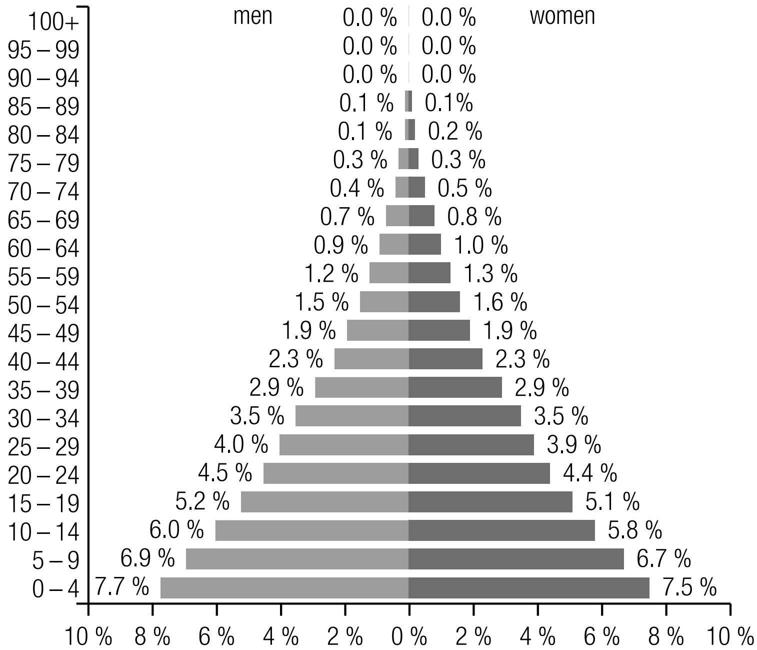
Task 27 (Part 2)

Population Growth in Africa

At the end of 2018, Africa had a population of around 1.3 billion people and displayed the highest level of population growth of all the continents.

Task:

a) The diagram below shows the age pyramid of the African population in the year 2018.
It can be read from the age pyramid, for example, that 4.5 % of the African population were men between the ages of 20 and 24 years and 4.4 % of the African population were women between the ages of 20 and 24 years in 2018. The *age* of a person is understood to be the number of complete years that a person has lived.



Data source: https://www.populationpyramid.net/de/afrika/2018 [10.05.2019].

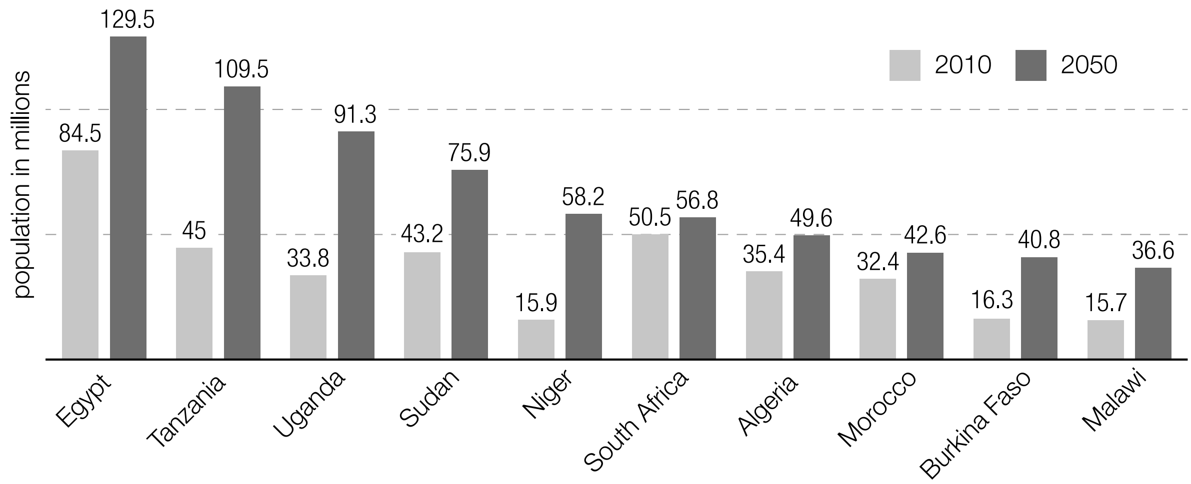
You should assume that each age within an age range occurs with equal frequency.

1) Using the age pyramid, determine the median m of the age of the African population in the year 2018.

$m =$ ____ years

2) Write down the number of African people who were younger than m years in 2018.

b) The diagram below shows the predicted population growth (given in millions) over the time period from 2010 to 2050 in a selection of African countries.



Data source: https://de.statista.com/statistik/daten/studie/159204/umfrage/prognose-zur-bevoelkerungsentwicklung-in-afrika-bis-2050/ [10.05.2019].

1) [A] Of the ten countries given, write down the country that will contribute most to the absolute population growth in Africa between 2010 and 2050 according to the prediction.

2) Of the ten countries given, write down the country in which there will be the largest relative population growth between 2010 and 2050 according to the prediction.

c) The table below shows the population growth for Nigeria in the time period from 1980 to 2010.

year	1980	1990	2000	2010
population in millions	73.5	95.3	122.4	158.6

1) Using the table, show that the population grew approximately exponentially in the time period from 1980 to 2010.

Assume that the population in Nigeria will continue to grow according to this exponential model.

2) Using the data from the years 2000 and 2010, write down the year in which the population of Nigeria will first be greater than 360 million.

d) The table below shows the development of the average life expectancy of the African population since 1953.

year	average life expectancy in years
1953	37.5
1958	40.0
1963	42.3
1968	44.4
1973	46.6
1978	48.7
1983	50.5
1988	51.7
1993	51.7
1998	52.3
2003	53.7
2008	57.0
2013	60.2
2018	62.4

1) Determine the average annual increase k of the average life expectancy in the time period from 1953 to 2018.

It is assumed that the average life expectancy in Africa after 2018 increases each year by the constant value k determined above.
In 2018, the average life expectancy in Europe was 78.5 years.

2) Based on this assumption, write down the year in which the average life expectancy in Africa would reach the value in Europe for 2018.

Task 28 (Part 2)

Security Check

At the entrance of a particular stadium, a security check of at most three stages is undertaken to check the items people are bringing in and to seize prohibited items. If the first stage of this security check does not yield a clear result, then the second stage of the security check is carried out. If there is still no clear result, the third stage of the security check is implemented.

The first and second stages of the security check both last 15 s. The third stage lasts 300 s. A clear result is obtained at the first stage with a probability of 90 %. The second stage gives a clear result with a probability of 60 %.

Task:

a) The random variable X describes the time taken d (in s) for the security check for one person. Any possible waiting times that may occur are not considered.

1) Complete the table below by writing down the probability distribution of the random variable X.

d			
$P(X = d)$			

2) Determine the expectation value $E(X)$.

b) The value p gives the probability that a person brings a prohibited item with them. The probability that 2 people who are selected at random and independently from each other are both carrying a prohibited item is 10 %.

1) [A] Determine the probability p.

2) Determine the probability that out of 10 people selected at random and independently from each other at least 5 people are carrying a prohibited item.

c) The instantaneous rate of change of the number of people in the stadium can be described in terms of the time t by the function A where $A(t) = a \cdot t^2 + b \cdot t + c$ with $a, b, c \in \mathbb{R}$ and $0 \leq t \leq 90$ (t in minutes, $A(t)$ in people per minute). Admittance to the stadium begins at time $t = 0$.

At time $t = 0$, no people are entering the stadium; 45 min after admittance begins, 15 people per minute are entering the stadium. At this time, the instantaneous rate of change of the number of people entering the stadium per minute is greatest.

1) Determine the values of a, b and c.

2) Write down the total number of people who have entered the stadium until the time $t = 90$.