Exemplar für Prüfer/innen

Supplementary Examination for the
Standardised Competence-Oriented
Written School-Leaving Examination

AHS

October 2023

Mathematics

Supplementary Examination 1
Examiner's Version

Bundesministerium
Bildung, Wissenschaft
und Forschung

Instructions for the standardized implementation of the supplementary examination

The following supplementary examination booklet contains four tasks that can be completed independently of one another as well as the corresponding solutions.

Each task comprises three competencies to be demonstrated.

The preparation time is to be at least 30 minutes; the examination time is at most 25 minutes.

The use of the official formula booklet that has been approved by the relevant government authority for use in the standardized school-leaving examination in mathematics is allowed. Furthermore, the use of electronic devices (e.g. graphic display calculators or other appropriate technology) is allowed provided there is no possibility to communicate (e.g. via the internet, intranet, Bluetooth, mobile networks etc.) and there is no access to an individual's data stored on the device.

After the examination, all materials (tasks, extra sheets of paper etc.) from the candidates are to be collected in. The examination materials (tasks, extra sheets of paper, data that has been produced digitally etc.) may only be made public after the time period allocated for the examination has passed.

Evaluation grid for the supplementary examination

The evaluation grid below may be used to assist in assessing the candidates' performances.

	Candidate 1			Candidate 2			Candidate 3			Candidate 4			Candidate 5		
Task 1															
Task 2															
Task 3															
Task 4															
Total															

Explanatory notes on assessment

Each task can be awarded zero, one, two or three points. A maximum of twelve points can be achieved.

Assessment scale for the supplementary examination

Total number of competencies demonstrated	Assessment of the oral supplementary examination
12	Very good
10–11	Good
8–9	Satisfactory
6–7	Pass
0–5	Fail

Task 1

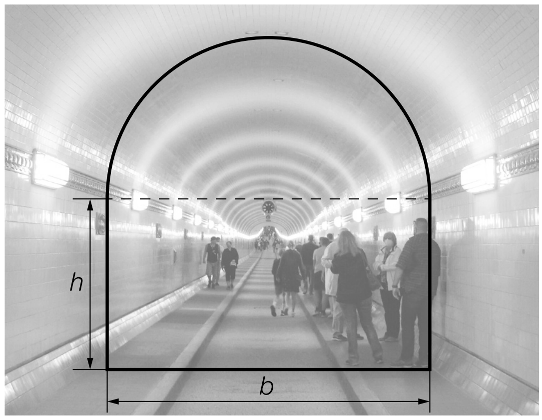
Old Elbe Tunnel

The Old Elbe Tunnel in Hamburg is an underpass for the Elbe.

a) The cross-section of the tunnel can be approximated by a rectangle with a semi-circle on top (see diagram on the right).

b ... width in m
h ... height in m

Daniel would like to calculate the volume of air V in the 426.5 m long Old Elbe Tunnel.

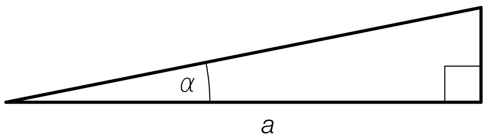


Source: BMBWF

1) Write down a formula that can be used to calculate V in terms of b and h.

V = ____

b) The diagram below shows a model of the gradient of a section of the cycle path in the tunnel.



a ... horizontal length of this section of the tunnel in m
α ... angle of elevation of this section of the tunnel

A cyclist cycles along this section of the tunnel with velocity v in m/s.

The following statement holds: $\frac{\frac{a}{\cos(\alpha)}}{v} = 12.5$

1) Interpret the value 12.5 in the given context including the corresponding unit.

c) In the first year after opening, 20 million people used the Old Elbe Tunnel. The number of people who used the Old Elbe Tunnel each year reduced by 97.5 % by 1985 and then increased again. In the year 2008, 40 % more people used the Old Elbe Tunnel than in the year 1985.

1) Determine the number of people who used the Old Elbe Tunnel in the year 2008.

Solution to Task 1

Old Elbe Tunnel

a1) $V = 426.5 \cdot \left(b \cdot h + \frac{1}{2} \cdot \left(\frac{b}{2}\right)^2 \cdot \pi\right)$

or:

$V = 426.5 \cdot \left(b \cdot h + \frac{b^2}{8} \cdot \pi\right)$

b1) The cyclist requires 12.5 s for this section of the tunnel.

c1) $20\,000\,000 \cdot 0.025 \cdot 1.4 = 700\,000$

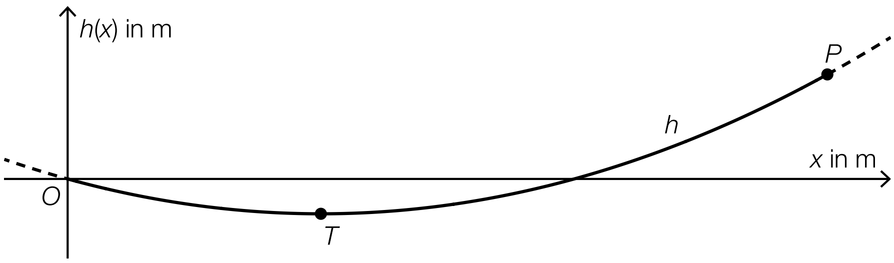
700 000 people used the Old Elbe Tunnel in the year 2008.

Task 2

Suspension Bridge

The shape of a particular suspension bridge for pedestrians can be modelled by quadratic functions.

a) In one model, the shape of the suspension bridge is described by the function h with $h(x) = a \cdot x^2 + b \cdot x$ (the side view is shown in the diagram below).



The graph of h goes through the point $P = (120, 6)$. The lowest point T of the bridge occurs when $x = 40$.

In order to determine the coefficients a and b, the system of equations shown below is created using the information about the points P and T.

1) Write down the missing numbers in the boxes provided.

I: $a \cdot \square^2 + b \cdot \square = \square$

II: $a \cdot \square + b = \square$

The following statement about the function h holds: $h(x) = 0.00125 \cdot x^2 - 0.1 \cdot x$

2) Determine the angle of elevation of the tangent to the graph of h at the point P.

In another coordinate system, the shape of the suspension bridge can be described by the function f with $f(x) = a \cdot x^2$.

3) In the diagram above, draw the coordinate axes for the graph of f.

Solution to Task 2

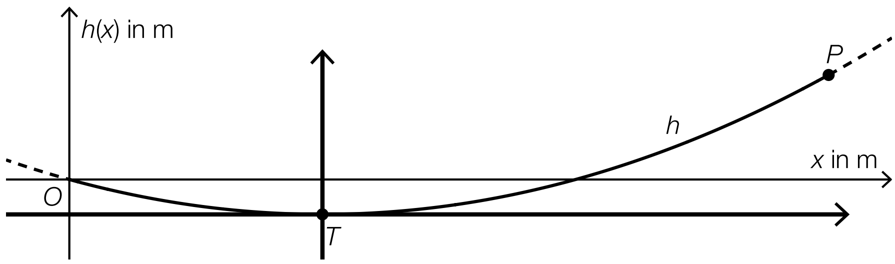
Suspension Bridge

a1) I: $a \cdot \boxed{120}^2 + b \cdot \boxed{120} = \boxed{6}$

II: $a \cdot \boxed{80} + b = \boxed{0}$

a2) $\alpha = \arctan(h'(120)) = \arctan(0.2)$
$\alpha = 11.30...°$

a3)



Task 3

Sporting Goods

a) For a particular item of sporting goods, the derivative K' of the cost function K is given.

$K'(x) = 3 \cdot x^2 - 8 \cdot x + 20$

x ... number of produced units of quantity in ME
$K'(x)$ ... 1st derivative of the cost function K for x ME in GE/ME, where GE are units of currency

The fixed costs are 4200 GE.

1) Write down an equation of the cost function K.

b) For another item of sporting goods, the cost function K_1 and the revenue function E_1 are given.

$K_1(x) = 0.01 \cdot x^2 + 10 \cdot x + 200$
$E_1(x) = -0.25 \cdot x^2 + 50 \cdot x$

x ... number of produced and sold units of quantity in ME
$K_1(x)$ ... total costs for x ME in units of currency, GE
$E_1(x)$ ... revenue for x ME in units of currency, GE

1) Determine the profit when $x = 70$ ME.

c) In a study it has been investigated how many units of a particular item of sporting goods can be sold long-term.
The number of units sold can be modelled by the function A in terms of time.

$A(t) = a - 30 \cdot b^t$ with $0 < b < 1$

t ... time in months with $t = 0$ for the start of sales
$A(t)$ ... number of units sold at time t
a, b ... parameters

1) Using the equation of the function of A, justify why no more than a units can ever be sold according to this model.

Solution to Task 3

Sporting Goods

a1) $K(x) = \int K'(x)\,\mathrm{d}x = 3 \cdot \frac{x^3}{3} - 8 \cdot \frac{x^2}{2} + 20 \cdot x + C = x^3 - 4 \cdot x^2 + 20 \cdot x + C$

$K(0) = 4\,200$

$C = 4\,200$

$K(x) = x^3 - 4 \cdot x^2 + 20 \cdot x + 4\,200$

b1) $G_1(x) = E_1(x) - K_1(x) = -0.26 \cdot x^2 + 40 \cdot x - 200$

$G_1(70) = 1\,326$

The profit for 70 ME is 1 326 GE.

c1) The expression $30 \cdot b^t$ is positive for all values of t and therefore the expression $a - 30 \cdot b^t$ can never take a greater value than a.

Task 4

Dice

In a particular game, players roll fair six-sided dice. The faces of these dice are labelled with the digits 1, 2, 3, ... , 6.

a) Andrea rolls a dice multiple times.

1) Write down a formula that can be used to calculate the probability P shown below.

P("Andrea does not roll a single 6 in n rolls") = ____________________

b) Ferdinand rolls 2 dice once.

He claims: "The probability that the sum of the faces is 5 is greater than the probability that the sum of the faces is 4."

1) Justify by calculation that Ferdinand's claim is correct.

c) Sabrina rolls 5 dice once.

1) Determine the probability that exactly 4 of the 5 dice show the same digit.

Solution to Task 4

Dice

a1) $P(E) = \left(\frac{5}{6}\right)^n$

b1) sum of the faces is 5: 1 + 4 *or* 2 + 3 *or* 3 + 2 *or* 4 + 1
sum of the faces is 4: 1 + 3 *or* 2 + 2 *or* 3 + 1
Therefore:
$P(X = 5) = \frac{4}{36}$
$P(X = 4) = \frac{3}{36}$
$\frac{4}{36} > \frac{3}{36}$

c1) $6 \cdot \binom{5}{4} \cdot \left(\frac{1}{6}\right)^4 \cdot \left(\frac{5}{6}\right) = 0.0192...$
The probability is around 1.9 %.