Exemplar für Prüfer/innen

Supplementary Examination for the
Standardised Competence-Oriented
Written School-Leaving Examination

AHS

January 2024

Mathematics

Supplementary Examination 1
Examiner's Version

Bundesministerium
Bildung, Wissenschaft
und Forschung

Instructions for the standardized implementation of the supplementary examination

The following supplementary examination booklet contains four tasks that can be completed independently of one another as well as the corresponding solutions.

Each task comprises three competencies to be demonstrated.

The preparation time is to be at least 30 minutes; the examination time is at most 25 minutes.

The use of the official formula booklet that has been approved by the relevant government authority for use in the standardized school-leaving examination in mathematics is allowed. Furthermore, the use of electronic devices (e.g. graphic display calculators or other appropriate technology) is allowed provided there is no possibility to communicate (e.g. via the internet, intranet, Bluetooth, mobile networks etc.) and there is no access to an individual's data stored on the device.

After the examination, all materials (tasks, extra sheets of paper etc.) from the candidates are to be collected in. The examination materials (tasks, extra sheets of paper, data that has been produced digitally etc.) may only be made public after the time period allocated for the examination has passed.

Evaluation grid for the supplementary examination

The evaluation grid below may be used to assist in assessing the candidates' performances.

	Candidate 1			Candidate 2			Candidate 3			Candidate 4			Candidate 5		
Task 1															
Task 2															
Task 3															
Task 4															
Total															

Explanatory notes on assessment

Each task can be awarded zero, one, two or three points. A maximum of twelve points can be achieved.

Assessment scale for the supplementary examination

Total number of competencies demonstrated	Assessment of the oral supplementary examination
12	Very good
10–11	Good
8–9	Satisfactory
6–7	Pass
0–5	Fail

Task 1

Crude Oil

a) On a particular day, the global consumption of crude oil was 15.1 billion litres.

A unit for the volume of crude oil is the barrel.

1 barrel corresponds to the volume of a cylindrical container with a diameter of 50 cm and a height of 81 cm.

1) Write down 15.1 billion litres in the unit barrels.

b) In the year 2018, 8.4 billion litres of diesel and 2.2 billion litres of petrol were sold in Austria.

The average price of 1 litre of diesel was x euros; the average price for 1 litre of petrol was y euros.
The total income from the sale of diesel and petrol was 13.02 billion euros.
The income from the sale of diesel was 7.476 billion euros higher than the income from the sale of petrol.

1) Write down a system of equations that can be used to determine x and y.

c) A system of equations in the variables x and y with the parameter c is shown below.

I: $c \cdot x + 4 \cdot y = 40$
II: $4 \cdot x + 2 \cdot y = 26$

1) Write down the value of c for which the system of equations has no solution.

$c =$ ____________________

Solution to Task 1

Crude Oil

a1) volume of a barrel in litres:

$V = 2.5^2 \cdot \pi \cdot 8.1 = 159.0...$

$\frac{15.1 \cdot 10^9}{159.0...} = 94.9... \cdot 10^6$

15.1 billion litres correspond to around 95 million barrels.

b1) I: $8.4 \cdot x + 2.2 \cdot y = 13.02$

II: $8.4 \cdot x = 7.476 + 2.2 \cdot y$

c1) $c = 8$

Task 2

Lighting

a) On a particular street in a town, new bulbs are to be installed in 174 streetlights. The town receives the following estimate:
A new bulb costs € 7.90, and exactly 1 bulb will be installed in each streetlight.
The running costs for all 174 streetlights are € 2.86 per hour.

The total costs for the lighting on this street are to be described by the function K in terms of the running time t.

t ... running time in h
$K(t)$ ... cost of the running time t in euros

1) Write down an equation of the function K. The time $t = 0$ corresponds to the time when the new bulbs are installed.

b) For the lighting on another street, a selection of two types of bulb, A and B, is available.
The lighting costs for bulb type A can be described by the function K_A.
The lighting costs for bulb type B can be described by the function K_B.

$K_A(t) = 600 + 429 \cdot t$
$K_B(t) = 1\,050 + 285 \cdot t$

t ... time in years
$K_A(t)$, $K_B(t)$ ... lighting costs after a total of t years in euros

1) Determine after how many years the lighting costs for each bulb type are the same.
2) Interpret the result of the calculation shown below in the given context.

   $K_A(10) - K_B(10) = 990$

Solution to Task 2

Lighting

a1) $K(t) = 174 \cdot 7.9 + 2.86 \cdot t$

or:

$K(t) = 1\,374.6 + 2.86 \cdot t$

b1) $1\,050 + 285 \cdot t = 600 + 429 \cdot t$

calculation using technology:

$t = 3.125$

After 3.125 years, the lighting costs for the two bulb types are the same.

b2) After a total of 10 years, the lighting costs for bulb type *A* are 990 euros higher than the lighting costs for bulb type *B*.

Task 3

Storms

In June 2012, there were heavy storms in Austria.

a) During one storm in Graz, the following data was collected:
At the beginning of the storm, the instantaneous precipitation per square metre was 150 ml per min.
The maximum value of the instantaneous precipitation per square metre of 400 ml per min was reached 50 min after the beginning of the storm.

The trend of the instantaneous precipitation per square metre over time can be approximated by the quadratic function f with $f(t) = a \cdot t^2 + b \cdot t + c$.

t ... time since the beginning of the storm in min
$f(t)$ ... instantaneous precipitation per square metre at time t in ml per min

1) Write down a system of equations that can be used to calculate the coefficients a, b and c.

b) In Mürzzuschlag, a storm lasted 2.5 h. For this storm, the instantaneous precipitation per square metre can be approximated by the function N shown below.

$N(t) = -\frac{44}{3} \cdot t^3 + 44 \cdot t^2 - \frac{103}{3} \cdot t + 40 \quad \text{with} \quad 0 \leq t \leq 2.5$
t ... time since the beginning of the storm in h
$N(t)$ ... instantaneous precipitation per square metre at time t in L per h

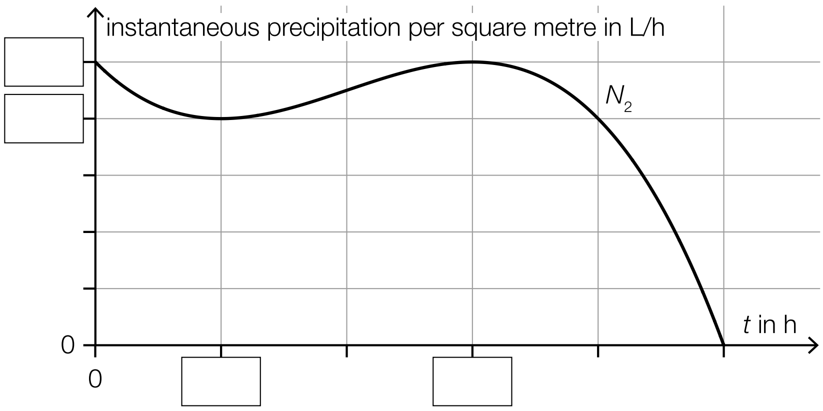
The total amount of precipitation per square metre in the time interval $[t_1, t_2]$ can be calculated using the expression shown below.
$\int_{t_1}^{t_2} N(t)\,\mathrm{d}t$

1) Determine the total amount of precipitation per square metre that fell during these 2.5 hours. Write down the result with the corresponding unit.

c) The instantaneous precipitation per square metre was also measured in a neighbouring town. Using the values collected, the graph of the 3rd degree polynomial function N_2 was created.

- $t_w = 1$ is the x-coordinate of the point of inflexion of the function N_2.
- At the x-coordinate t_m of the minimum of the function N_2, the following statements hold: $f(t_m) = 32$ and $f'(t_m) = 0$

1) In the diagram below, write down the missing numbers in the boxes provided.



Solution to Task 3

Storms

a1) $f(t) = a \cdot t^2 + b \cdot t + c$
$f'(t) = 2 \cdot a \cdot t + b$

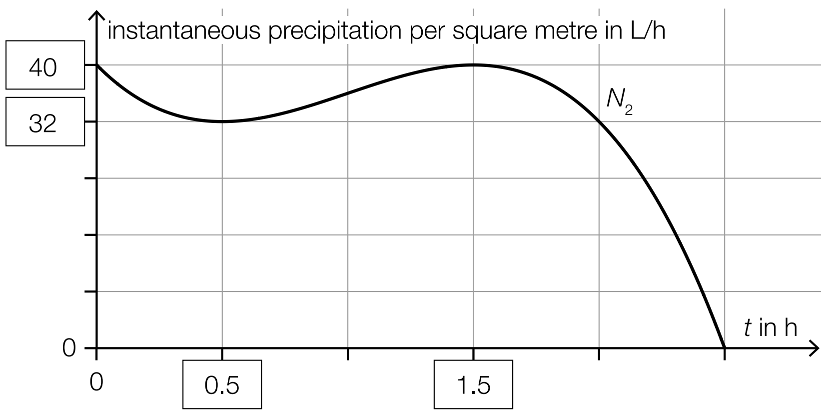
$f(0) = 150$
$f'(50) = 0$
$f(50) = 400$

or:

$c = 150$
$100 \cdot a + b = 0$
$2500 \cdot a + 50 \cdot b + c = 400$

b1) $\int_0^{2.5} N(t)\,dt = 78.645...$
The total amount of precipitation per square metre was around 78.6 L.

c1)



Task 4

Soft Drinks

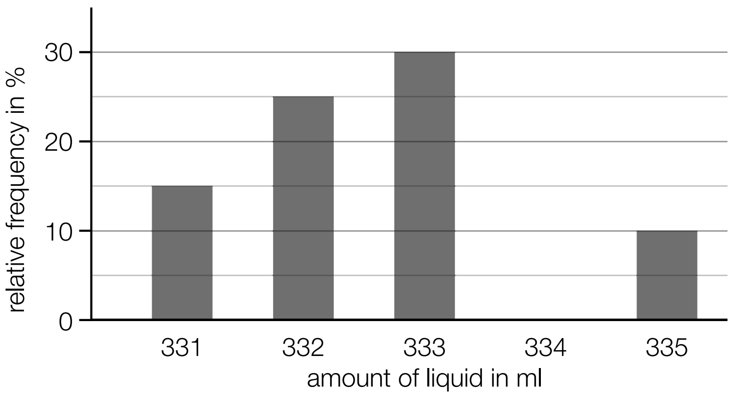
Soft drinks are offered in a variety of bottles and cans.

a) Markus investigates the amount of liquid in 5 glass bottles and measures the following values:

amount of liquid in ml: 753, 754, 752, 754, 753

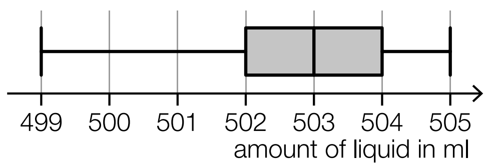
1) Determine the mean and the standard deviation of the amount of liquid in these 5 glass bottles.

b) Using a machine, Nina investigates the amount of liquid in plastic bottles. She measures the following amounts in ml: 331, 332, 333, 334 and 335.
The results of her investigation are represented in the bar chart below. One bar on the chart is missing.



1) Complete the bar chart above by drawing the missing bar.

c) Antonia investigates the amount of liquid in 37 cans.
The results of her investigation are shown in the box plot on the right.



1) Put a cross next to each of the two correct statements. *[2 out of 5]*

The range is 2 ml.	☐
The median is 503 ml.	☐
The interquartile range (the difference between the 3rd quartile and the 1st quartile) is 1 ml.	☐
At least 18 cans contain at most 503 ml of liquid.	☐
At least 25 cans contain at least 504 ml of liquid.	☐

Solution to Task 4

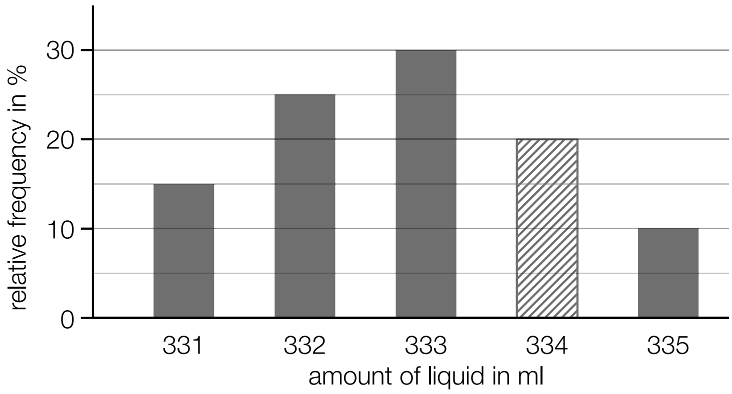
Soft Drinks

a1) calculation using technology:

mean: $\bar{x} = 753.2$ ml
standard deviation: $s = 0.748...$ ml

A standard deviation calculated to be $s_{n-1} = 0.836...$ ml *is also to be considered correct.*

b1)



c1)

The median is 503 ml.	☒
At least 18 cans contain at most 503 ml of liquid.	☒