Name:	
Class:	

Standardised Competence-Oriented
Written School-Leaving Examination

AHS

19th September 2023

Mathematics

Bundesministerium
Bildung, Wissenschaft
und Forschung

Advice for Completing the Tasks

Dear candidate,

The following booklet contains Part 1 and Part 2 tasks (divided into sub-tasks). The tasks can be completed independently of one another. You have a total of *270 minutes* available in which to work through this booklet.

Please do all of your working out solely in this booklet and on the paper provided to you. Write your name and that of your class on the cover page of the booklet in the spaces provided. Please also write your name on any separate sheets of paper used and number these pages consecutively. When responding to the instructions of each task, write the task reference (e.g. 25a1) on your sheet.

In the assessment of your work, everything that is not crossed out will be considered.

The use of the official formula booklet for this examination that has been approved by the relevant government authority is permitted. Furthermore, the use of electronic device(s) (e.g. graphic display calculators or other appropriate technology) is allowed provided there is no possibility of communicating via the internet, Bluetooth, mobile networks etc. and there is no access to your own data stored on the device.

An explanation of the task types is displayed in the examination room.

Instructions for Completing the Tasks

- Solutions must be unambiguous and clearly recognisable.
- Solutions must be given alongside their corresponding units if this has been explicitly required in the task instructions.

For tasks with open answer formats, evidence of the targeted core competency is required for the award of the point. When completing tasks with open answer formats, it is recommended that you:

- document how the solution was reached, even if electronic devices were used,
- explain any variables you have chosen yourself and give their corresponding units,
- avoid rounding prematurely,
- label diagrams or sketches.

Changing an answer for a task that requires a cross:

1. Fill in the box that contains the cross.
2. Put a cross in the box next to your new answer.

In this instance, the answer “5 + 5 = 9” was originally chosen. The answer was later changed to be “2 + 2 = 4”.

1 + 1 = 3	☐
2 + 2 = 4	☒
3 + 3 = 5	☐
4 + 4 = 4	☐
5 + 5 = 9	■
6 + 6 = 10	☐

Selecting an item that has been filled in:

1. Fill in the box that contains the cross for the answer you do not wish to give.
2. Put a circle around the filled-in box you would like to select.

In this instance, the answer “2 + 2 = 4” was filled in and then selected again.

1 + 1 = 3	☐
2 + 2 = 4	■
3 + 3 = 5	☐
4 + 4 = 4	■
5 + 5 = 9	☐
6 + 6 = 10	☐

Grading System

points awarded	grade
32–36 points	very good
27–31.5 points	good
22–26.5 points	satisfactory
17–21.5 points	pass
0–16.5 points	fail

Best-of Assessment: A best-of assessment approach will be applied to tasks 26, 27 and 28. Of these three Part 2 tasks, the task with the lowest point score will not be included in the total point score.

Good luck!

Task 1

Integers and Irrational Numbers

Four properties of numbers as well as six numbers are shown below.

Task:

Match each of the four properties of numbers to the number with this property from A to F.

negative integer	
negative irrational number	
positive integer	
positive irrational number	

A	$2 - \sqrt{10}$
B	10^{-2}
C	$-\sqrt{10^2}$
D	$2 \div (-10)$
E	$\sqrt{10} \div 2$
F	$(-\sqrt{10})^2$

[0/½/1 p.]

Task 2

Taxi Journey

A particular taxi company sets their daily rate as follows:
In addition to the predetermined base fare G, a charge K is to be paid per kilometre of distance travelled.

For a journey that starts at night between 22:00 and 6:00, an extra charge of 30 % of the daily rate is to be paid.

A passenger gets into a taxi from this taxi company at 22:00 and covers a distance of S kilometres.

Task:

Write down an equation that can be used to calculate the total travel costs F for this journey. Write the equation in terms of G, S and K.

$F =$ ____

[0/1 p.]

Task 3

Apple Juice and Orange Juice

At an event, the only drinks that are sold are apple juice and orange juice in cups.
In total, 375 cups are sold at the event, of which *a* cups contain apple juice at € 0.80 each and *b* cups contain orange juice at € 1.00 each.
The revenue earned from these sales is € 339.00.

Task:

Write down a system of equations that can be used to calculate *a* and *b*.

I: ____

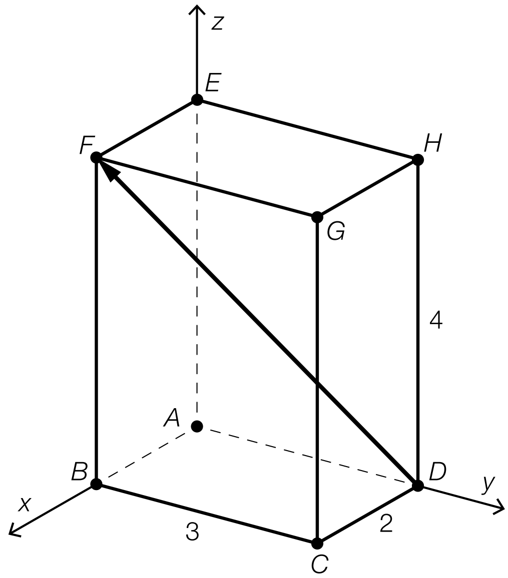
II: ____

[0/½/1 p.]

Task 4

Cuboid

The diagram below shows a cuboid *ABCDEFGH* in a three-dimensional coordinate system. The lengths of the edges of the cuboid can be read from the diagram (lengths in centimetres).



Task:

Write down the coordinates of the vector $\overrightarrow{DF}$.

$$\overrightarrow{DF} = \begin{pmatrix} \square \\ \square \\ \square \end{pmatrix}$$

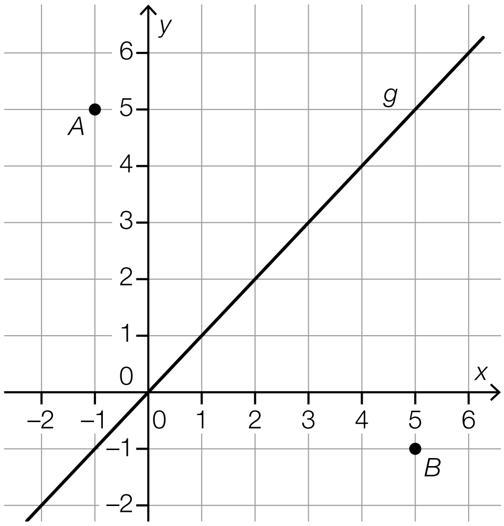
[0/1 p.]

Task 5

Vector and Line

The diagram below shows the points A and B as well as the line g: $y = x$.

The points A and B have integer coordinates.



Task:

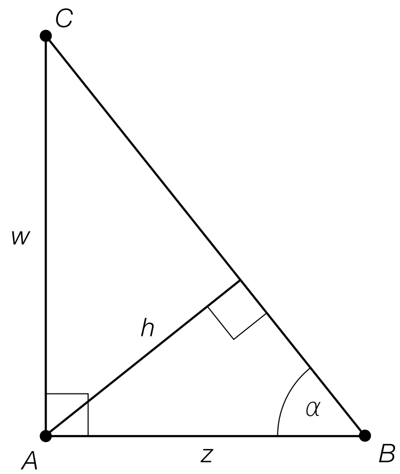
Show by calculation that the vector $\overrightarrow{AB}$ is perpendicular to the line g.

[0/1 p.]

Task 6

Triangle

The diagram below shows a right-angled triangle *ABC*.



Task:

Put a cross next to the equation that is definitely true. *[1 out of 6]*

$h = \frac{w}{\sin(\alpha)} \cdot \cos(\alpha)$	☐
$h = \frac{w}{\cos(\alpha)} \cdot \sin(\alpha)$	☐
$h = \frac{w}{\sin(\alpha)} \cdot \tan(\alpha)$	☐
$h = \frac{w}{\tan(\alpha)} \cdot \sin(\alpha)$	☐
$h = \frac{\sin(\alpha)}{w} \cdot \tan(\alpha)$	☐
$h = \frac{\sin(\alpha)}{w} \cdot \cos(\alpha)$	☐

[0/1 p.]

Task 7

Exponential Functions

Let $f\colon \mathbb{R} \to \mathbb{R}$ be an exponential function of the form $f(x) = a \cdot b^x$ with $a, b \in \mathbb{R}$ with $a, b > 0$ and $b \neq 1$.

Task:

Put a cross next to each of the two statements that are true about every exponential function of the form given above. *[2 out of 5]*

f has no zeros.	☐
f is strictly monotonically increasing.	☐
f has at least one local extremum (maximum or minimum).	☐
The graph of *f* is concave up.	☐
For $x \to \infty$, the graph of *f* tends towards the positive *x*-axis.	☐

[0/1 p.]

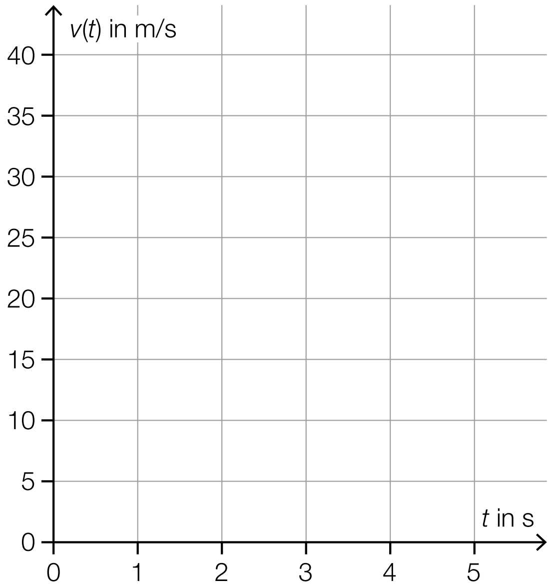
Task 8

Acceleration

A vehicle moves forwards at a velocity of 20 m/s along a straight path.
At time $t = 0$, it accelerates uniformly at 3 m/s^2 for 5 s. The direction of the movement remains unchanged.
The function v describes the velocity of the vehicle (in m/s) after t seconds in the time interval [0, 5].

Task:

In the coordinate system shown below, draw the graph of v.



[0/1 p.]

Task 9

Quadratic Function

Let f be a quadratic function of the form $f(x) = a \cdot x^2 + b$ with $a, b \in \mathbb{R}\setminus\{0\}$.

Task:

Write down a condition that the parameters a and b must satisfy such that f has two real zeros.

[0/1 p.]

Task 10

Number of Zeros of a Polynomial Function

There is a relationship between the number of possible real zeros and the degree of a polynomial function.

Task:

Complete the gaps in the following sentence by putting a cross next to one of the given possibilities for each gap so that the sentence becomes a correct statement.

Every ______①______ degree polynomial function has ______②______ one real zero.

①	
second	☐
third	☐
fourth	☐

②	
exactly	☐
at least	☐
more than	☐

[0/1 p.]

Task 11

Doubling Time

The number of bacteria in each of six bacterial cultures grows exponentially. Each of the corresponding doubling times is different.

The number of bacteria in each bacterial culture is modelled in terms of time t by
$N_i\colon \mathbb{R}_0^+ \to \mathbb{R}^+,\ t \mapsto N_i(t)$ with $i \in \{1, 2, \dots, 6\}$ (t in hours).

Task:

Match each of the four statements about the doubling times to the corresponding equation of a function from A to F.

The number of bacteria doubles once an hour.	
The number of bacteria doubles twice an hour.	
The number of bacteria doubles 3 times an hour.	
The number of bacteria doubles 4 times an hour.	

A	$N_1(t) = N_1(0) \cdot 1.5^t$
B	$N_2(t) = N_2(0) \cdot 4^t$
C	$N_3(t) = N_3(0) \cdot 2^t$
D	$N_4(t) = N_4(0) \cdot 16^t$
E	$N_5(t) = N_5(0) \cdot 3^t$
F	$N_6(t) = N_6(0) \cdot 8^t$

[0/½/1 p.]

Task 12

Period Length

Let $f\colon \mathbb{R} \to \mathbb{R}$ be a function with $f(x) = \sin\left(\frac{\pi}{c} \cdot x\right)$ with $c \in \mathbb{R}^+$.

The (shortest) period length of f is $\frac{3}{2}$.

Task:

Determine c.

[0/1 p.]

Task 13

Bitcoin

Bitcoin is a digital artificial currency. On 17.12.2017, the exchange rate per bitcoin was € 16,198.60.
The table below shows the exchange rate per bitcoin over the course of a year.

date	exchange rate per bitcoin
17.12.2017	€ 16,198.60
17.03.2018	€ 6,422.98
17.06.2018	€ 5,571.62
17.09.2018	€ 5,362.46
17.12.2018	€ 3,145.20

During one of the three-month time periods, the value of the absolute change in the exchange rate was the greatest.

Task:

Determine the relative change of the exchange rate of bitcoin in this time period.

relative change: ____

[0/1 p.]

Task 14

Average Speed

The movement of a particular body is modelled by the distance-time function s with $s(t) = d \cdot t^2$ (t in s, $s(t)$ in m).
The average speed of this body in the time interval [0 s, 2 s] is 10 m/s.

Task:

Determine d.

[0/1 p.]

Task 15

Antiderivative of a Sine Function

The function $F\colon \mathbb{R} \to \mathbb{R}$ with $F(x) = -1.25 \cdot \cos(b \cdot x)$ is an antiderivative of the function $f\colon \mathbb{R} \to \mathbb{R}$ with $f(x) = 2 \cdot \sin(b \cdot x)$ with $b \in \mathbb{R}\setminus\{0\}$.

Task:

Determine b.

[0/1 p.]

Task 16

Value of a Definite Integral

The function g is an antiderivative of the polynomial function f. Some pairs of values of the function g are shown below:

x	$g(x)$
–2	3
–1	0
0	–1
1	0
2	3
3	8
4	15

Task:

Write down the value of the integral shown below.

$\int_0^3 f(x)\,dx =$ ____

[0/1 p.]

Task 17

Properties of a Polynomial Function

A 4th degree polynomial function f has a local maximum at each of the x-coordinates $a \in \mathbb{R}$ and $b \in \mathbb{R}$ with $a < b$. Six statements about $c \in \mathbb{R}$ with $a < c < b$ are shown below.

Task:

Put a cross next to the statement that is definitely true. *[1 out of 6]*

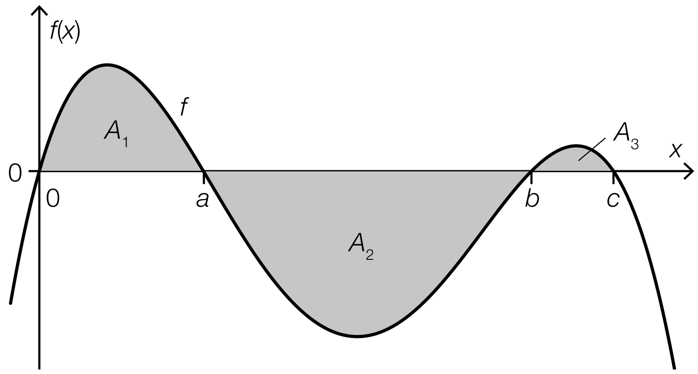
There is exactly one c for which $f'(c) = 0$ holds.	☐
There is exactly one c for which $f''(c) = 0$ holds.	☐
There is no c for which $f(c) = 0$ holds.	☐
There is no c for which $f'(c) = 0$ holds.	☐
There is exactly one c for which $f(c) = 0$ holds.	☐
There is no c for which $f''(c) = 0$ holds.	☐

[0/1 p.]

Task 18

Integral and Area

The diagram below shows the graph of the function f, which crosses the x-axis at 0, a, b and c.
The graph of f and the x-axis bound three regions with areas $A_1 = 17$, $A_2 = 50$ and $A_3 = 2$.



Task:

Match each of the four expressions to the corresponding value from A to F.

$\int_0^c f(x)\,\mathrm{d}x$	
$\int_0^a f(x)\,\mathrm{d}x + \int_a^b f(x)\,\mathrm{d}x$	
$\int_0^a f(x)\,\mathrm{d}x - \int_a^b f(x)\,\mathrm{d}x + \int_b^c f(x)\,\mathrm{d}x$	
$\int_a^c f(x)\,\mathrm{d}x + 100$	

A	–31
B	69
C	–33
D	52
E	67
F	152

[0/½/1 p.]

Task 19

List of Data

A list of data with n natural numbers ($n \in \mathbb{N}$, $n \geq 2$) is given.

Task:

Complete the gaps in the following sentence by putting a cross next to one of the given possibilities for each gap so that the sentence becomes a correct statement.

If all the values of the list of data are increased by a ($a \in \mathbb{R}^+$), then _____①_____ also increases by a, while _____②_____ remains unchanged.

①	
the range	☐
the median	☐
the variance	☐

②	
the mean	☐
the mode	☐
the standard deviation	☐

[0/½/1 p.]

Task 20

Number of Births

The following sentence about a particular region appears in a local newspaper:

"In 2019, the number of births in the region was higher than the average value in the 4-year period from 2015 to 2018."

Task:

Put a cross next to each of the two statements that are definitely true according to the sentence above. *[2 out of 5]*

The number of births in 2019 was higher than in each year of the period from 2015 to 2018.	☐
The total number of births in the period from 2015 to 2018 was lower than four times the number of births in 2019.	☐
The number of births in at least one year in the period from 2015 to 2018 was higher than in 2019.	☐
The number of births in at most three years in the period from 2015 to 2018 was higher than in 2019.	☐
The number of births in at least two years in the period from 2015 to 2018 was lower than in 2019.	☐

[0/1 p.]

Task 21

Game

The probability of winning 1 round of a particular game has the value p.
The probability of winning 2 consecutive rounds of this game has the value p_1.
Consecutive rounds are independent of each other.

Task:

Put a cross next to each of the two statements that are definitely true about the game described above. *[2 out of 5]*

$p_1 = 2 \cdot p$	☐
$p_1 = (1 - p)^2$	☐
$p_1 = p \cdot (1 - p)$	☐
$p_1 \leq p$	☐
$p_1 = p^2$	☐

[0/1 p.]

Task 22

Ice Cream Parlour

In an ice cream parlour, 24 flavours of ice cream are sold.

Task:

Write down the number of possible ways of choosing 3 different flavours of ice cream out of the 24 flavours sold. (The order of the choices is not to be considered.)

[0/1 p.]

Task 23

Probability of an Event

A random experiment is conducted n times ($n \in \mathbb{N}$ with $n \geq 12$).

The random variable X corresponds to how often a particular event occurs in these n experiments.
The probability that this event occurs at least 10 times is 35 %.

Task:

Put a cross next to each of the two statements that are definitely true. *[2 out of 5]*

$P(X = 0) = 0$	☐
$P(X \leq 10) = 0.35$	☐
$P(X < 9) \leq 0.65$	☐
$P(X \geq 10) = 0.35$	☐
$P(X > 11) > 0.4$	☐

[0/1 p.]

Task 24

Quality Assurance

As part of the quality assurance in the production of porcelain figures, the figures are checked for faults once they are finished. It is known from experience that 2 % of the porcelain figures are faulty.

A random sample of n porcelain figures is taken ($n \in \mathbb{N}$ with $n \geq 2$). The number of faulty porcelain figures is assumed to be binomially distributed.
The event that at least 1 of the n porcelain figures is faulty is given by E.

Task:

Write down a formula in terms of n that can be used to calculate the probability $P(E)$.

$P(E) =$ ______________________________________

[0/1 p.]

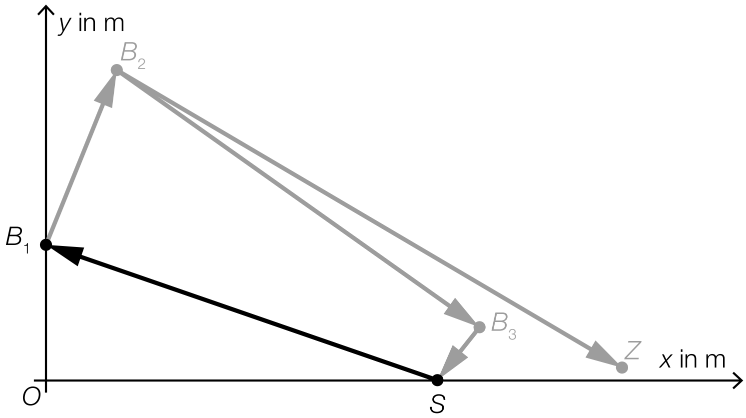
Task 25 (Part 2)

Triathlon

Triathlon is a competition in which sportspeople complete a swimming race, a cycling race and a running race in exactly this order.

Task:

a) The path of the swimming race of a particular triathlon is modelled in the diagram below. The swimming race starts at point S and ends at point Z. Between these points, the checkpoints B_1, B_2, B_3, S, B_1 and B_2 must be passed in exactly this order.



The distance from point $S = (600, 0)$ to point B_1 is 700 m.

1) Determine the y-coordinate of B_1.

$B_1 = \left(0, \boxed{}\right)$ *[0/1 p.]*

b) In the cycling race of a particular triathlon, Stefanie starts 1.45 min before Tanja.

t ... time in min
$t = 0$ ... time at which Stefanie starts
$v_{\text{Stefanie}}(t)$ ... Stefanie's velocity at time t in km/min
$v_{\text{Tanja}}(t)$ ... Tanja's velocity at time t in km/min

Stefanie completes the cycling race in 291 min. At this time, Tanja is still cycling.

1) Interpret what can be calculated by the expression shown below in the given context.

$\int_0^{291} v_{\text{Stefanie}}(t)\,\mathrm{d}t - \int_{1.45}^{291} v_{\text{Tanja}}(t)\,\mathrm{d}t$ *[0/1 p.]*

c) Michael takes part in a particular triathlon.

Michael starts the final 42.195 km long running race with a total time of 5 h 12 min 38 s up to that point.

Michael finishes the triathlon with a total time of 7 h 36 min 56 s.

1) Determine Michael's average speed in the running race in km/h. *[0/1 p.]*

d) The most famous triathlon competition is the *Ironman World Championship* in Hawaii.

The function $f\colon \mathbb{N} \to \mathbb{R}$ with $f(t) = 0.1275 \cdot t^3 - 8.525 \cdot t^2 + 198.425 \cdot t + 15$ models the number of participants in this competition in terms of time t in the period from 1978 to 2018 (t in years with $t = 0$ for the year 1978).

Source: https://www.tri226.de/ironman-ergebnisse.php?language=ge&table=start_finish [09.08.2022].

The number of participants in this competition increased by an average of n people per year from 1978 to 2018.

1) Determine n. *[0/1 p.]*

Task 26 (Part 2, Best-of Assessment)

Ludo

Ludo is a board game for at least two people. The aim of the game is to move your 4 playing pieces of the same colour from the starting position to the finishing position as quickly as possible.

Task:

a) There are 4 red, 4 yellow and 4 blue playing pieces for Ludo in a material bag. Isabella removes 4 playing pieces at random and without replacement.

1) Determine the probability that all 4 of the playing pieces removed are red. *[0/1 p.]*

Isabella has removed all of the red playing pieces. Therefore, the material bag now contains only the 4 yellow and 4 blue playing pieces.
Now Fatima removes 1 playing piece at a time, without replacement, until she has removed all 4 yellow playing pieces.

The random variable X describes the number of draws k that Fatima requires until she has drawn all 4 yellow playing pieces. The probability distribution of the random variable X is shown in the table below.

k	4	5	6	7	8
$P(X = k)$	$\frac{1}{70}$	u	$\frac{10}{70}$	$\frac{20}{70}$	v

2) Determine u and v.

u = ____________________

v = ____________________ *[0/1 p.]*

b) Isabella wins an average of 3 out of 5 games of Ludo against her friend Fatima. In the upcoming summer holidays, the two girls will play n games against each other (n is even, $n > 2$).
The binomially distributed random variable Y gives the number of games out of n that will be won by Isabella.

Four probabilities and six events are shown below.

1) Match each of the four probabilities to the event that occurs with that probability from A to F. *[0/½/1 p.]*

$\binom{n}{\frac{n}{2}} \cdot 0.6^{\frac{n}{2}} \cdot 0.4^{\frac{n}{2}}$	
$1 - 0.4^n - n \cdot 0.6 \cdot 0.4^{n-1}$	
$1 - 0.6^n$	
$n \cdot 0.6^{n-1} \cdot 0.4$	

A	Isabella wins exactly half of the n games.
B	Isabella wins at least 2 of the n games.
C	Isabella loses more than half of the n games.
D	Isabella loses exactly 1 of the n games.
E	Isabella loses at least 1 of the n games.
F	Isabella wins at most 1 of the n games.

The expectation value of Y is given by μ; the standard deviation of Y is given by σ.

2) Determine the probability $P(\mu - \sigma < Y < \mu + \sigma)$ for $n = 14$. *[0/1 p.]*

Task 27 (Part 2, Best-of Assessment)

Beekeeping in Austria

The table below shows the number of beekeepers and their bee colonies in Austria in the time from 2015 to 2019.

year	number of beekeepers	number of bee colonies
2015	26063	347128
2016	26609	354080
2017	27580	353267
2018	28432	373412
2019	30237	390607

Source: https://www.biene-oesterreich.at/daten-und-zahlen+2500++1000247 [10.08.2020].

Task:

a) Maja completes the following calculation using values from the table above:

$$\frac{353\,267}{27\,580} \approx 13$$

1) Interpret the result of this calculation in the given context. *[0/1 p.]*

b) The number of beekeepers in Austria is modelled in terms of time t by the quadratic function f of the form $f(t) = c \cdot t^2 + d$ with $c, d \in \mathbb{R}$ (t in years with $t = 0$ for the year 2015).
The values of the function of f match the values in the table above for the years 2015 and 2019.

1) Determine c and d. *[0/1 p.]*

c) Lower temperatures lead to winter mortality among the bee colonies. The number of bee colonies would reduce by an average of 16 % per year without the beekeepers breeding new colonies.

The number of bee colonies that would exist in Austria if there were no extra breeding is described by the exponential function g.

The following conditions hold:

t ... time in years with $t = 0$ for the year 2015
$g(t)$ ... number of bee colonies in Austria at time t

1) Write down an equation of the function g.

$g(t) =$ ____________________________________ *[0/1 p.]*

2) Determine the time it would take for the number of bee colonies in Austria to halve according to the exponential function g. *[0/1 p.]*

Task 28 (Part 2, Best-of Assessment)

Pond

There are 129 m^3 of water in an artificial pond.

Task:

a) The pond can be completely emptied through two drains.

If only one drain is opened, then the complete drainage of the pond takes 10 h.
If only the other drain is opened, then the complete drainage of the pond takes 6 h.
Each of the drainage velocities is constant over the whole time period.
The time it takes for the pond to be completely drained when both drains are opened at the same time is given by T.

1) Determine T. *[0/1 p.]*

b) The completely empty pond is filled again with 129 m^3 of water.

The function $d\colon \mathbb{R}^+ \to \mathbb{R}^+$ gives the filling time $d(z)$ in terms of the constant inflow velocity z (z in m^3/h, $d(z)$ in h).

1) Write down an equation of the function d.

$d(z) =$ ____ *[0/1 p.]*

The function h describes the height of the surface of the water above the deepest point of the pond in terms of the time t for a constant inflow velocity $z = 6$ m^3/h (t in h, $h(t)$ in m).

The following holds for the instantaneous rate of change of the height of the surface of the water:

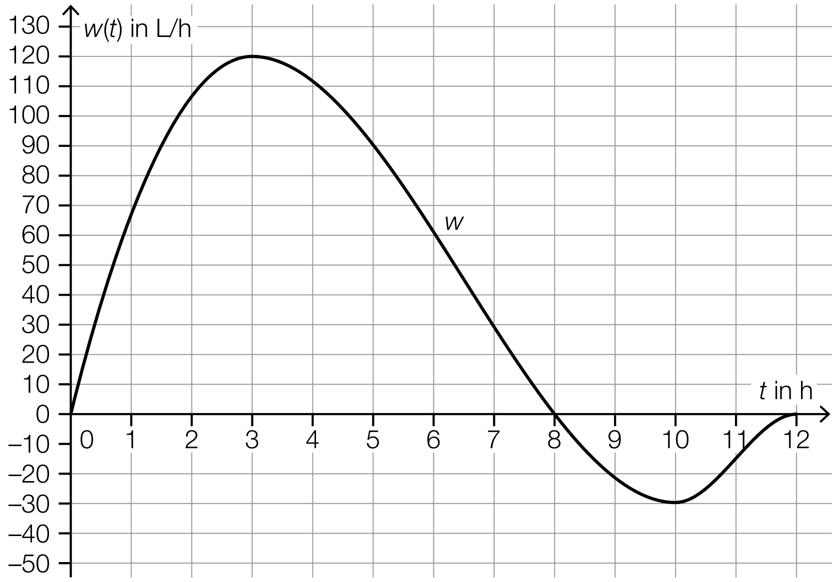
$h'(t) = \dfrac{15}{\sqrt{2\,738 \cdot \pi \cdot t}}$ with $t > 0$

2) Determine by how many metres the height of the surface of the water rises in the last 10 h of filling. *[0/1 p.]*

c) The amount of water in the pond changes due to rain and evaporation.

The function w: $[0, 12] \to \mathbb{R}$ approximates the instantaneous rate of change of the amount of water in the pond in terms of the time t (t in h, $w(t)$ in L/h).

The diagram below shows the graph of w.



1) Match each of the four statements to the corresponding largest possible time period from A to F. *[0/½/1 p.]*

The amount of water in the pond is decreasing.	
The amount of water in the pond is increasing faster and faster.	
The instantaneous rate of change of the amount of water in the pond is decreasing.	
The amount of water in the pond is increasing.	

A	(0, 3)
B	(3, 10)
C	(8, 12)
D	(3, 12)
E	(8, 10)
F	(0, 8)