Name:	
Class:	

Standardised Competence-Oriented
Written School-Leaving Examination

AHS

10th January 2024

Mathematics

Bundesministerium
Bildung, Wissenschaft
und Forschung

Advice for Completing the Tasks

Dear candidate,

The following booklet contains Part 1 and Part 2 tasks (divided into sub-tasks). The tasks can be completed independently of one another. You have a total of *270 minutes* available in which to work through this booklet.

Please do all of your working out solely in this booklet and on the paper provided to you. Write your name and that of your class on the cover page of the booklet in the spaces provided. Please also write your name on any separate sheets of paper used and number these pages consecutively. When responding to the instructions of each task, write the task reference (e. g. 25a1) on your sheet.

In the assessment of your work, everything that is not crossed out will be considered.

The use of the official formula booklet for this examination that has been approved by the relevant government authority is permitted. Furthermore, the use of electronic device(s) (e. g. graphic display calculators or other appropriate technology) is allowed provided there is no possibility of communicating via the internet, Bluetooth, mobile networks etc. and there is no access to your own data stored on the device.

An explanation of the task types is displayed in the examination room.

Instructions for Completing the Tasks

– Solutions must be unambiguous and clearly recognisable.
– Solutions must be given alongside their corresponding units if this has been explicitly required in the task instructions.

For tasks with open answer formats, evidence of the targeted core competency is required for the award of the point. When completing tasks with open answer formats, it is recommended that you:

– document how the solution was reached, even if electronic devices were used,
– explain any variables you have chosen yourself and give their corresponding units,
– avoid rounding prematurely,
– label diagrams or sketches.

Changing an answer for a task that requires a cross:

1. Fill in the box that contains the cross.
2. Put a cross in the box next to your new answer.

In this instance, the answer “5 + 5 = 9” was originally chosen. The answer was later changed to be “2 + 2 = 4”.

1 + 1 = 3	☐
2 + 2 = 4	☒
3 + 3 = 5	☐
4 + 4 = 4	☐
5 + 5 = 9	■
6 + 6 = 10	☐

Selecting an item that has been filled in:

1. Fill in the box that contains the cross for the answer you do not wish to give.
2. Put a circle around the filled-in box you would like to select.

In this instance, the answer “2 + 2 = 4” was filled in and then selected again.

1 + 1 = 3	☐
2 + 2 = 4	◉
3 + 3 = 5	☐
4 + 4 = 4	■
5 + 5 = 9	☐
6 + 6 = 10	☐

Grading System

points awarded	grade
32 – 36 points	very good
27 – 31.5 points	good
22 – 26.5 points	satisfactory
17 – 21.5 points	pass
0 – 16.5 points	fail

Best-of Assessment: A best-of assessment approach will be applied to tasks 26, 27 and 28. Of these three Part 2 tasks, the task with the lowest point score will not be included in the total point score.

Good luck!

Task 1

Linear Equation

An equation in the variable $x \in \mathbb{Z}$ is shown below:
$2 \cdot x - c = 0 \quad \text{with} \quad c \in \mathbb{R}$

Task:

Write down all real numbers c for which this equation has a solution in $\mathbb{Z}$.

[0 / 1 p.]

Task 2

Donations

Anton donates an amount of a euros to each of 3 research institutes and an amount of $(a + 10)$ euros to each of 5 animal welfare organisations.

Task:

Write down the average amount G (in euros) that Anton has donated in terms of a.

$G =$ _____ euros

[0/1 p.]

Task 3

Force and Acceleration

If a force acts on a body at rest, this body accelerates in the direction of the force. The absolute value of the force is given by $F = m \cdot a$, where m is the mass and a is the acceleration of the body (F in Newtons (N), m in kg, a in m/s²).

A force of $F_1 = 5$ N acts on a particular sphere at rest. This sphere thus accelerates at a rate of $a_1 = 0.625$ m/s². A second sphere at rest with the same mass is acted on by a force F_2 so that this sphere accelerates at a rate of $a_2 = 0.5$ m/s².

Task:

Determine F_2 in N.

[0 / 1 p.]

Task 4

Position of a Ship

A ship travels with constant velocity along a straight path on a particular day from 8:10 am to 8:30 am.

In a Cartesian coordinate system, the position of this ship at 8:10 am is given by the point $A = (2, 3)$; its position at 8:30 am is given by the point $B = (10, 5)$.

The vector $\vec{s}$ gives the change in the position of this ship in a time period of 5 minutes.

Task:

Write down the components of the vector $\vec{s}$.

$\vec{s} = \begin{pmatrix} \square \\ \square \end{pmatrix}$

[0 / 1 p.]

Task 5

Parallel Vector

The vector $\vec{a} = \begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix}$ is given.

A vector $\vec{b}$ is parallel to the vector $\vec{a}$ and has a greater length than $\vec{a}$.

Task:

Write down the components of a possible vector $\vec{b}$.

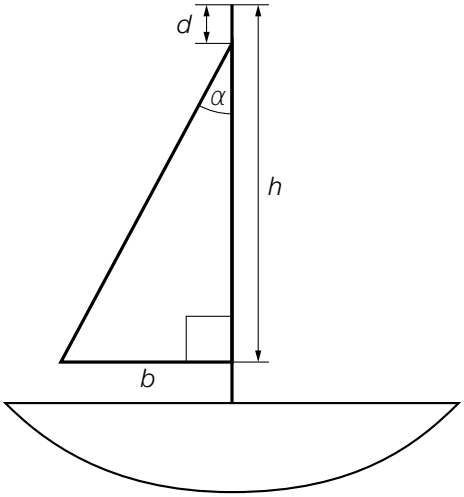
$\vec{b} = \begin{pmatrix} \square \\ \square \\ \square \end{pmatrix}$

[0/1 p.]

Task 6

Sailing Boat

A model of a sailing boat is shown in the diagram below.



Task:

Write down a formula in terms of h, d and b that can be used to calculate the size of the angle α.

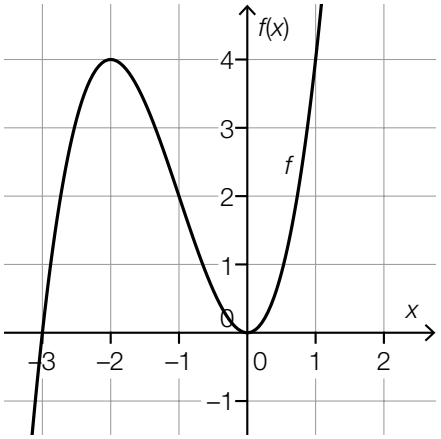
$\alpha =$ _____

[0 / 1 p.]

Task 7

Monotonicity and Concavity of a Polynomial Function

The graph of a 3rd degree polynomial function f is shown below. All the characteristic points of this graph (intersections with the axes, maxima, minima, points of inflexion) have integer coordinates.



Task:

Complete the gaps in the following sentence by putting a cross next to one of the given possibilities for each gap so that the sentence becomes a correct statement.

The function f is strictly monotonically increasing in the interval _____①_____ and its concavity changes when _____②_____.

①	
$(-\infty, -2)$	☐
$(-1, 1)$	☐
$(-2, 0)$	☐

②	
$x = -2$	☐
$x = -1$	☐
$x = 0$	☐

[0 / ½ / 1 p.]

Task 8

Swimming Pool

Water is let out of a swimming pool.

The function h: $[0, 6] \to \mathbb{R}$ with $h(t) = 180 - 30 \cdot t$ models the height of the surface of the water in terms of the time t (t in h, $h(t)$ in cm).

Task:

Interpret the coefficients 180 and –30 in the given context with the corresponding units.

180: _____

–30: _____

[0 / ½ / 1 p.]

Task 9

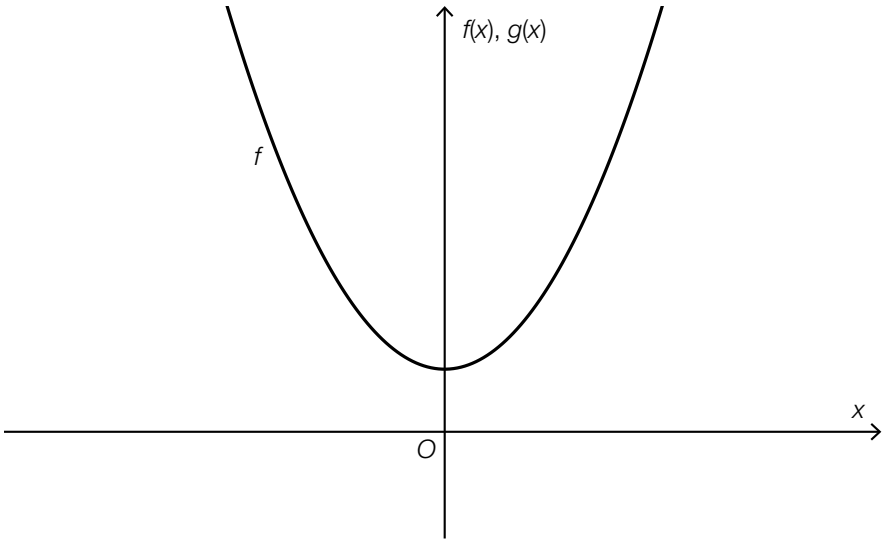
Graph of a Quadratic Function

The graph of a function $f\colon \mathbb{R} \to \mathbb{R}$ of the form $f(x) = a \cdot x^2 + b$ with $a, b \in \mathbb{R}$ is shown below.

For a function $g\colon \mathbb{R} \to \mathbb{R}$ of the form $g(x) = c \cdot x^2 + d$ with $c, d \in \mathbb{R}$, the following statements hold: $c < -a$ and $d > b$

Task:

On the diagram below, sketch the graph of one such function g.



[0 / 1 p.]

Task 10

Number of Zeros, Maxima, Minima and Points of Inflexion

Let *f* be a 4th degree polynomial function.

Statements about the exact number of distinct zeros, local extrema (maxima or minima), and points of inflexion are shown below.

Task:

Put a cross next to each of the two statements that could be true about *f*. *[2 out of 5]*

The function *f* can have 0 real zeros, 1 local extremum (maximum or minimum) and 0 points of inflexion.	☐
The function *f* can have 1 real zero, 3 local extrema (maxima or minima) and 2 points of inflexion.	☐
The function *f* can have 2 distinct real zeros, 2 local extrema (maxima or minima) and 2 points of inflexion.	☐
The function *f* can have 3 distinct real zeros, 2 local extrema (maxima or minima) and 0 points of inflexion.	☐
The function *f* can have 4 distinct real zeros, 3 local extrema (maxima or minima) and 1 point of inflexion.	☐

[0 / 1 p.]

Task 11

Video Views

A video is uploaded to an internet platform. At the beginning of the observations, the video had 500 views. In the time period [0, t_1], the number of views is described by an exponential function.

The table below shows pairs of values of this exponential function.

time since the beginning of the observations in h	number of views
0	500
1	700
2	980
3	1372
t_1	10330

Task:

Determine t_1.

[0/1 p.]

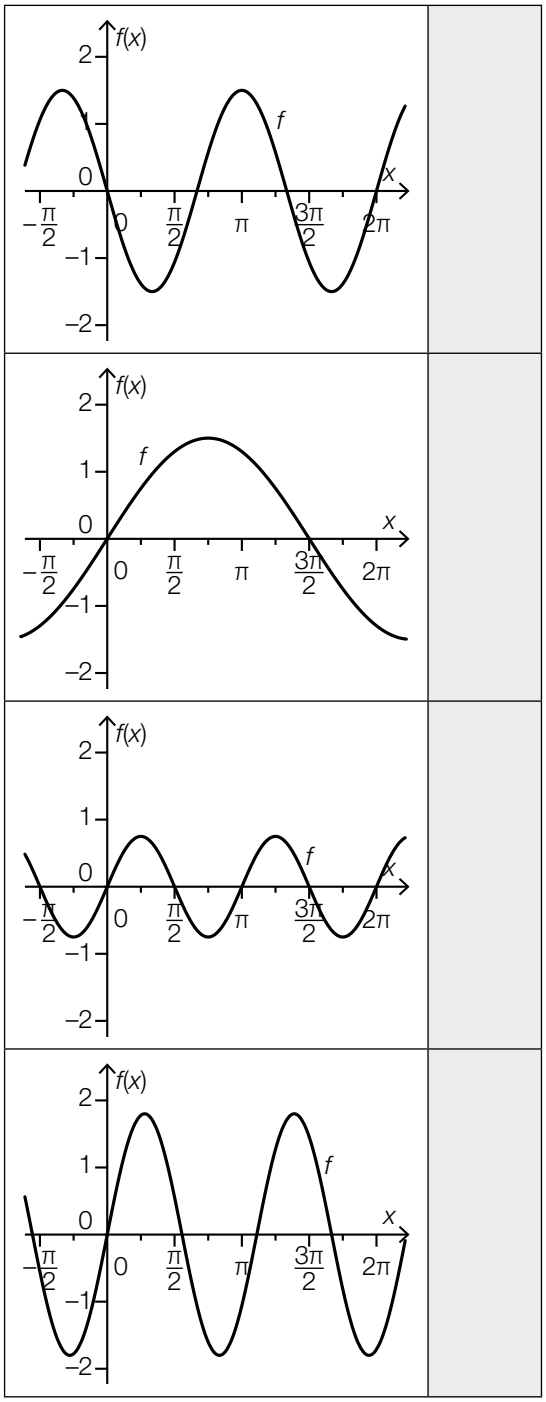
Task 12

Sine Functions

Four graphs of functions of the form $f(x) = a \cdot \sin(b \cdot x)$ with $a \in \mathbb{R}$ and $b \in \mathbb{R}^+$ are shown in the diagrams below.

Task:

Match each of the four graphs to the corresponding condition for a and b from A to F.



A	$a < 0$ and $b < 1$
B	$a < 0$ and $b > 1$
C	$0 < a < 1$ and $b < 1$
D	$0 < a < 1$ and $b > 1$
E	$a > 1$ and $b < 1$
F	$a > 1$ and $b > 1$

[0/½/1 p.]

Task 13

Relay Marathon

Every year teams of four people participate in the relay marathon in Linz. Four consecutive sections comprise the marathon (in total around 42.2 km).

A particular team is made up of the people *A*, *B*, *C* and *D*. The table below shows the running times they achieved in the years 2017 and 2018 for each of the sections.

	1st section	2nd section	3rd section	4th section
year	person *A*	person *B*	person *C*	person *D*
2017	43 min	1 h 4 min	41 min	1 h 8 min
2018	41 min	58 min	42 min	1 h 2 min

Task:

Write down the person who has seen the greatest percentage change in their running time and calculate this percentage change.

person: _____

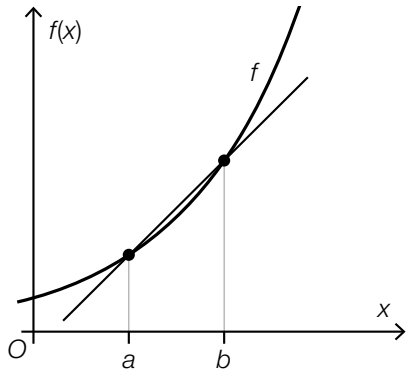
percentage change: _____ %

[0 / ½ / 1 p.]

Task 14

Graph and Secant Line

The diagram below shows the graph of the differentiable function f as well as the secant line through the points $(a, f(a))$ and $(b, f(b))$.



Task:

Complete the gaps in the following sentence by putting a cross next to one of the given possibilities for each gap so that the sentence becomes a correct statement.

The expression $\lim\limits_{a \to b} \frac{f(b) - f(a)}{b - a}$ is _____①_____ and corresponds to the _____②_____.

①	
the difference quotient in the interval $[a, b]$	☐
the differential quotient at the point where $x = b$	☐
the average rate of change in the interval $[a, b]$	☐

②	
gradient of the secant line in the interval $[a, b]$	☐
gradient of the tangent at $(a, f(a))$	☐
gradient of the tangent at $(b, f(b))$	☐

[0 / 1 p.]

Task 15

Air Pressure

Air pressure decreases as the height above sea level becomes greater.

The function $p: \mathbb{R}_0^+ \to \mathbb{R}^+$ models the air pressure p in terms of the height above sea level h (h in m, $p(h)$ in hectopascals (hPa)).
It is known that $h_1 = 300$ m and $h_2 = 500$ m.

Task:

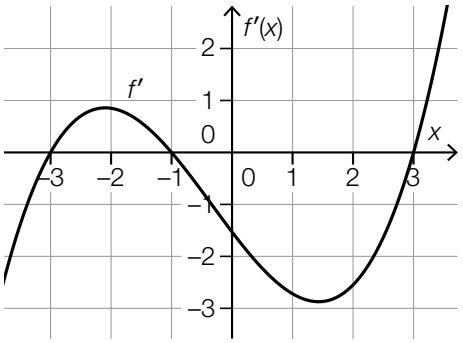
Interpret the expression $\frac{p(h_2) - p(h_1)}{h_2 - h_1}$ in the given context and write down the corresponding unit.

[0/1 p.]

Task 16

Graph of a Derivative

The graph of the derivative f' of a polynomial function f is shown below. The derivative f' is a 3rd degree polynomial function and has 3 integer zeros.



Task:

Put a cross next to each of the two statements that are definitely true about the polynomial function f. *[2 out of 5]*

f is strictly monotonically increasing in the interval [2, 3].	☐
f is concave up in the interval [2, 3].	☐
The statement $f(-3) \leq f(3)$ holds.	☐
f has exactly 2 points of inflexion.	☐
f has exactly 2 local maxima.	☐

[0 / 1 p.]

Task 17

Third Degree Polynomial Function

Let f be a 3rd degree polynomial function with $f(x) = a \cdot x^3 + b \cdot x^2 + c \cdot x + d$ with $a, b, c, d \in \mathbb{R}$, $a \neq 0$ and $d \neq 0$.

Task:

Complete the gaps in the following sentence by putting a cross next to one of the given possibilities for each gap so that the sentence becomes a correct statement.

For $b = 0$ and $c \neq 0$ the point where $x = 0$ is definitely a _____①_____ and for $c = 0$ and $b \neq 0$ it is definitely a _____②_____.

①	
zero	☐
maximum or minimum	☐
point of inflexion	☐

②	
zero	☐
maximum or minimum	☐
point of inflexion	☐

[0 / ½ / 1 p.]

Task 18

Marginal Costs and Total Costs

The marginal costs for the production of a particular product are modelled by the function K'. The following statements hold:

$$K'(x) = \frac{1}{100} \cdot \left(x^3 - \frac{x^2}{2} + 3 \cdot x + 4\right)$$

x ... amount produced in units
$K'(x)$ ... marginal costs for the amount produced x in euros per unit

The total costs are given in euros.

Task:

Determine the amount by which the total costs rise if 110 units are produced instead of 100 units.

[0 / 1 p.]

Task 19

Stem-and-Leaf Diagram

Data are displayed below in a stem-and-leaf diagram.

1	2 2 5 5 6
2	2 3 7 7
3	1 1 1 2 2 2 2
4	1 2 7 7 9 9

Task:

Match each of the four values listed below to the corresponding statistical parameter from A to F.

31	
32	
37	
49	

A	median
B	mode
C	mean
D	range
E	standard deviation
F	maximum

[0/½/1 p.]

Task 20

Sign of Statistical Parameters

The list of data with values $x_1 < ... < x_n$ with $x_1 < 0$ and $x_n > 0$ is given.

Task:

Put a cross next to the two statistical parameters that are definitely positive for the list of data given above. *[2 out of 5]*

range	☐
mean	☐
standard deviation	☐
minimum	☐
median	☐

[0/1 p.]

Task 21

Heads or Tails

A coin has a heads side *H* and a tails side *T*.

This coin is tossed 3 times. An example of a possible result of this random experiment is *THH*.

In this string of letters, the 1st letter gives the result of the 1st toss, the 2nd letter the result of the 2nd toss, and the 3rd letter the result of the 3rd toss.

The event that the 2nd toss results in *T* is given by *E*.

Task:

Write down the event *E* as a subset of the corresponding sample space of this random experiment.

$E = \{ \_\_\_\_\_ \}$

[0 / 1 p.]

Task 22

Balls

There are 5 red and n green balls in a container ($n \geq 2$).

3 balls are removed from the container without replacement.

The probability that exactly 2 green balls are removed is given by p.

Task:

Put a cross next to the correct statement. *[1 out of 6]*

$p = \frac{n}{n+5} \cdot \frac{n-1}{n+5} \cdot \frac{5}{n+5} \cdot 3$	☐
$p = \left(\frac{n}{n+5}\right)^2 \cdot \frac{5}{n+5}$	☐
$p = \frac{n}{n+5} \cdot \frac{n-1}{n+4} \cdot \frac{5}{n+3} \cdot 3$	☐
$p = \frac{5}{n+5} \cdot \left(\frac{n}{n+5}\right)^2 \cdot 3$	☐
$p = \frac{5}{n+5} \cdot \frac{n}{n+4} \cdot \frac{n-1}{n+3}$	☐
$p = \frac{5}{n+5} \cdot \frac{n}{n+5} \cdot \frac{n-1}{n+5}$	☐

[0/1 p.]

Task 23

Penalties

Johanna regularly shoots penalties with her football team.
She has 5 attempts. The random variable X gives the number of goals k scored.

The table below shows the probability distribution of X based on empirical values.

k	0	1	2	3	4	5
$P(X = k)$	0.001	0.008	0.131	0.310	0.372	$P(X = 5)$

Task:

Determine the probability that Johanna scores more than 3 goals in 5 attempts.

$P(X > 3) =$ _____

[0 / 1 p.]

Task 24

Therapy

The application of a particular therapy is successful in 90 % of people.
A specialist doctor applies this therapy to 30 people.
The random variable X is assumed to be binomially distributed and gives the number of people for whom the therapy is successful.

Task:

Determine the probability that the number of people for whom the therapy is successful is greater than the expected value $E(X)$.

[0 / 1 p.]

Task 25 (Part 2)

Flights

Task:

a) In Austria in 2018, the parking fees near the airports listed below were different.

airport	parking fees per week in euros
Klagenfurt	*K*
Salzburg	54
Linz	*L*
Graz	*G*
Vienna-Schwechat	*W*
Innsbruck	147

Source: https://www.derstandard.at/story/2000079383984/ranking-wo-das-parken-teurer-ist-als-der-flug [09.08.2022].

1) Determine the percentage by which the parking fees per week at Innsbruck airport were higher than the fees at Salzburg airport. *[0 / 1 p.]*

The mean of these 6 parking fees is *D* (in euros).

2) Write down a formula that can be used to calculate the parking fees *G* at Graz airport in terms of *D* and the entries in the table above.

G = _____ *[0 / 1 p.]*

b) An airplane accelerates along the runway and takes off after 32 s. The velocity of the airplane is modelled by a linear function v in terms of the time t. The graph of the function v is shown in the diagram below.

1) Determine the distance traveled by the airplane before take-off in metres. *[0/1 p.]*

c) For a particular flight, 124 people have booked a seat.

It is assumed that the seats booked on a flight are occupied by passengers independently of each other with the probability p.

The probability of the event E is given by:

$$P(E) = 1 - \binom{124}{123} \cdot p^{123} \cdot (1 - p) - \binom{124}{124} \cdot p^{124}$$

1) Complete the gaps in the following sentence by putting a cross next to one of the given possibilities for each gap so that the sentence becomes a correct statement. *[0/1 p.]*

The event E is: "_____① _____② of the seats booked are occupied."

①	
At most	☐
Exactly	☐
At least	☐

②	
122	☐
123	☐
124	☐

Task 26 (Part 2, Best-of Assessment)

Passwords

Passwords are comprised of symbols in a pre-defined order. In a password, symbols can appear more than once.

The number of characters in a password is defined as the password length k ($k \in \mathbb{N}$, $k \geq 2$). For each of these characters, a symbol is chosen out of n different symbols ($n \in \mathbb{N}$, $n \geq 2$).

The number A of all possible passwords can be calculated using the formula $A = n^k$.

Task:

a) A particular computer can check 1 billion passwords per second. In order to check n^k passwords, the computer requires t hours.

1) Write down a formula in terms of k and n that can be used to calculate t.

$t =$ _____ *[0/1 p.]*

This formula that can be used to calculate t can be written as a function of k and n.

2) Complete the gaps in the following sentence by putting a cross next to one of the given possibilities for each gap so that the sentence becomes a correct statement. *[0/½/1 p.]*

If k is constant, then t in terms of n is _____①_____; if n is constant, then t in terms of k is _____②_____.

①	
a linear function	☐
a power function	☐
an exponential function	☐

②	
a linear function	☐
a power function	☐
an exponential function	☐

b) The password to access a particular website is created automatically by a random password generator. The random password generator chooses each symbol from 26 letters and 10 digits ($n = 36$) independently of the other symbols and with equal probability. The password length is 8 symbols ($k = 8$).

1) Determine the probability that the password only contains letters. *[0/1 p.]*

2) Determine the probability that the password contains at most 1 digit. *[0/1 p.]*

Task 27 (Part 2, Best-of Assessment)

Dogs in Austria

Dogs are very popular pets in Austria.

Task:

a) The diagram below shows the distribution of the number of dogs in Austria by federal state in the year 2018.

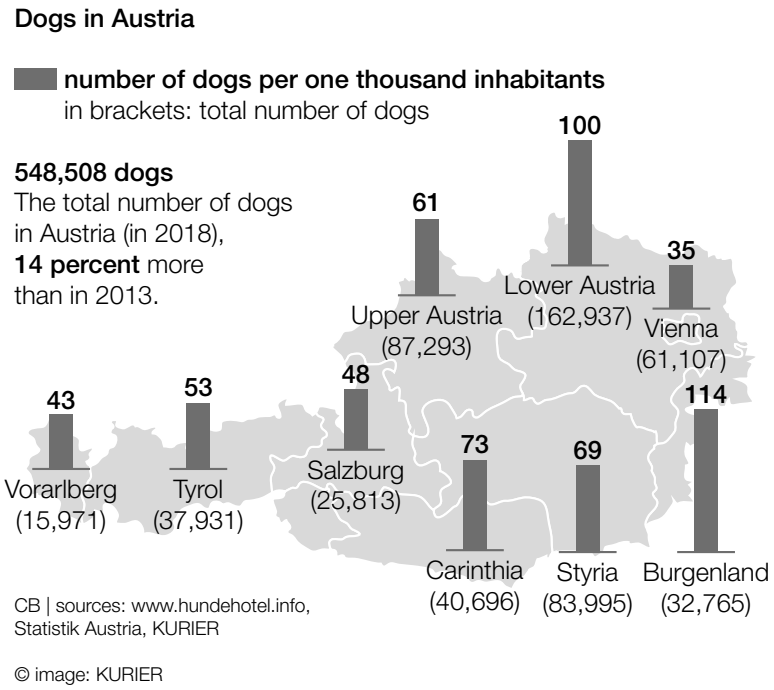
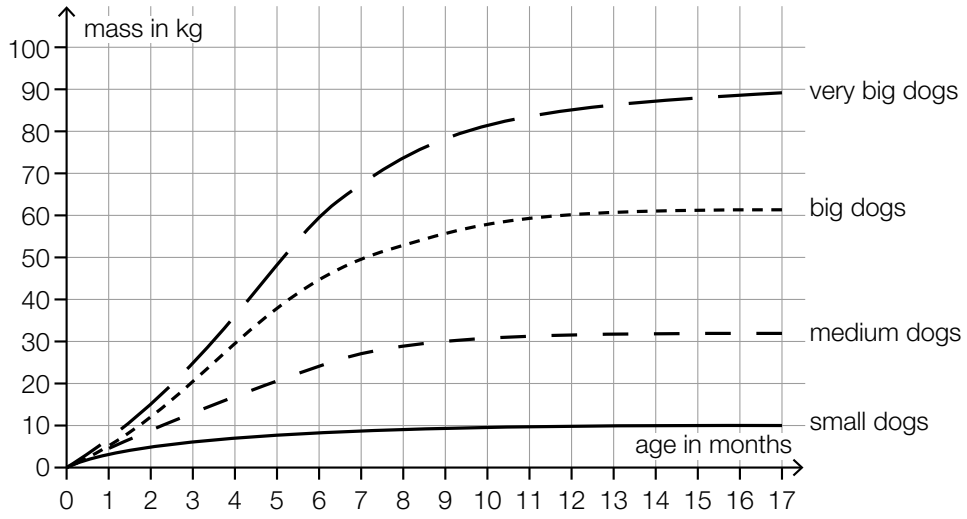


Image source: https://kurier.at/chronik/oesterreich/plus-14-prozent-hunde-liegen-voll-im-trend/400573877 [16.03.2021] (adapted).

The median number of dogs per one thousand inhabitants in the 9 federal states is equal to the number of dogs per one thousand inhabitants in one particular federal state.

1) Write down this federal state. *[0 / 1 p.]*

b) The diagram below shows the development of the masses of dogs of different sizes in the first 17 months of life.



Source: https://www.dasgesundetier.de/magazin/artikel/welpenerziehung-teil-2 [15.03.2021] (adapted).

1) Complete the two sentences below using the data from the diagram above so that they become correct statements. *[0/½/1 p.]*

"Big dogs" have a mass of around _____ kg at an age of 4 months.

"Very big dogs" have a mass of 80 kg at an age of around _____ months.

c) A *Labrador* is a breed of dog.

The *minimum mass* is the smallest mass that a healthy female Labrador should have at a given age.

The table below shows the minimum masses of female Labradors in terms of age.

age in months	3	4	5	6	7	8	9	10	11	12
minimum mass in kg	10	13	16	18	20	22	22	23	24	24

Data source: https://tierpal.de/labrador-wachstum/ [06.09.2022].

It can be assumed that the minimum masses of female Labradors grow linearly between the ages of 1 and 5 months.

1) Determine the percentage by which the minimum mass of female Labradors increases between the ages of 2 and 3 months. *[0/1 p.]*

The minimum masses of female Labradors between the ages of 7 and 15 months can be modelled by the function m: $[7, 15] \rightarrow \mathbb{R}^+$.

$m(t) = 25 - 24.7 \cdot e^{-k \cdot t}$ with $k \in \mathbb{R}^+$

t ... age in months
$m(t)$ ... minimum mass at age t in kg

For the age of 7 months, the value of the function m matches the corresponding value in the table.

For the age of 12 months, the value of the function m deviates from the corresponding value in the table above.

2) Determine this deviation in kg. *[0/1 p.]*

Task 28 (Part 2, Best-of Assessment)

Growth of Animal Populations

Task:

a) The size of the population (number of individuals) of a particular animal species can be modelled by the function N: $\mathbb{R}_0^+ \to \mathbb{R}^+$ in terms of the time t.

For this model, the following information is given:

$$N(t) = \frac{500}{1 + 4 \cdot e^{-0.2 \cdot t}}$$

t ... time in weeks
$N(t)$ ... size of the population at time t

At time t_v, the population has doubled in size since the time $t = 0$.

1) Determine t_v. *[0/1 p.]*

b) The speed of growth of a different animal population can be modelled by the polynomial function f with $f(t) = a \cdot t^2 + b \cdot t + c$ with $a, b, c \in \mathbb{R}$ and $a \neq 0$. The instantaneous rate of change of the number of individuals in terms of the time t is given by $f(t)$ (t in weeks, $f(t)$ in individuals per week).

The speed of growth at time $t = 0$ is 15 individuals per week and reaches a maximum after 7 weeks. After 35 weeks, the speed of growth is 0 individuals per week.

1) Write down an equation of the function f.

$f(t) =$ _____ *[0/1 p.]*

It is assumed that the animal population comprises 50 individuals at the beginning of the observations.

2) Interpret $50 + \int_0^7 f(t)\,dt$ in the given context. *[0/1 p.]*

One of the expressions below describes the average rate of change of the size of the animal population in the time interval $[t_1, t_2]$ with $t_1 < t_2$.

3) Put a cross next to the expression that is always correct. *[1 out of 6]* *[0/1 p.]*

$\frac{f(t_2) - f(t_1)}{t_2 - t_1}$	☐
$\frac{\int_{t_1}^{t_2} f(t)\,dt}{t_2 - t_1}$	☐
$\int_0^{t_1} f(t)\,dt - \int_0^{t_2} f(t)\,dt$	☐
$\frac{f(t_2) - f(t_1)}{t_2}$	☐
$f(t_2) - f(t_1)$	☐
$\frac{f(t_2) - f(t_1)}{f(t_1)}$	☐