Name:	
Class:	

Standardised Competence-Oriented
Written School-Leaving Examination

AHS

12th January 2017

Mathematics

Part 2 Tasks

BMB
Bundesministerium
für Bildung

Advice for Completing the Tasks

Dear candidate,

The following booklet for Part 2 contains five tasks, each of which contains between two and four sub-tasks. All sub-tasks can be completed independently of one another. You have *150 minutes* available in which to work on these tasks.

Please use a blue or black pen that cannot be rubbed out. You may use a pencil for tasks that require you to draw a graph, vectors or a geometric construction.

When completing these tasks please use this booklet and the paper provided. Write your name on each piece of paper you use as well as on the first page of this task booklet in the space provided. Please show clearly which sub-task each answer relates to.

In the assessment of your work, everything that is not crossed out will be considered. Your solutions must be clearly marked. If a solution is not clearly marked or if more than one solution is given, the task will be considered to be unsolved. Draw a line through any notes you make.

You may use a pre-approved formula book as well as your usual electronic device(s).

Please hand in both the task booklet and the separate sheets you have used at the end of the examination.

Assessment

Every task in Part 1 will be awarded either 0 points or 1 point. Every sub-task in Part 2 will be awarded 0, 1 or 2 points. The tasks marked with an [A] will be awarded either 0 points or 1 point.

– If at least 16 of the 24 tasks in Part 1 are solved correctly, you will pass the examination.
– If fewer than 16 of the 24 tasks in Part 1 are solved correctly, then the tasks marked with an [A] from Part 2 may compensate for the shortfall (as part of the "range of essential skills" outlined by the LVBO).
  If, including the tasks marked with an [A] from Part 2, at least 16 tasks are solved correctly, you will pass the examination.
  If, including the tasks marked with an [A] from Part 2, fewer than 16 tasks are solved correctly, you will not be awarded enough points to pass the examination.
– If at least 16 tasks are solved correctly (including the compensation tasks marked with an [A] from Part 2), a grade will be awarded as follows:

Pass	16–23 points
Satisfactory	24–32 points
Good	33–40 points
Very Good	41–48 points

Explanation of the Task Types

Some tasks require a ***free answer***. For these tasks, you should write your answer directly underneath each task in the task booklet or on the paper provided. Other task types used in the examination are as follows:

Matching tasks: For this task type you will be given a number of statements, tables or diagrams, which will appear alongside a selection of possible answers. To correctly answer these tasks, you will need to match each statement, table or diagram to its corresponding answer. You should write **the letter of the correct answer** next to the statement, table or diagram in the space provided.

Example:
You are given two equations.

Task:
Match the two equations to their corresponding description (from A to D).

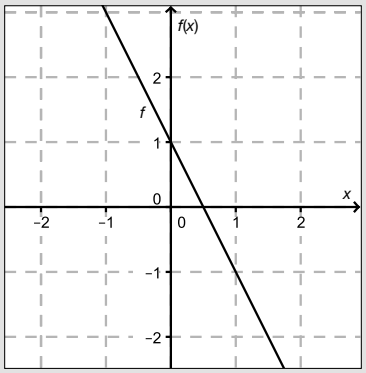
$1 + 1 = 2$	*A*
$2 \cdot 2 = 4$	*C*

A	Addition
B	Division
C	Multiplication
D	Subtraction

Construction tasks: This task type requires you to draw points, lines and/or curves in the task booklet.

Example:
Below you will see a linear function f where $f(x) = k \cdot x + d$.

Task:
On the axes provided below, draw the graph of a linear function for which $k = -2$ and $d > 0$.



Multiple-choice tasks of the form "1 out of 6": This task type consists of a question and six possible answers. Only **one answer** should be selected. You should put a cross next to the only correct answer in the space provided.

Example:
Which equation is correct?

Task:
Put a cross next to the correct equation.

1 + 1 = 1	☐
2 + 2 = 2	☐
3 + 3 = 3	☐
4 + 4 = 8	☒
5 + 5 = 5	☐
6 + 6 = 6	☐

Multiple-choice tasks of the form "2 out of 5": This task type consists of a question and five possible answers, of which **two answers** should be selected. You should put a cross next to each of the two correct answers in the space provided.

Example:
Which equations are correct?

Task:
Put a cross next to each of the two correct equations.

1 + 1 = 1	☐
2 + 2 = 4	☒
3 + 3 = 3	☐
4 + 4 = 8	☒
5 + 5 = 5	☐

Multiple-choice tasks of the form "x out of 5": This task type consists of a question and five possible answers, of which **one, two, three, four *or* five answers** may be selected. The task will require you to: "Put a cross next to each correct statement/equation …". You should put a cross next to each correct answer in the space provided.

Example:
Which of the equations given are correct?

Task:
Put a cross next to each correct equation.

1 + 1 = 2	☒
2 + 2 = 4	☒
3 + 3 = 6	☒
4 + 4 = 4	☐
5 + 5 = 10	☒

Gap-fill: This task type consists of a sentence with two gaps, i.e. two sections of the sentence are missing and must be completed. For each gap you will be given the choice of three possible answers. You should put a cross next to **each of the two answers** that are necessary to complete the sentence correctly.

Example:
Below you will see 3 equations.

Task:
Complete the following sentence by putting a cross next to one of the given possibilities for each gap so that the sentence becomes a correct statement.

The operation in equation _____ ① _____ is known as summation or _____ ② _____ .

①	
1 – 1 = 0	☐
1 + 1 = 2	☒
1 · 1 = 1	☐

②	
Multiplication	☐
Subtraction	☐
Addition	☒

Changing an answer for a task that requires a cross:

1. Fill in the box that contains the cross for your original answer.
2. Put a cross in the box next to your new answer.

1 + 1 = 3	☐
2 + 2 = 4	☒
3 + 3 = 5	☐
4 + 4 = 4	☐
5 + 5 = 9	■

In this instance, the answer "5 + 5 = 9" was originally chosen. The answer was later changed to be "2 + 2 = 4".

Selecting an answer that has been filled in:

1. Fill in the box that contains the cross for the answer you do not wish to give.
2. Put a circle around the filled-in box you would like to select.

1 + 1 = 3	☐
2 + 2 = 4	(■)
3 + 3 = 5	☐
4 + 4 = 4	■
5 + 5 = 9	☐

In this instance, the answer "2 + 2 = 4" was filled in and then selected again.

If you still have any questions, please ask your teacher.

Good Luck!

Task 1

Graphs of Third Degree Polynomial Functions

The shape of a graph of a third degree polynomial function f with equation $f(x) = a \cdot x^3 + b \cdot x^2 + c \cdot x + d$ where $a, b, c, d \in \mathbb{R}, a \neq 0$ depends of the values of the coefficients a, b, c, d .

By choosing values for the coefficients, the number and position of the zeros, maxima, minima and points of inflexion of f, among other things, are determined.

Task:

a) What is the greatest number of local maxima and minima that f can have? Write down this number and justify your answer on the basis of the derivative of f.

   A Show that for the particular case $a = 1, b = -3, c = 3, d = 0$, the function f has no local maxima or minima.

b) If the relationship $x \in \mathbb{R}$ holds for all $f(-x) = -f(x)$ then the graph of f is symmetrical about the origin.

   Write down the values of the coefficients b and d for which the graph of f with equation $f(x) = a \cdot x^3 + b \cdot x^2 + c \cdot x + d$ would be symmetric about the origin.

   For one such function f, determine the value of the integral $\int_{-x_1}^{x_1} f(x)\,\mathrm{d}x$ for an arbitrary $x_1 > 0$ and give an explanation for your solution.

c) The graph of f definitely has a point of inflexion. Which of the coefficients a, b, c, d determines that the point of inflexion of the function f lies on the vertical axis? Write down this coefficient and the corresponding condition.

   Write down an additional condition that is necessary for the graph of f to have a tangent parallel to the x-axis at the point of inflexion $W = (0|f(0))$.

Task 2

Ebola

Ebola is a contagious disease that is caused by a virus. The Ebola epidemic that broke out in many West African countries in 2014 is recognised by the World Health Organisation (WHO) as the worst Ebola epidemic to date. The course of the epidemic was precisely observed and documented by the WHO.

The table below shows an extract of the documentation from the WHO for the countries Guinea, Liberia and Sierra Leone for three days in September 2014. For each day, the total number of people infected (cases) is shown.

Date	6th September	13th September	20th September
Total Number of Cases	4,269	4,963	5,843

Data source: http://www.who.int/csr/disease/ebola/situation-reports/en/ [20.09.2014].

Task:

a) Write down the meaning of the expressions $4{,}963 - 4{,}269$ and $\frac{4{,}963 - 4{,}269}{4{,}269}$ in the given context.

Using these expressions, based on the number of cases on the 6th September 2014 and the 13th September 2014, the number of cases on the 20th September 2014 can be predicted on the basis of a linear or an exponential growth model.

Determine the values of both growth models for the 20th September 2014, compare the results with the actual data and write down which of the two models is more appropriate for modelling the number of cases within this timeframe.

b) In mid-September 2014, the *New York Times* cited the claim from scientists that the epidemic could last 12 to 18 months, and that by mid-October 2014 there could be 20,000 people infected.
Data source: http://www.nytimes.com/2014/09/13/world/africa/us-scientists-see-long-fight-against-ebola.html [29.06.2016].

The development of the number of cases in an epidemic can, for a limited time, be described by an exponential function. On the basis of the number of people infected on the 6th September 2014 and on the 20th September 2014, the number of cases should be modelled by an exponential function f where $f(t) = a \cdot b^t$. The time, t, is measured in days from the 6th September 2014, and the time $t = 0$ corresponds to the 6th September 2014.

A Determine the value of b.

Determine after how many days from the 6th September 2014 the number of cases is expected to exceed 20,000 according to the model, and compare your result with the scientists' claim.

Task 3

Net Monthly Income

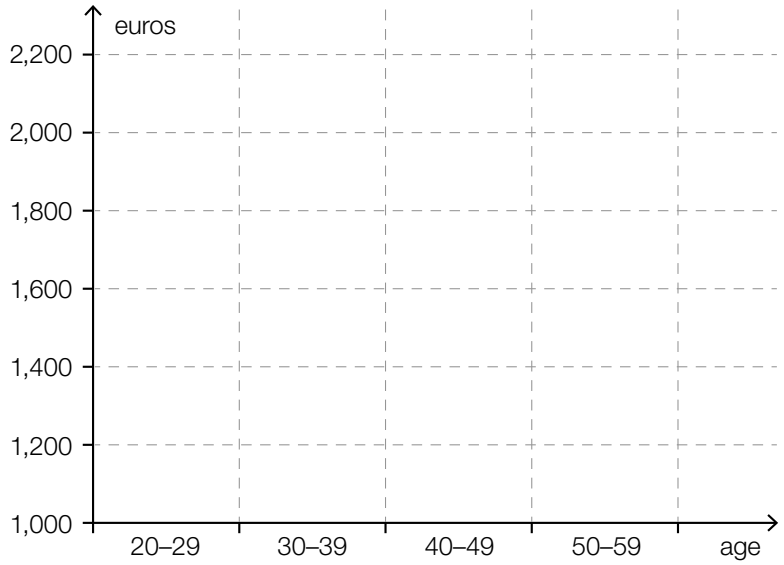
The net monthly income of employed people depends on socioeconomic factors such as age, nationality, education, level of employment and professional position. The table below shows data about the annual average of net monthly incomes of employees in Austria in 2010, broken down by socioeconomic factors. All of the following tasks are related to this data from the year 2010.

Characteristics	Employees	Arithmetic Mean	10 %	Quartile			90 %
				25 %	50 % (median)	75 %	
	in 1,000 people	in euros	earn less than or the same amount as ... (in euros)				
Total	3,407.9	1,872.7	665.0	1,188.0	1,707.0	2,303.0	3,122.0
Age							
15–19 years	173.5	799.4	399.0	531.0	730.0	1,020.0	1,315.0
20–29 years	705.1	1,487.0	598.0	1,114.0	1,506.0	1,843.0	2,175.0
30–39 years	803.1	1,885.7	770.0	1,252.0	1,778.0	2,306.0	2,997.0
40–49 years	1,020.4	2,086.1	863.0	1,338.0	1,892.0	2,556.0	3,442.0
50–59 years	632.8	2,205.0	893.0	1,394.0	1,977.0	2,779.0	3,710.0
60+ years	73.0	2,144.7	258.0	420.0	1,681.0	3,254.0	4,808.0
Highest Level of Education Completed							
Compulsory Education	523.4	1,183.0	439.0	677.0	1,104.0	1,564.0	1,985.0
Apprenticeship	1,385.2	1,789.3	833.0	1,303.0	1,724.0	2,143.0	2,707.0
Vocational Schools	454.4	1,777.1	733.0	1,199.0	1,677.0	2,231.0	2,824.0
College	557.2	2,061.6	590.0	1,218.0	1,824.0	2,624.0	3,678.0
University	487.7	2,723.4	1,157.0	1,758.0	2,480.0	3,376.0	4,567.0
Professional Position							
Apprentice	134.2	775.3	466.0	551.0	705.0	930.0	1,167.0
White Collar Worker	1,800.3	2,018.1	705.0	1,222.0	1,771.0	2,489.0	3,550.0
Blue Collar Worker	1,030.9	1,539.3	627.0	1,135.0	1,554.0	1,922.0	2,274.0
Official	442.5	2,391.4	1,377.0	1,800.0	2,295.0	2,848.0	3,492.0

Data source: Statistik Austria (ed.) (2012). Arbeitsmarktstatistik. Jahresergebnisse 2011. Mikrozensus-Arbeitskräfteerhebung. Vienna: Statistik Austria, p. 81 (adapted).

Task:

a) In the space below, draw a diagram that shows the median incomes of 20 to 59 year olds. For your diagram, use the median incomes rounded to two decimal places.



Is it possible to represent the net monthly income of 20 to 29 year olds and the 30 to 39 year olds in boxplots on the basis of the data given in the table? Justify your answer.

b) Someone calculated the arithmetic mean of all net monthly incomes by using the means of the six age groups in the following way:

$$\frac{799.4 + 1{,}487.0 + 1{,}885.7 + 2{,}086.1 + 2{,}205.0 + 2{,}144.7}{6} \approx 1{,}768.0$$

However, in the table given the arithmetic mean of all incomes is shown as 1,872.7.
Explain why the calculation shown above does not give the correct result and write down the correct method for this calculation.

For the age group 60+, the arithmetic mean of the net monthly incomes is considerably larger (by almost € 500) than the median income for this age group. Write down the inference that can be made on the basis of the very low or very high net monthly incomes in this age group.

c) [A] Write down the values of the 1st and 3rd quartiles of the net monthly incomes of the employees whose highest completed level of education is compulsory education.

1st quartile: _____

3rd quartile: _____

The interquartile range is the difference of the 3rd and 1st quartiles.
An expert claims: “With increasing levels of completed education that exceed compulsory education, the interquartile range of the net monthly incomes increases.” Verify or contradict this statement by using the data given in the table.

d) The data given in the table shows that around 53 % of the employees are white collar workers and around 30 % are blue collar workers.

A comment about the labour market report says: “The relative proportion of white collar workers is approximately 23 % higher than the relative proportion of blue collar workers.” Is this statement correct? Justify your answer.

Check the following statements about net monthly incomes on the basis of the given data. Put a cross next to each of the two correct statements.

White collar workers earn, on average, over € 500 more than blue collar workers.	☐
At most a quarter of blue collar workers earn more than € 1,922.	☐
From the data of the table, the range of net monthly income cannot be stated exactly.	☐
Three quarters of apprentices earn at least € 930.	☐
Precisely half of officials earn exactly € 1,800.	☐

Task 4

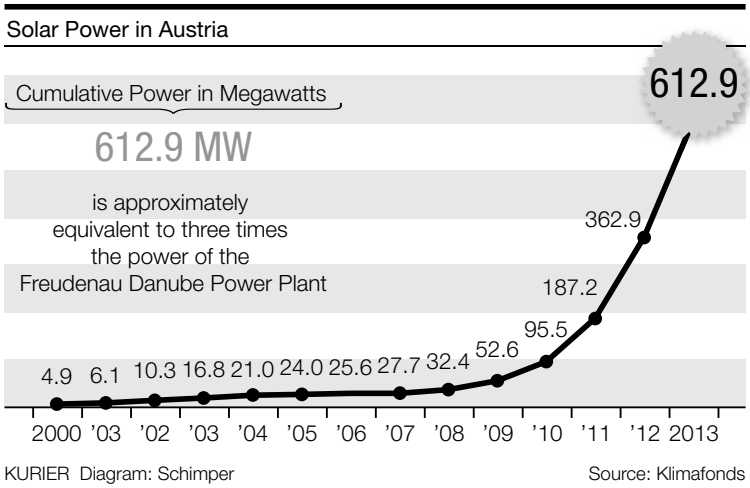
Solar Power in Austria

In a photovoltaic plant, sunlight is converted into electric energy by solar cells and thereby "solar power" is created. In Austria, solar panels work most efficiently if they are directed towards the south.

For this purpose, there are two different systems: the supported systems, which mean that the solar panels can be moved depending on the direction of the Sun's rays, and the cheaper roof-mounted systems that remain parallel to the roof.

Excess electricity can be fed into the public grid, which further lowers the electricity costs of a household.

The diagram below shows the increase in solar power since the year 2000.



Source: http://kurier.at/wirtschaft/ein-oel-mann-wird-zum-solar-fan/42.474.775 [28.06.2016] (adapted).

The number of years since the year 2000 is given by t and $f(t)$ represents the power (in MW) after t years shown in the diagram above.

Task:

a) [A] Determine and interpret the difference quotient $\frac{f(13) - f(0)}{13}$ in the given context.

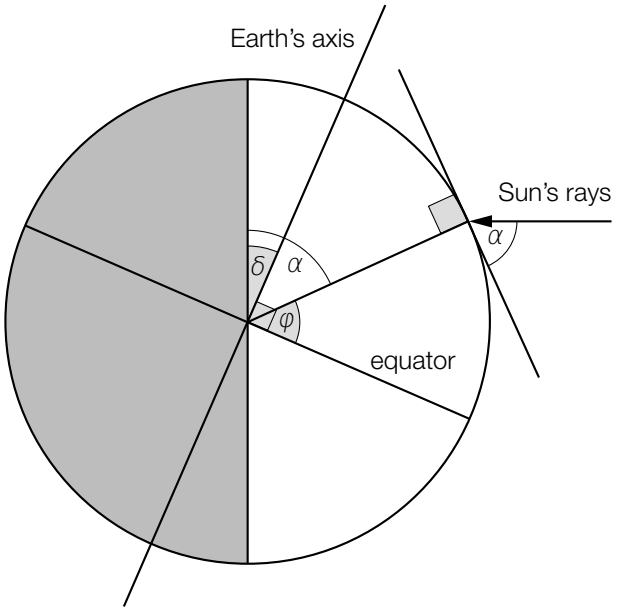
Write down the meaning of the integral $\int_0^{13} f(t)\,dt$ with respect to the creation of solar power.

b) Justify why the power (in MW) shown in the diagram above for the time period [9 years, 12 years] can be approximated well by an exponential function g with equation $g(t) = a \cdot b^t$.

Determine the the expression $\frac{f(12) - f(9)}{f(9)} + 1$ by using the parameter b of the function g.

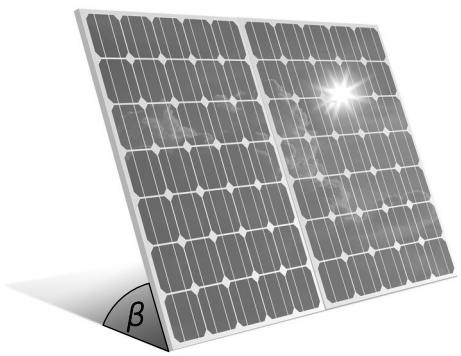
c) Energy production from a photovoltaic plant is highest if the Sun's rays fall at right angles onto the solar cells.

Thus, the optimal tilt of a photovoltaic panel depends on the angle of incidence of the Sun's rays. For the northern hemisphere, this angle is largest on the 21st June.



In the diagram above, the angle of incidence of the Sun's rays, α, is shown on 21st June at noon. The angle $\delta \approx 23.5°$ gives the tilt of the Earth's axis to the orbital plane of the Sun and the angle φ the latitude.

Determine a formula for the angle of incidence α of the Sun's rays dependent on the latitude φ.



Source: http://www.solaranlage.eu/sites/default/files/bilder/reflexionsverluste-solarmodule.jpg [14.11.2016] (adapted).

The diagram above shows photovoltaic panels that are tilted at an angle of β to the horizontal. These solar panels are facing towards the Sun.

Determine a formula for the optimal tilting angle of the photovoltaic panels, β_{opt}, in terms of α so that the Sun's rays are perpendicular to the panel. Write down how this angle changes in winter or at higher latitudes. Justify your answer.