Name:	
Class:	

Standardised Competence-Oriented
Written School-Leaving Examination

AHS

10th May 2017

Mathematics

Part 1 Tasks

Bundesministerium
für Bildung

Advice for Completing the Tasks

Dear candidate,

The following booklet for Part 1 contains 24 tasks. The tasks can be completed independently of one another. You have *120 minutes* available in which to work through this booklet.

Please use a blue or black pen that cannot be rubbed out. You may use a pencil for tasks that require you to draw a graph, vectors or a geometric construction.

Please do all of your working out solely in this booklet. Write your name on the first page of the booklet in the space provided.

All answers must be written in this booklet. In the assessment of your work, everything that is not crossed out will be considered. Your solutions must be clearly marked. If a solution is not clearly marked or if more than one solution is given, the task will be considered to be unsolved. Draw a line through any notes you make.

You may use a pre-approved formula book as well as your usual electronic device(s).

Please hand in the task booklet at the end of the examination.

Assessment

Every task in Part 1 will be awarded either 0 points or 1 point. Every sub-task in Part 2 will be awarded 0, 1 or 2 points. The tasks marked with an $\boxed{\text{A}}$ will be awarded either 0 points or 1 point.

– If at least 16 of the 24 tasks in Part 1 are solved correctly, you will pass the examination.
– If fewer than 16 of the 24 tasks in Part 1 are solved correctly, then the tasks marked with an $\boxed{\text{A}}$ from Part 2 may compensate for the shortfall (as part of the “range of essential skills” outlined by the LVBO).
  If, including the tasks marked with an $\boxed{\text{A}}$ from Part 2, at least 16 tasks are solved correctly, you will pass the examination.
  If, including the tasks marked with an $\boxed{\text{A}}$ from Part 2, fewer than 16 tasks are solved correctly, you will not be awarded enough points to pass the examination.
– If at least 16 tasks are solved correctly (including the compensation tasks marked with an $\boxed{\text{A}}$ from Part 2), a grade will be awarded as follows:

Pass	16–23 points
Satisfactory	24–32 points
Good	33–40 points
Very Good	41–48 points

Explanation of the Task Types

Some tasks require a ***free answer***. For these tasks, you should write your answer directly underneath each task in the task booklet. Other task types used in the examination are as follows:

Matching tasks: For this task type you will be given a number of statements, tables or diagrams, which will appear alongside a selection of possible answers. To correctly answer these tasks, you will need to match each statement, table or diagram to its corresponding answer. You should write **the letter of the correct answer** next to the statement, table or diagram in the space provided.

Example:
You are given two equations.

Task:
Match the two equations to their corresponding description (from A to D).

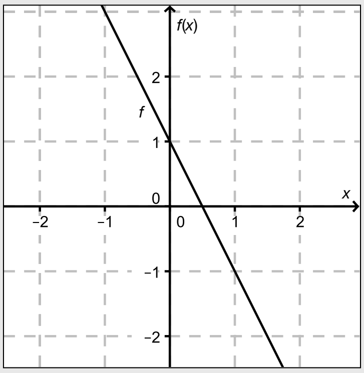
$1 + 1 = 2$	A
$2 \cdot 2 = 4$	C

A	Addition
B	Division
C	Multiplication
D	Subtraction

Construction tasks: This task type requires you to draw points, lines and/or curves in the task booklet.

Example:
Below you will see a linear function f where $f(x) = k \cdot x + d$.

Task:
On the axes provided below, draw the graph of a linear function for which $k = -2$ and $d > 0$.



Multiple-choice tasks of the form "1 out of 6": This task type consists of a question and six possible answers. Only one answer should be selected. You should put a cross next to the only correct answer in the space provided.

Example:
Which equation is correct?

Task:
Put a cross next to the correct equation.

1 + 1 = 1	☐
2 + 2 = 2	☐
3 + 3 = 3	☐
4 + 4 = 8	☒
5 + 5 = 5	☐
6 + 6 = 6	☐

Multiple-choice tasks of the form "2 out of 5": This task type consists of a question and five possible answers, of which two answers should be selected. You should put a cross next to each of the two correct answers in the space provided.

Example:
Which equations are correct?

Task:
Put a cross next to each of the two correct equations.

1 + 1 = 1	☐
2 + 2 = 4	☒
3 + 3 = 3	☐
4 + 4 = 8	☒
5 + 5 = 5	☐

Multiple-choice tasks of the form "x out of 5": This task type consists of a question and five possible answers, of which one, two, three, four *or* five answers may be selected. The task will require you to: "Put a cross next to each correct statement/equation …". You should put a cross next to each correct answer in the space provided.

Example:
Which of the equations given are correct?

Task:
Put a cross next to each correct equation.

1 + 1 = 2	☒
2 + 2 = 4	☒
3 + 3 = 6	☒
4 + 4 = 4	☐
5 + 5 = 10	☒

Gap-fill: This task type consists of a sentence with two gaps, i.e. two sections of the sentence are missing and must be completed. For each gap you will be given the choice of three possible answers. You should put a cross next to **each of the two answers** that are necessary to complete the sentence correctly.

Example:
Below you will see 3 equations.

Task:
Complete the following sentence by putting a cross next to one of the given possibilities for each gap so that the sentence becomes a correct statement.

The operation in equation ____①____ is known as summation or ____②____ .

①	
1 – 1 = 0	☐
1 + 1 = 2	☒
1 · 1 = 1	☐

②	
Multiplication	☐
Subtraction	☐
Addition	☒

Changing an answer for a task that requires a cross:

1. Fill in the box that contains the cross for your original answer.
2. Put a cross in the box next to your new answer.

1 + 1 = 3	☐
2 + 2 = 4	☒
3 + 3 = 5	☐
4 + 4 = 4	☐
5 + 5 = 9	■

In this instance, the answer “5 + 5 = 9” was originally chosen. The answer was later changed to be “2 + 2 = 4”.

Selecting an answer that has been filled in:

1. Fill in the box that contains the cross for the answer you do not wish to give.
2. Put a circle around the filled-in box you would like to select.

1 + 1 = 3	☐
2 + 2 = 4	■
3 + 3 = 5	☐
4 + 4 = 4	■
5 + 5 = 9	☐

In this instance, the answer “2 + 2 = 4” was filled in and then selected again.

If you still have any questions now, please ask your teacher.

Good Luck!

Task 1

Integers

Let a be a positive integer.

Task:

Which of the following expressions for $a \in \mathbb{Z}^+$ always result in an integer?
Put a cross next to each of the two correct expressions.

a^{-1}	☐
a^2	☐
$a^{\frac{1}{2}}$	☐
$3 \cdot a$	☐
$\frac{a}{2}$	☐

Task 2

Investment

An annual interest rate of 1.2 % is paid on an investment K for 5 years.

Task:

The following expression is given:

$K \cdot 1.012^5 - K$

Write down the meaning of this expression in the given context.

Task 3

Animal Feed

A farmer has bought two types of pre-prepared animal feed for his cattle.
Feed *A* has a protein content of 14 %, whereas feed *B* has a protein content of 35 %.
The farmer would like to create 100 kg of mixed feed for his male calves by mixing the two types of pre-prepared feeds. The resulting feed should have a protein content of 18 %. The mixture will contain *a* kg of feed *A* and *b* kg of feed *B*.

Task:

Write down two equations in terms of the variables *a* and *b* that can be used to calculate the necessary amounts for the mixture.

1st equation: ____

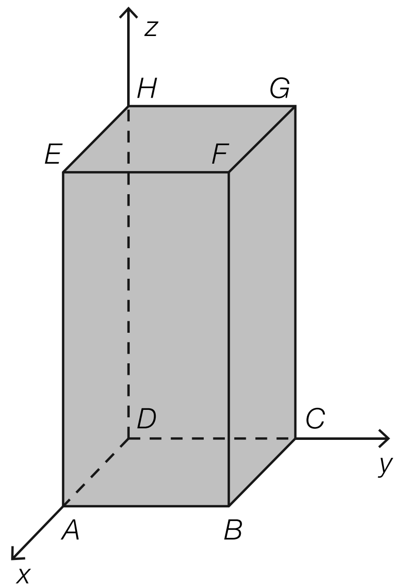
2nd equation: ____

Task 4

Cuboid with a Square Base

The diagram below shows a cuboid whose square base lies in the xy-plane. The length of an edge of the base is 5 units; the height of the solid is 10 units. The vertex D is located at the origin, and the vertex C lies on the positive y-axis.

Thus the vertex E has coordinates $E = (5,0,10)$.



Task:

Determine the coordinates (components) of the vector $\overrightarrow{HB}$.

Task 5

Parallel Lines

Let g and h be lines with vector equations:

$$g\colon X = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} + t \cdot \begin{pmatrix} -3 \\ 1 \\ 2 \end{pmatrix} \quad \text{where } t \in \mathbb{R}$$

$$h\colon X = \begin{pmatrix} 3 \\ 1 \\ 1 \end{pmatrix} + s \cdot \begin{pmatrix} 6 \\ h_y \\ h_z \end{pmatrix} \quad \text{where } s \in \mathbb{R}$$

Task:

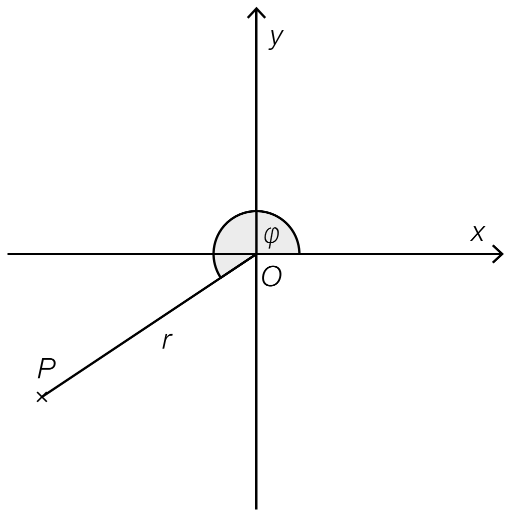
Determine the components h_y and h_z of the direction vector of line h such that line h is parallel to line g.

Task 6

Coordinates of a Point

In the diagram below the point $P = (-3,-2)$ is shown.

The location of the point P can also be uniquely determined by the distance $r = \overline{OP}$ and the size of the angle φ.



Task:

Determine the size of the angle φ.

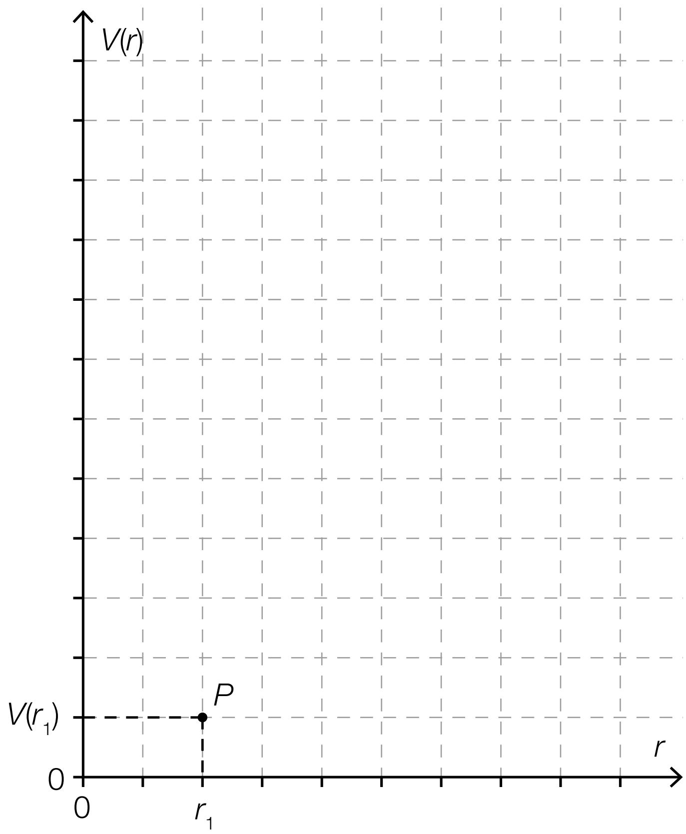
Task 7

Volume of a Cylinder

The base of a cylinder has radius r and the height of the cylinder is given by h. If the height of the cylinder is constant, then the function V where $V(r) = r^2 \cdot \pi \cdot h$ describes the volume of the cylinder with respect to the radius.

Task:

The point $P = (r_1, V(r_1))$ is shown in the coordinate system given below. Plot the point $Q = (3 \cdot r_1, V(3 \cdot r_1))$ in the coordinate system.



Task 8

Concavity of a Polynomial Function

The graph of a third degree polynomial function has a local minimum at point $T = (-3,1)$, a local maximum at point $H = (-1,3)$ and a point of inflexion at point $W = (-2,2)$.

Task:

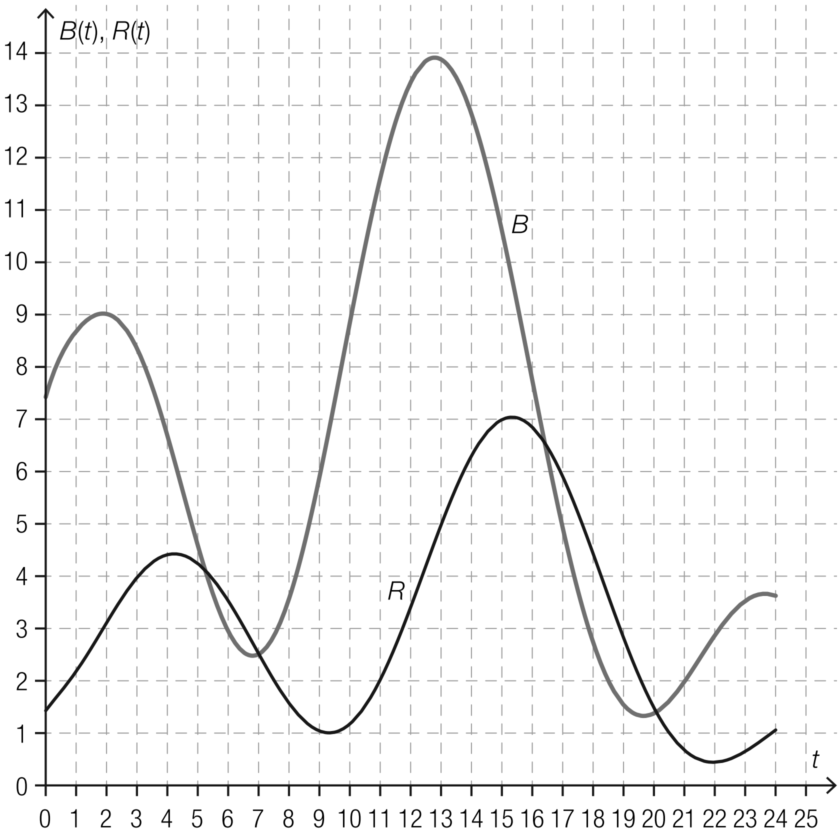
In which interval is the function concave up?
Put a cross next to the correct interval.

$(-\infty, 2)$	☐
$(-\infty, -2)$	☐
$(-3, -1)$	☐
$(-2, 2)$	☐
$(-2, \infty)$	☐
$(3, \infty)$	☐

Task 9

Predator-Prey Model

The predator-prey model gives a simplified model of population fluctuations of a predator group (e.g. the number of Canadian lynxes) and a prey group (e.g. the number of snowshoe hares). The functions R and B shown in the diagram below model the number of predators $R(t)$ and the number of prey animals $B(t)$ for an observed period of 24 years ($B(t)$, $R(t)$ are given in 10,000 individuals, t in years).



Task:

Write down all of the intervals in the observed time period for which both the predator and prey populations are decreasing.

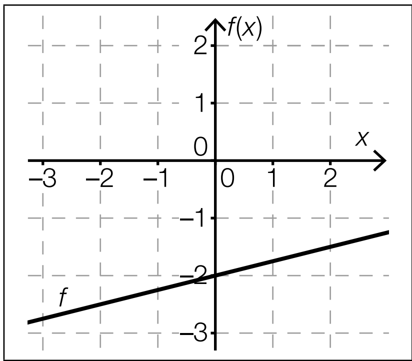
Task 10

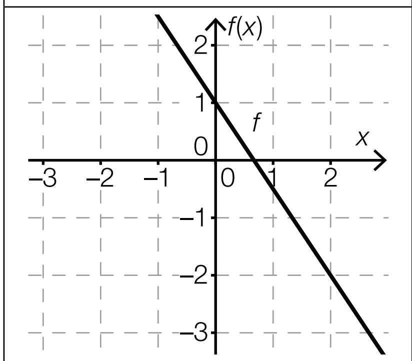
Linear Functions

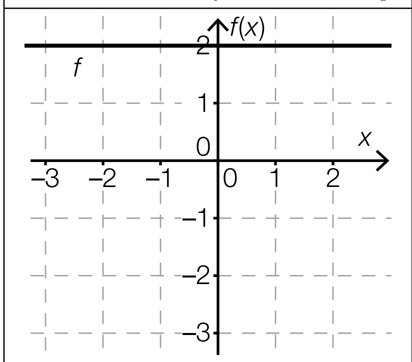
The graphs of four different linear functions f where $f(x) = m \cdot x + c$ and $m, c \in \mathbb{R}$ are shown in the diagram below.

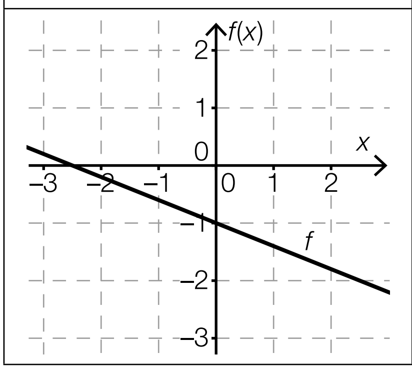
Task:

Match each of the four graphs to the corresponding statement about the parameters m and c (from A to F).









A	$m = 0, c < 0$
B	$m > 0, c > 0$
C	$m = 0, c > 0$
D	$m < 0, c < 0$
E	$m > 0, c < 0$
F	$m < 0, c > 0$

Task 11

Negative Values of a Function

Let f be a real function with equation $f(x) = x^2 - x - 6$. A value of the function $f(x)$ is negative if $f(x) < 0$.

Task:

Determine all $x \in \mathbb{R}$ for which the corresponding value of the function $f(x)$ is negative.

Task 12

Half-Life of Cobalt-60

The radioactive isotope cobalt-60 is used, among other purposes, for preserving food and in medicine.
The law of decay for cobalt-60 is $N(t) = N_0 \cdot e^{-0.13149 \cdot t}$ where t is measured in years. In this expression, N_0 gives the amount of the isotope at time $t = 0$ and $N(t)$ the amount at time $t \geq 0$.

Task:

Determine the half-life of cobalt-60.

Task 13

Performance Improvement

Three people, *A*, *B* and *C*, undertake a coordination test both before and after a special training programme. The numbers of points achieved in the test are shown in the table below.

	Person *A*	Person *B*	Person *C*
Number of points achieved before the special training programme	5	15	20
Number of points achieved after the special training programme	8	19	35

A good performance is shown by a high number of points. It is evident from the table that all three people achieve more points after the special training programme than they did previously.

Task:

List the two people from the group *A*, *B*, *C* that satisfy the conditions given below.

- For the first person, the absolute change in the number of points is greater than for the second person.
- For the second person, the relative change in the number of points is greater than for the first person.

First person: ___________________

Second person: ___________________

Task 14

Financial Debt

The financial debt of Austria increased in the time period from 2000 to 2010. In the year 2000, Austria's financial debt amounted to F_0; ten years later it amounted to F_1 (each in billions of euros).

Task:

Interpret the expression $\frac{F_1 - F_0}{10}$ in the context of the development of Austria's financial debt.

Task 15

Difference Equation

The table below contains the values of a sequence at time n ($n \in \mathbb{N}$).

n	x_n
0	10
1	21
2	43
3	87

The development of this sequence over time can be described by a difference equation of the form $x_{n+1} = a \cdot x_n + b$.

Task:

Write down the values of the (real) parameters a and b that describe the sequence shown in the table above.

a = ____

b = ____

Task 16

Depth of a Channel

As a preventative measure in the event of high water, a town has installed a channel (or water course).

The function f describes the depth of the water in the channel in the event of high water in terms of the time t at a particular point in the channel in the time interval [0, 2].

The equation of the function f is $f(t) = t^3 + 6 \cdot t^2 + 12 \cdot t + 8$ where $t \in [0, 2]$.

In this function, $f(t)$ is measured in dm and t in days.

Task:

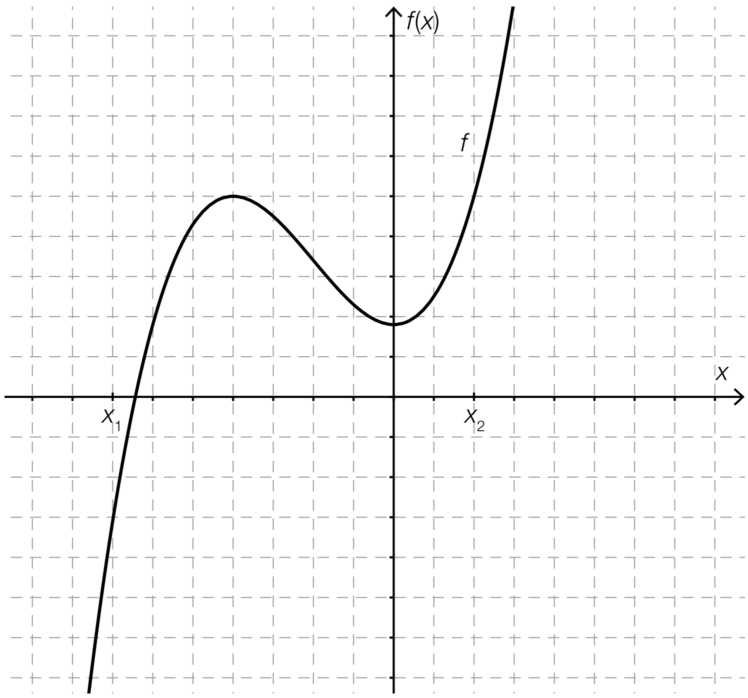
Write down an equation of the function g that gives the instantaneous rate of change of the depth of the water in the channel (in dm per day) in terms of time t.

$g(t) =$ ____

Task 17

Differentiating Graphically

The graph of a third degree polynomial function f is shown below.



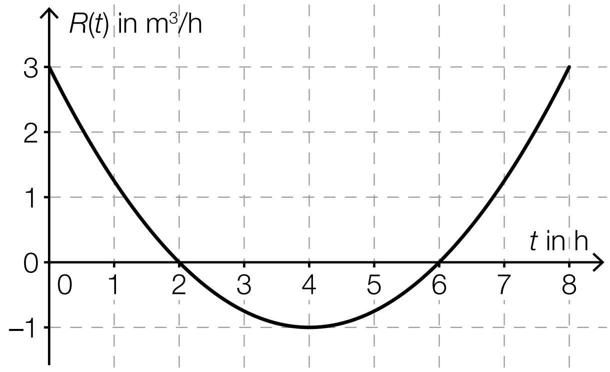
Task:

Sketch the graph of the first derivative f' in the interval $[x_1, x_2]$ on the diagram above and mark the zeros as necessary.

Task 18

Amount of Water in a Container

In the diagram shown below, the instantaneous rate of change R of the amount of water in a container (in m^3/h) is shown in terms of the time t.



Task:

Put a cross next to each of the two correct statements about the amount of water in the container.

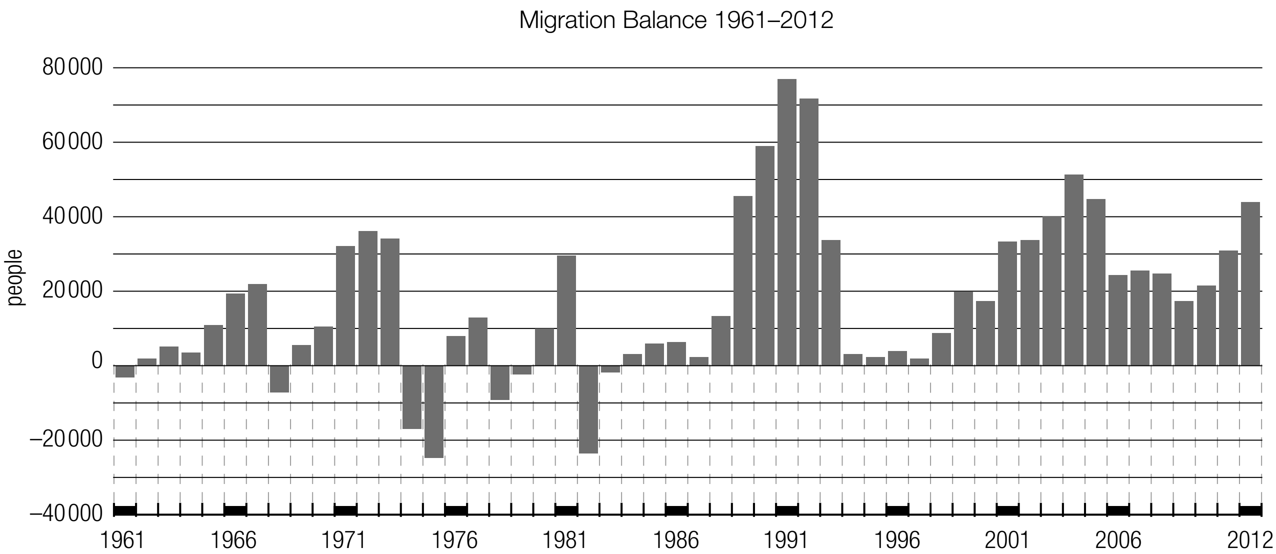
At time $t = 6$ there is less water in the container than at time $t = 2$.	☐
In the time interval (6, 8) the amount of water in the container increases.	☐
At time $t = 2$ there is no water in the container.	☐
In the time interval (0, 2) the amount of water in the container decreases.	☐
At time $t = 4$ there is the least water in the container.	☐

Task 19

Migration Balance for Austria

In a given time period, the difference between the number of immigrants that come into a country and the number of emigrants that leave a country in this period is known as the *migration balance*.

In the diagram below, the annual migration balance for Austria is shown for the years from 1961 to 2012.



Source: STATISTIK AUSTRIA, Errechnete Wanderungsbilanz 1961–1995; Wanderungsstatistik 1996–2012; 2007–2011: revidierte Daten. Wanderungsbilanz: Zuzüge aus dem Ausland minus Wegzüge in das Ausland (adapted).

Task:

Put a cross next to each of the two statements that give a correct interpretation of the diagram.

From the value given for the year 2003, the diagram shows that around 40,000 more people immigrated than emigrated in that year.	☐
The growth in the migration balance from the year 2003 to the year 2004 is around 50 %.	☐
In the time period from 1961 to 2012, there are eight years for which the number of immigrants was smaller than the number of emigrants.	☐
In the time period from 1961 to 2012, there are three years for which the number of immigrants was equal to the number of emigrants.	☐
The migration balance for the year 1981 is approximately twice as large as it was for the year 1970.	☐

Task 20

Alarm Systems

In the case of a break-in, a particular alarm system is triggered with a probability of 0.9. A family has two of these systems installed in their house in such a way that the alarms are triggered independently of one another.

Task:

Determine the probability that at least one of the alarms sounds if the house is broken into.

Task 21

Youth Group

A youth group has 21 members. For a game, teams should be formed.

Task:

Complete the following sentence by putting a cross next to one of the given possibilities for each gap so that the sentence becomes a correct statement.

The binomial coefficient $\binom{21}{3}$ gives ____①____; its value is ____②____.

①	
how many of the 21 members are in a team if three equally-sized teams are formed	☐
how many different possibilities there are to form teams of three from the 21 members	☐
in how many ways three different tasks can be allocated to three members of the youth group	☐

②	
7	☐
1,330	☐
7,980	☐

Task 22

Statements about a Random Variable

The random variable X can only take the values 10, 20 and 30. The table below shows the probability distribution of X, where a and b are positive real numbers.

k	10	20	30
$P(X = k)$	a	b	a

Task:

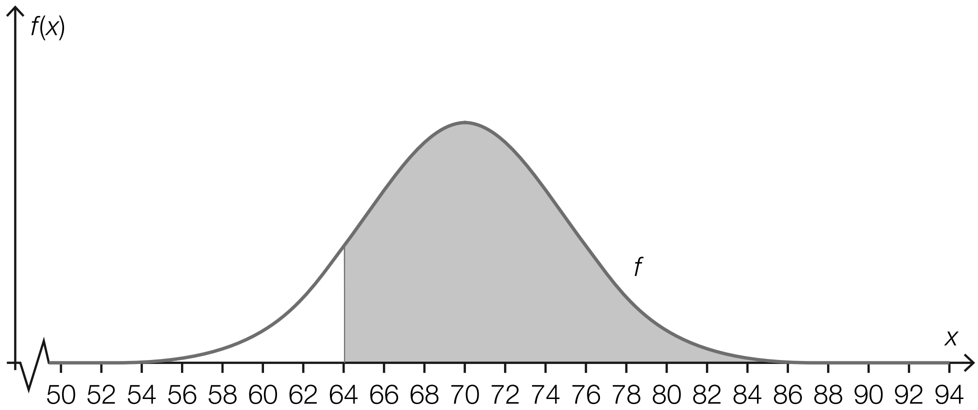
Put a cross next to each of the two correct statements.

The expectation value of X is 20.	☐
The standard deviation of X is 20.	☐
$a + b = 1$	☐
$P(10 \leq X \leq 30) = 1$	☐
$P(X \leq 10) = P(X \geq 10)$	☐

Task 23

Graphical Interpretation

The density function f of the approximated normal distribution of a binomially distributed random variable X is shown in the diagram below.



Task:

Interpret the meaning of the area of the section shaded in grey with respect to the calculation of a probability.

Task 24

Election Forecast

In order to predict the proportion of votes for a particular party *A*, a sample of randomly selected voters is surveyed.

The survey results in a 95 % confidence interval for the proportion of votes of [9.8 %, 12.2 %].

Task:

Which of the following statements are definitely true in this context?
Put a cross next to each of the two correct answers.

The probability that a randomly selected voter votes for party *A* definitely lies between 9.8 % and 12.2 %.	☐
For the data collected, a 90 % confidence interval would have had a smaller range.	☐
Under the condition that the proportion of voters for party *A* stays the same in the sample, then an increase in the sample size would result in a reduction in size of the 95 % confidence interval.	☐
95 out of 100 people say that they will vote for party *A* with a probability of 11 %.	☐
The probability that party *A* receives more than 12.2 % of the votes is 5 %.	☐