Name:	
Class:	

Standardised Competence-Oriented
Written School-Leaving Examination

AHS

10th May 2017

Mathematics

Part 2 Tasks

BMB
Bundesministerium
für Bildung

Advice for Completing the Tasks

Dear candidate,

The following booklet for Part 2 contains four tasks, each of which contains between two and four sub-tasks. All sub-tasks can be completed independently of one another. You have *150 minutes* available in which to work on these tasks.

Please use a blue or black pen that cannot be rubbed out. You may use a pencil for tasks that require you to draw a graph, vectors or a geometric construction.

When completing these tasks please use this booklet and the paper provided. Write your name on each piece of paper you use as well as on the first page of this task booklet in the space provided. Please show clearly which sub-task each answer relates to.

In the assessment of your work, everything that is not crossed out will be considered. Your solutions must be clearly marked. If a solution is not clearly marked or if more than one solution is given, the task will be considered to be unsolved. Draw a line through any notes you make.

You may use a pre-approved formula book as well as your usual electronic device(s).

Please hand in both the task booklet and the separate sheets you have used at the end of the examination.

Assessment

Every task in Part 1 will be awarded either 0 points or 1 point. Every sub-task in Part 2 will be awarded 0, 1 or 2 points. The tasks marked with an A will be awarded either 0 points or 1 point.

– If at least 16 of the 24 tasks in Part 1 are solved correctly, you will pass the examination.
– If fewer than 16 of the 24 tasks in Part 1 are solved correctly, then the tasks marked with an A from Part 2 may compensate for the shortfall (as part of the "range of essential skills" outlined by the LVBO).
  If, including the tasks marked with an A from Part 2, at least 16 tasks are solved correctly, you will pass the examination.
  If, including the tasks marked with an A from Part 2, fewer than 16 tasks are solved correctly, you will not be awarded enough points to pass the examination.
– If at least 16 tasks are solved correctly (including the compensation tasks marked with an A from Part 2), a grade will be awarded as follows:

Pass	16–23 points
Satisfactory	24–32 points
Good	33–40 points
Very Good	41–48 points

Explanation of the Task Types

Some tasks require a ***free answer***. For these tasks, you should write your answer directly underneath each task in the task booklet or on the paper provided. Other task types used in the examination are as follows:

Matching tasks: For this task type you will be given a number of statements, tables or diagrams, which will appear alongside a selection of possible answers. To correctly answer these tasks, you will need to match each statement, table or diagram to its corresponding answer. You should write **the letter of the correct answer** next to the statement, table or diagram in the space provided.

Example:
You are given two equations.

Task:
Match the two equations to their corresponding description (from A to D).

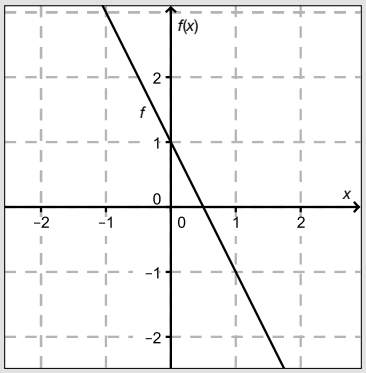
$1 + 1 = 2$	*A*
$2 \cdot 2 = 4$	*C*

A	Addition
B	Division
C	Multiplication
D	Subtraction

Construction tasks: This task type requires you to draw points, lines and/or curves in the task booklet.

Example:
Below you will see a linear function f where $f(x) = k \cdot x + d$.

Task:
On the axes provided below, draw the graph of a linear function for which $k = -2$ and $d > 0$.



Multiple-choice tasks of the form "1 out of 6": This task type consists of a question and six possible answers. Only **one answer** should be selected. You should put a cross next to the only correct answer in the space provided.

Example:
Which equation is correct?

Task:
Put a cross next to the correct equation.

1 + 1 = 1	☐
2 + 2 = 2	☐
3 + 3 = 3	☐
4 + 4 = 8	☒
5 + 5 = 5	☐
6 + 6 = 6	☐

Multiple-choice tasks of the form "2 out of 5": This task type consists of a question and five possible answers, of which **two answers** should be selected. You should put a cross next to each of the two correct answers in the space provided.

Example:
Which equations are correct?

Task:
Put a cross next to each of the two correct equations.

1 + 1 = 1	☐
2 + 2 = 4	☒
3 + 3 = 3	☐
4 + 4 = 8	☒
5 + 5 = 5	☐

Multiple-choice tasks of the form "x out of 5": This task type consists of a question and five possible answers, of which **one, two, three, four *or* five answers** may be selected. The task will require you to: "Put a cross next to each correct statement/equation …". You should put a cross next to each correct answer in the space provided.

Example:
Which of the equations given are correct?

Task:
Put a cross next to each correct equation.

1 + 1 = 2	☒
2 + 2 = 4	☒
3 + 3 = 6	☒
4 + 4 = 4	☐
5 + 5 = 10	☒

Gap-fill: This task type consists of a sentence with two gaps, i.e. two sections of the sentence are missing and must be completed. For each gap you will be given the choice of three possible answers. You should put a cross next to **each of the two answers** that are necessary to complete the sentence correctly.

Example:
Below you will see 3 equations.

Task:
Complete the following sentence by putting a cross next to one of the given possibilities for each gap so that the sentence becomes a correct statement.

The operation in equation _____ ① _____ is known as summation or _____ ② _____ .

①	
1 – 1 = 0	☐
1 + 1 = 2	☒
1 · 1 = 1	☐

②	
Multiplication	☐
Subtraction	☐
Addition	☒

Changing an answer for a task that requires a cross:

1. Fill in the box that contains the cross for your original answer.
2. Put a cross in the box next to your new answer.

1 + 1 = 3	☐
2 + 2 = 4	☒
3 + 3 = 5	☐
4 + 4 = 4	☐
5 + 5 = 9	■

In this instance, the answer "5 + 5 = 9" was originally chosen. The answer was later changed to be "2 + 2 = 4".

Selecting an answer that has been filled in:

1. Fill in the box that contains the cross for the answer you do not wish to give.
2. Put a circle around the filled-in box you would like to select.

1 + 1 = 3	☐
2 + 2 = 4	(■)
3 + 3 = 5	☐
4 + 4 = 4	■
5 + 5 = 9	☐

In this instance, the answer "2 + 2 = 4" was filled in and then selected again.

If you still have any questions, please ask your teacher.

Good Luck!

Please turn over the page.

Task 1

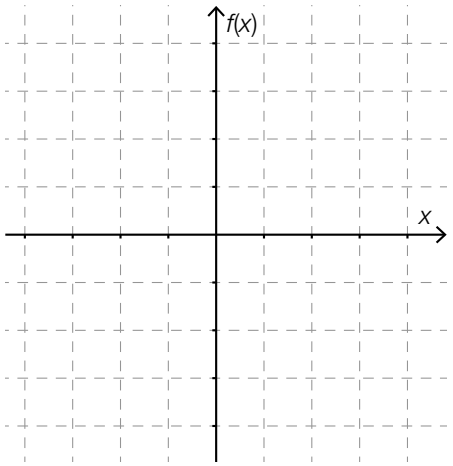
Quadratic Function

This task deals with quadratic functions of the form $x \mapsto a \cdot x^2 + b \cdot x + c$ where $a, b, c \in \mathbb{R}$ and $a \neq 0$. The choice of the coefficients a, b and c influences various properties of the function such as the sign of the function, changes in sign, symmetry about the axes and points of intersection with the axes.

Task:

a) The graph of a quadratic function f is symmetric about the vertical axis and crosses the x-axis at x_1 and x_2 with $x_1 < x_2$. For f, the expression $\int_{x_1}^{x_2} f(x)\,\mathrm{d}x = d$ where $d \in \mathbb{R}^+$ holds.

Sketch an appropriate graph of a function f in the coordinate system given below that shows the value d.



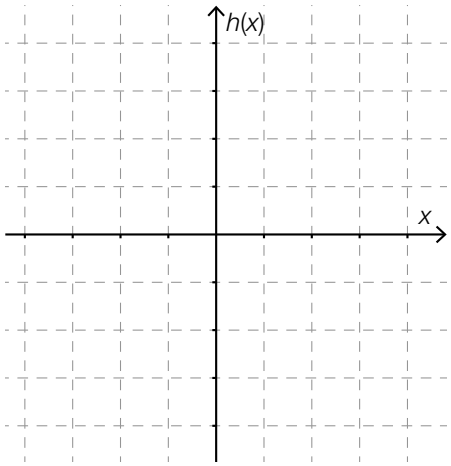
For each of the coefficients a, b and c of the function f, write down whether the coefficient must be positive, negative or exactly zero.

b) The graph of a quadratic function g has a minimum and crosses the x-axis at $x_1 = 0$ and $x_2 > 0$. The zero x_2 can be calculated by using the coefficients of the function g. Write down an appropriate formula.

The graph of the function g creates an area bounded by the curve and the x-axis. Write down a definite integral that can be used to calculate the size of this finite area.

c) For the point where $x = k$ ($k \in \mathbb{R}$) of the graph of a quadratic function h, the conditions $h(k) = 0$ and $h'(k) = 0$ hold.

A Sketch a possible graph of h and mark where $x = k$ in the coordinate system below.



Show by calculation that a function h with equation $h(x) = x^2 - 2 \cdot k \cdot x + k^2$ satisfies the conditions $h(k) = 0$ and $h'(k) = 0$.

Task 2

Muscular Strength

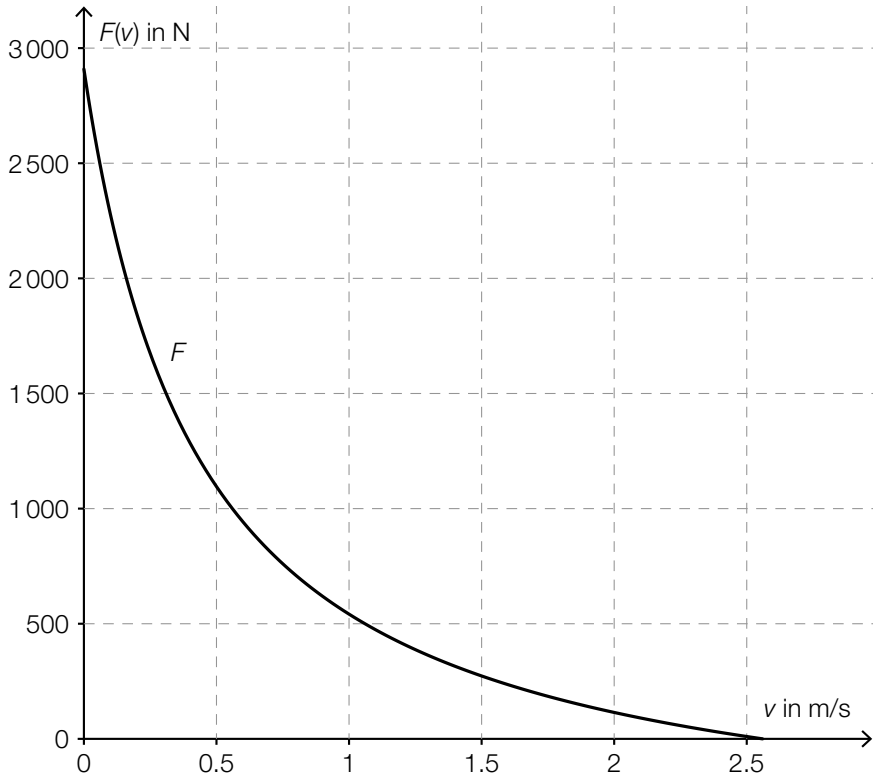
The behaviour of muscles is often compared to that of (metal) springs. However, unlike elastic force, muscular strength also depends on the velocity with which the muscle contracts (i. e. actively shortens or tenses).

This relationship can be modelled by the formula $F = \frac{c}{v + b} - a$.

In this formula, F gives the possible strength (in newtons) of the muscular force under ideal conditions with a particular contraction velocity v (in metres per second). The parameters a (in N), b (in m/s) and c (in watts) are positive real quantities that describe properties of the muscle.

The formula given above can be considered as the equation of a function F that describes the muscular strength, $F(v)$, in terms of the velocity of the muscular contraction, v. The values of a, b and c are constant for a particular muscle.

The graph of the function F is shown in the diagram below.



Task:

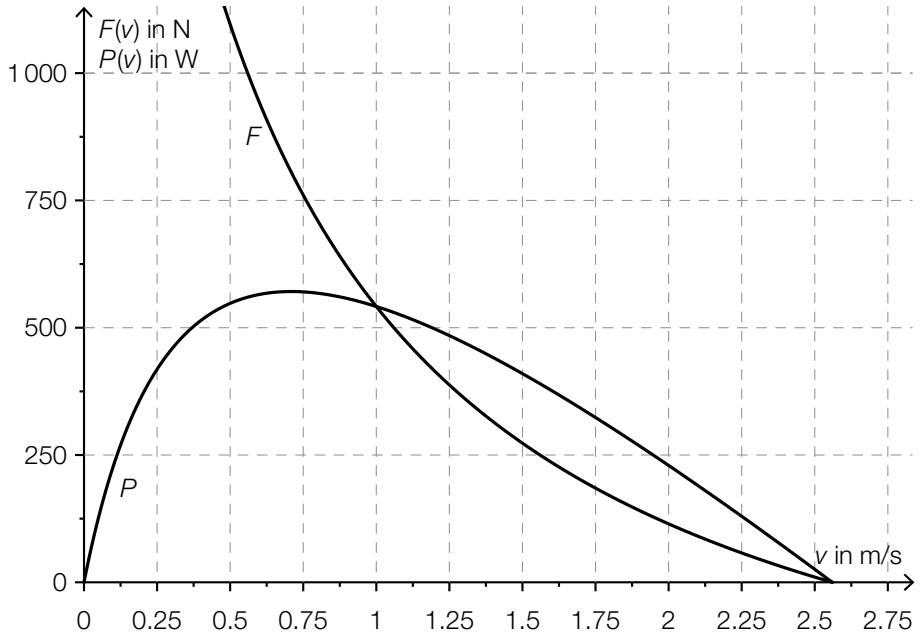
a) From the diagram, write down the value $F(0)$ and how this value should be interpreted in the given context.

Write down whether the function F describes an indirectly proportional relationship between F and v and justify your answer.

b) The power of a muscle can be determined by the formula $P = F \cdot v$.

Given a constant force, F, this formula can be considered as the equation of a function P in which the power, $P(v)$, is described in terms of the velocity of the muscle contraction, v ($P(v)$ in W, v in m/s and F in N).

In the diagram below, the graphs of the functions F and P are shown in terms of the velocity of the muscle contraction, v, for a particular muscle.



A From the diagram, determine an approximate value for the strength (in N) that gives the maximum power for this muscle.

From the diagram, determine an approximate value of the velocity of the muscle contraction v_1 for which $P'(v_1) = 0$ holds.

Task 3

Destruction of the Rainforest

Various studies are concerned with the destruction of the rainforest.

In 1992, a team led by the American economist Dennis Meadows published the study *Beyond the Limits*.

In this study, the rainforest coverage of the Earth at the end of 1990 was determined to be 800 million hectares. In the following year, around 17 million hectares was cleared. The three "catastrophe scenarios" listed below were suggested by the study:

Scenario 1: The annual relative reduction of approximately 2.1 % remains constant.
Scenario 2: The deforestation of 17 million hectares per year remains constant.
Scenario 3: The rate of deforestation (in millions of hectares per year) increases exponentially.

Diagram 1 below shows the graphs of the functions f_1 and f_3, which describe the rainforest coverage according to scenarios 1 and 3 as described above.

Diagram 2 below shows the graph of the first derivative f_3' of the function f_3 shown in diagram 1.

Diagram 1:

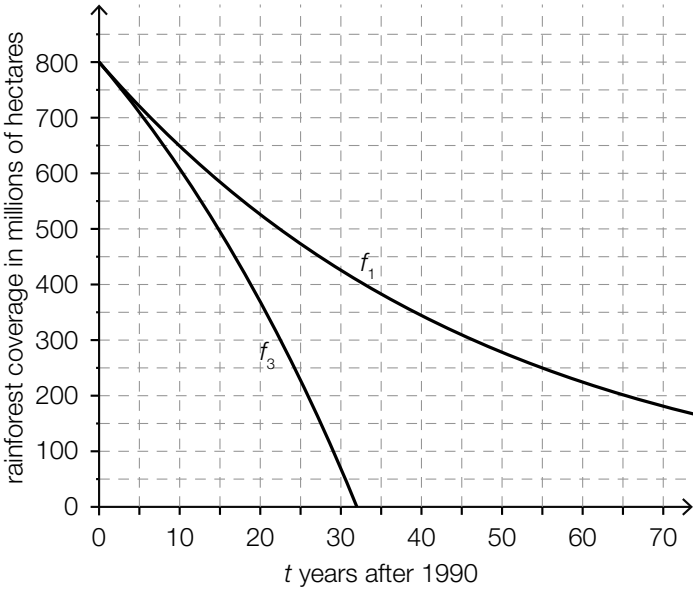
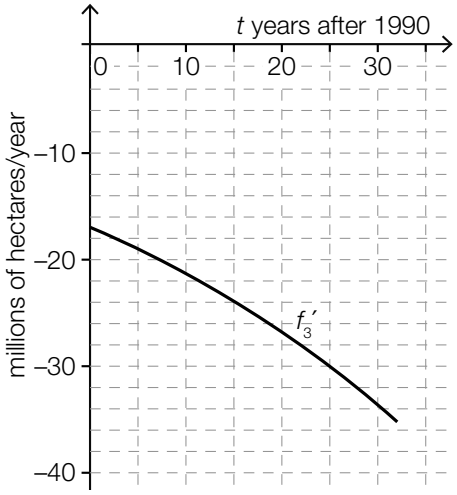


Diagram 2:



Task:

a) A Determine the equation of the function f_1 in which the variable t corresponds to the years passed since after 1990.

Determine when the rainforest coverage will have sunk to below 100 million hectares according to scenario 1.

b) Write down the equation of the function f_2 that models the rainforest coverage t years after 1990 assuming that the coverage will reduce by a constant amount of 17 million hectares per year.

Determine in which year the rainforest would disappear from the Earth's surface according to this model and draw the graph of this function on diagram 1.

c) In the following sub-tasks, you should base your solutions on Meadow's assumption of an exponentially increasing rate of deforestation and give your answers based on the corresponding diagrams.

Write down an approximate value for the time t_1 at which the instantaneous rate of deforestation has reached 24 million hectares per year.

Determine an approximate value for the definite integral $\int_0^{t_1} f_3'(t)\,\mathrm{d}t$ by reading from the graph and write down the meaning of this result in the context of the deforestation of the rainforest.

d) An international research team led by geographer Matthew Hansen from the University of Maryland determined the change in the number of trees in the rainforest from 2000 to 2012 by using satellite photos. From the data, it can be established that, on average, each year a million hectares ($a > 0$) more are deforested each year than in the previous year.

Justify why scenario 3 as suggested by Meadows best corresponds to Matthew Hansen's observations.

Hansen's team reports 0.2101 million hectares per year for the value a. Write down whether Meadow's model predicting the rate of change of deforestation for the time period from 2000 to 2012 is larger or smaller than the values observed by Hansen and justify your answer.

Task 4

Buccolam

Buccolam is a liquid pharmaceutical used in the treatment of acute, persistent cases of seizures in people who are at least three months old and younger than 18 years old (hereafter referred to as “children”). It contains the active ingredient midazolam, a highly effective sedative.
In the course of a clinical trial, Buccolam is administered to 440 children with seizures. 22 of the children experienced side effects of nausea and vomiting. In 308 of the children, visible signs of seizure disappeared within 10 minutes of taking the medication.

Task:

a) There are four types of Buccolam injections that contain appropriate doses of midazolam for children of different ages:

Age Range	Midazolam Dose	Label Colour
up to < 1 year	2.5 mg	yellow
1 year up to < 5 years	5 mg	blue
5 years up to < 10 years	7.5 mg	purple
10 years up to < 18 years	10 mg	orange

Data source: http://www.ema.europa.eu/docs/de_DE/document_library/EPAR_-_Product_Information/human/002267/WC500112310.pdf [02.12.2016].

These injections each contain a solution with an age-appropriate midazolam dose. For example, the injections with a yellow label contain a solution with a volume of 0.5 ml.
In general, the volume V (in ml) of a solution and the midazolam dose D (in mg) are directly proportional.

Write down an equation that describes the relationship between the volume of a solution, V, and the midazolam dose, D.

Write down whether the relationship between the patient’s age (in years) and the midazolam dose is linear. Justify your answer based on the data provided in the table above.

b) The relative frequency H of side effects after administering a medication is classified as follows:

common	$0.01 \leq H < 0.1$
uncommon	$0.001 \leq H < 0.01$
rare	$0.0001 \leq H < 0.001$
very rare	$H < 0.0001$

Data source: https://www.vfa.de/de/patienten/patientenratgeber/ratgeber031.html [02.12.2016] (adapted).

A Write down how the relative frequency of the side effect "nausea and vomiting" of Buccolam should be classified according to the clinical trial described in the introduction.

In the information about Buccolam included with the medication, the frequency of experiencing a "skin rash" is given as "uncommon".
The random variable X describes how many of the 440 children treated with Buccolam during the course of the trial experienced the side effect of a "skin rash". This random variable can be taken to be a binomially distributed random variable with parameter $p = 0.01$, expectation value μ, and standard deviation σ.
Write down how many children in the trial could have experienced the side effect of a "skin rash" so that the number of children affected lies in the interval $[\mu - \sigma; \mu + \sigma]$.

c) The actual proportion of patients whose visible signs of seizure disappear within 10 minutes after consuming the medication is given by p.
Using the data provided in the introduction about the clinical trial, determine a symmetrical confidence interval for p with confidence level $\gamma = 0.95$.

In a separate trial investigating the efficacy of Buccolam, n_1 children were involved. Using the same method, the results led to the symmetrical confidence interval [0.67, 0.73] with confidence level γ_1.
Justify why the values $n_1 < 400$ and $\gamma_1 = 0.99$ could not have been the basis for calculating this confidence interval.