Exemplar für Prüfer/innen

Supplementary Examination for the
Standardised Competence-Oriented
Written School-Leaving Examination

AHS

June 2018

Mathematics

Supplementary Examination 6
Examiner's Version

BMBWF
BUNDESMINISTERIUM
FÜR BILDUNG, WISSENSCHAFT
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Instructions for the supplementary examination

The following supplementary examination is comprised of five tasks that can be completed independently of one another.

Each task contains two parts: The statement of the task requires the candidate to demonstrate core competencies, and the guiding question that follows it requires the candidate to show their ability to communicate their ideas.

In the following document, the examiner will find the tasks as well as the expected solutions and the answer key.

The preparation time is to be at least 30 minutes and the examination time is to be at most 25 minutes.

Assessment

Each task can be awarded zero, one or two points. There is one point available for each demonstration of core competencies as well as for each guiding question. A maximum of 10 points can be achieved.

For the grading of the examination the following scale should be used:

Grade	Minimum number of points
Pass	4 points for the core competencies + 0 points for the guiding questions 3 points for the core competencies + 1 point for the guiding questions
Satisfactory	5 points for the core competencies + 0 points for the guiding questions 4 points for the core competencies + 1 point for the guiding questions 3 points for the core competencies + 2 points for the guiding questions
Good	5 points for the core competencies + 1 point for the guiding questions 4 points for the core competencies + 2 points for the guiding questions 3 points for the core competencies + 3 points for the guiding questions
Very good	5 points for the core competencies + 2 points for the guiding questions 4 points for the core competencies + 3 points for the guiding questions

The examination board will decide on the final grade based on the candidate's performance in the supplementary examination as well as the result of the written examination.

Evaluation grid for the supplementary examination

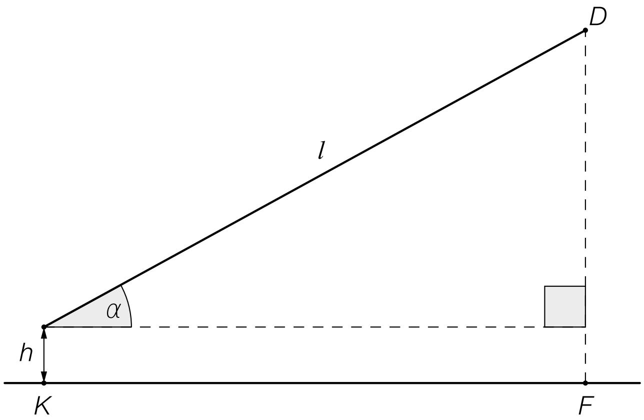
This evaluation grid may be used to assist the examiner in assessing the candidate's performance.

	Point for core competencies reached	Point for the guiding question reached
Task 1		
Task 2		
Task 3		
Task 4		
Task 5		

Task 1

Flying a Kite

A child is flying a kite. An approximation of the positions of the child (K) and the kite (D) at a particular time is shown in the diagram below.



The position of the child, K, as well as the point F lie in a horizontal plane. The child is holding the kite at a height h of 1.5 metres above the ground. The length of the taut string is $l = 50$ m.

Task:

Write down a formula that can be used to calculate the height $\overline{FD}$ of the kite above the horizontal plane (in metres) in terms of the angle α.

Guiding question:

Write down a function to calculate the size of the angle α for which the horizontal distance $\overline{KF}$ is equal to the height $\overline{FD}$ of the kite and determine the size of α.

Solution to Task 1

Flying a Kite

Expected solution to the statement of the task:

$\overline{FD} = 50 \cdot \sin(\alpha) + 1.5$

Answer key:

The point for the core competencies is to be awarded if a correct formula has been given. Equivalent formulae are also to be accepted.

Expected solution to the guiding question:

$\overline{KF} = \overline{FD} \quad \Rightarrow \quad 50 \cdot \cos(\alpha) = 50 \cdot \sin(\alpha) + 1.5$
$\alpha \approx 43.78°$

Answer key:

The point for the guiding question is to be awarded if a correct equation to the calculation of α and the angle α has been given correctly.
Tolerance interval: [43°, 44°]

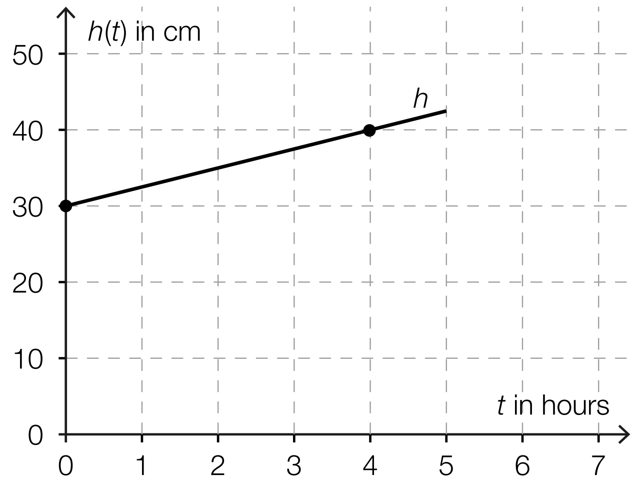
Task 2

Snowfall

The height of the snow level during a five-hour snowfall can be modelled by a linear function, h. The height of the snow, $h(t)$, is measured in cm and the time, t, is measured in hours where $0 \leq t \leq 5$.

Task:

The graph shown below shows the height of the snow level during this five-hour snowfall. The points shown in bold have integer coordinates.



Write down the equation of the function that gives the height of the snow level, h, in terms of the time t and write down the meaning of the numbers that appear in the equation.

Guiding question:

For a function h_1, write down all of the conditions that must be fulfilled so that h_1 describes a directly proportional relationship between the height of the snow level, $h_1(t)$ (in cm), and the time t (in hours).

Write down the equation of the directly proportional function h_1 if the snow level is 20 cm high after a five-hour snowfall.

Solution to Task 2

Snowfall

Expected solution to the statement of the task:

$h(t) = 30 + 2.5 \cdot t$

At the beginning of the observations ($t = 0$), the height of the snow level is 30 cm. The height of the snow level increases by 2.5 cm per hour.

Answer key:

The point for the core competencies is to be given if a correct equation for the function has been given and the meaning of the numbers has been given correctly.

Expected solution to the guiding question:

At the beginning of the observations (when $t = 0$), the height of the snow level has to be zero and the (absolute) height must increase at a constant rate.

$h_1(t) = 4 \cdot t$

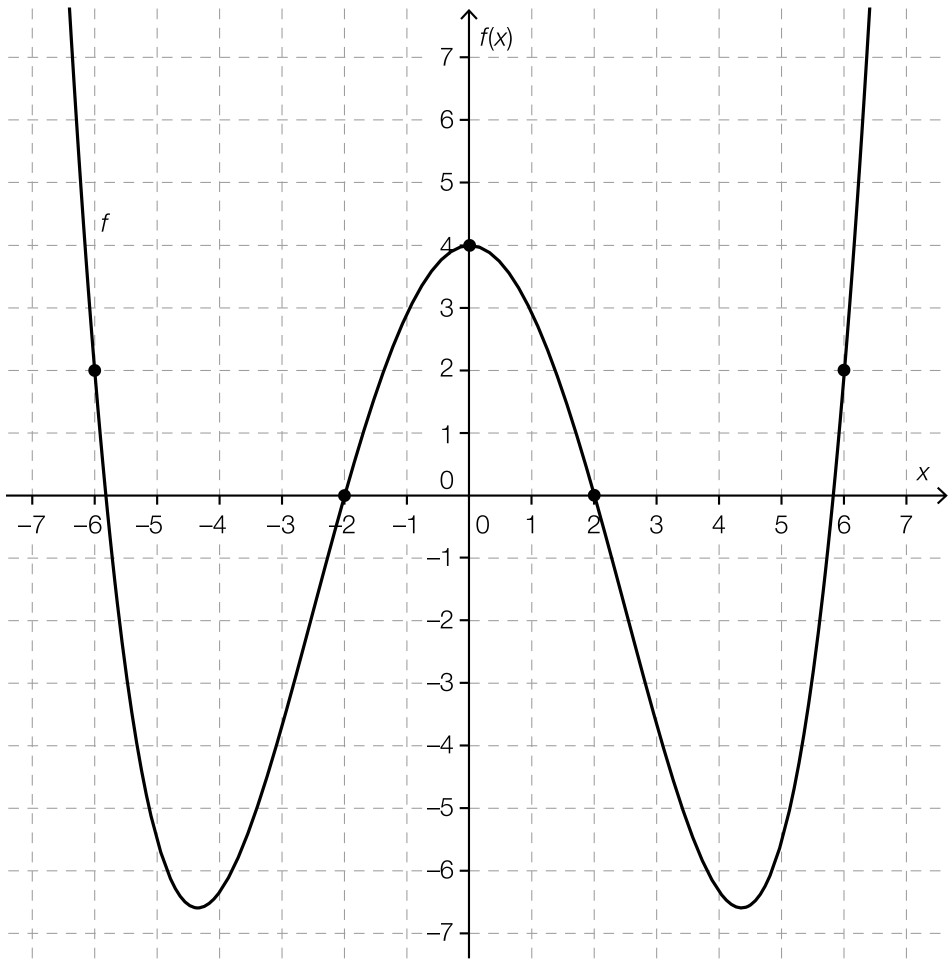
Answer key:

The point for the guiding question is to be awarded if the conditions given in the expected solution have been given correctly and the equation of the function has been given correctly.

Task 3

Fourth Degree Polynomial Function

The diagram below shows the graph of a fourth degree polynomial function, f, with equation $f(x) = a \cdot x^4 + b \cdot x^2 + c$ where $a, b, c \in \mathbb{R}$. The points shown in bold have integer coordinates.



Task:

Determine the parameters a, b and c of the function f.
Write down the intervals for which $f'(x) > 0$ holds and explain your method.

Guiding question:

Write down a value of $k \in \mathbb{R}$ where $k > 2$ such that the equation shown below is generally valid and explain your reasoning.

$$\int_{-3}^{0} f(x)\,\mathrm{d}x - \int_{0}^{k} f(x)\,\mathrm{d}x = f'(0)$$

There is a further value $h \in \mathbb{R}$, $0 \leq h \leq 2$ for which the equation $\int_{-3}^{0} f(x)\,\mathrm{d}x - \int_{0}^{h} f(x)\,\mathrm{d}x = f'(0)$ is satisfied. Determine this value.

Solution to Task 3

Fourth Degree Polynomial Function

Expected solution to the statement of the task:

$a \approx 0.0295$ $b \approx -1.1181$ $c = 4$
intervals: $(-4.35, 0)$ and $(4.35, \infty)$

Possible explanation:
In an interval (x_1, x_2) $f'(x) > 0$ holds, if for all $x \in (x_1, x_2)$ the tangent to the graph of the function f is increasing. By finding the maxima and minima of the function, the boundaries of the intervals can be determined.

Answer key:

The point for the core competencies is to be given if the parameters a, b and c and also both intervals have been given correctly and a correct method has been explained.
Half-open or closed intervals as well as equivalent notation are also to be considered correct.
Tolerance intervals for the lower interval boundaries: $[-4.4, -4.3]$ and $[4.3, 4.4]$ respectively

Expected solution to the guiding question:

For $k = 3$ the equation is generally valid.

Possible explanation:
As $x = 0$ is a local maximum, $f'(0) = 0$ holds. So that the difference between the definite integrals is also zero, both of the non-zero integrals (under the condition that $k > 2$) have to be symmetrical about the origin (as the function f is an even function and the graph of the function is symmetrical about the vertical axis).

Also for $h \approx 0.91$ the equation is generally valid.

Answer key:

The point for the guiding question is to be awarded if the parameters k and h have been given correctly and correct methods have been explained.

Task 4

Number of Inhabitants

The number of inhabitants in a particular country in year t is represented by $B(t)$.

Task:

Interpret both of the equations below with respect to the number of inhabitants in this country.

- $\frac{B(2015)}{B(1950)} = 2$
- $\frac{B(2015) - B(2000)}{B(2000)} = 0.1$

Guiding question:

Interpret the equation $\frac{B(2015) - B(2000)}{15} = 100\,000$ in the given context.

Using the equations given, determine the number of inhabitants in this country in the year 2015 and explain your reasoning.

Solution to Task 4

Number of Inhabitants

Expected solution to the statement of the task:

Possible interpretations:

- The number of inhabitants in the country is twice as high in 2015 than in 1950.
- The number of inhabitants in the country is 10 % higher in 2015 than in 2000.

Answer key:

The point for the core competencies is to be given if both equations have been interpreted correctly.

Expected solution to the guiding question:

The number of inhabitants in the country has increased on average by 100 000 inhabitants per year from 2000 to 2015.

The number of inhabitants in 2015 was 16.5 million.

Possible method:

Over the 15 years, the number of inhabitants increased by 1.5 million.

As the equation $\frac{B(2015) - B(2000)}{B(2000)} = 0.1$ corresponds to an increase of 10 %, $B(2000) = 15$ million must hold.

$B(2015) = B(2000) + 1.5 = 16.5$

Answer key:

The point for the guiding question is to be awarded if the equation has been correctly interpreted within the given context, the number of inhabitants in the year 2015 has been determined correctly, and a correct method has been explained.

Task 5

Discount Dice

A shop has a game that customers can play to win discounts. The aim of the game is to roll the highest possible number with a (fair) dice. (A dice is considered to be “fair” if the probability of the dice showing any of its faces after being thrown is equal for all six faces.)
If a person rolls a number from 1 to 5, they win a percentage discount equal to the number shown. If a person rolls a six on their first roll, they are allowed to roll again. The sum of the numbers from both rolls corresponds to the percentage discount the customer receives.

Task:

Determine the probability, P, that a customer gets a 10 % discount.
Explain your reasoning.

Guiding question:

The random variable X describes the percentage discount that a customer can receive.

Write down all of the possible values of the random variable X and their corresponding probabilities.

Determine the expectation value $E(X)$ of the random variable X and explain the meaning of the value obtained in the given context.

Solution to Task 5

Discount Dice

Expected solution to the statement of the task:

A customer receives a 10 % discount if they roll a six on their first roll and a four on their second roll.

$P = \frac{1}{6} \cdot \frac{1}{6} = \frac{1}{36} \approx 0.0278 = 2.78\ \%$

Answer key:

The point for the core competencies is to be given if the probability has been given correctly and a correct method has been explained.

Expected solution to the guiding question:

Values of the random variable:

1, 2, 3, 4, 5 each with a probability of $\frac{1}{6} \approx 0.1667 = 16.67\ \%$

7, 8, 9, 10, 11, 12 each with a probability of $\frac{1}{36} \approx 0.0278 = 2.78\ \%$

$E(X) = (1 + 2 + 3 + 4 + 5) \cdot \frac{1}{6} + (7 + 8 + 9 + 10 + 11 + 12) \cdot \frac{1}{36} \approx 4.08$

On average, a discount of 4 % is to be expected.

Answer key:

The point for the guiding question is to be awarded if the possible values of the random variable and the corresponding probabilities have been given correctly. Also, the expectation value must have been both determined and explained correctly.