Exemplar für Prüfer/innen

Supplementary Examination for the
Standardised Competence-Oriented
Written School-Leaving Examination

AHS

January 2019

Mathematics

Supplementary Examination 1
Examiner's Version

Bundesministerium
Bildung, Wissenschaft
und Forschung

Instructions for the supplementary examination

The following supplementary examination is comprised of five tasks that can be completed independently of one another.

Each task contains two parts: The statement of the task requires the candidate to demonstrate core competencies, and the guiding question that follows it requires the candidate to show their ability to communicate their ideas.

In the following document, the examiner will find the tasks as well as the expected solutions and the answer key.

The preparation time is to be at least 30 minutes and the examination time is to be at most 25 minutes.

Assessment

Each task can be awarded zero, one or two points. There is one point available for each demonstration of core competencies as well as for each guiding question. A maximum of 10 points can be achieved.

For the grading of the examination the following scale should be used:

Grade	Minimum number of points
Pass	4 points for the core competencies + 0 points for the guiding questions 3 points for the core competencies + 1 point for the guiding questions
Satisfactory	5 points for the core competencies + 0 points for the guiding questions 4 points for the core competencies + 1 point for the guiding questions 3 points for the core competencies + 2 points for the guiding questions
Good	5 points for the core competencies + 1 point for the guiding questions 4 points for the core competencies + 2 points for the guiding questions 3 points for the core competencies + 3 points for the guiding questions
Very good	5 points for the core competencies + 2 points for the guiding questions 4 points for the core competencies + 3 points for the guiding questions

The examination board will decide on the final grade based on the candidate's performance in the supplementary examination as well as the result of the written examination.

Evaluation grid for the supplementary examination

This evaluation grid may be used to assist the examiner in assessing the candidate's performance.

	Point for core competencies reached	Point for the guiding question reached
Task 1		
Task 2		
Task 3		
Task 4		
Task 5		

Task 1

Three Vectors in $\mathbb{R}^3$

Below, you will see three vectors.

$\vec{a} = \begin{pmatrix} 1 \\ -2 \\ 3 \end{pmatrix}, \ \vec{b} = \begin{pmatrix} 2 \\ 1 \\ b_z \end{pmatrix}, \ \vec{c} = \begin{pmatrix} -3 \\ c_y \\ 5 \end{pmatrix}$

Task:

Determine the components b_z and c_y so that the vectors $\vec{b}$ and $\vec{c}$ are both perpendicular to $\vec{a}$.

Show that the vectors $\vec{b}$ and $\vec{c}$ are also perpendicular to one another for the components you have calculated. Show your method.

Guiding question:

For each of the lines *g*, *h* and *i*, determine a vector equation such that each of the conditions below is fulfilled.

I: The line *g* has the vector $\vec{a}$ as its direction vector and goes through the origin.

II: The line *h* has the vector $\vec{b}$ as its direction vector and crosses the line *g* at exactly one point.

III: The line *i* is parallel to the line *h* and skew to the line *g* (i. e. it does not cross the line *g*).

Explain your method and show that *i* is skew to *g*.

Solution of Task 1

Three Vectors in $\mathbb{R}^3$

Expected solution of the statement of the task:

A pair of vectors, neither of which is the zero vector, is perpendicular if and only if their scalar product is zero.

$\vec{a} \cdot \vec{b} = 0 \;\Rightarrow\; 2 - 2 + 3 \cdot b_z = 0 \;\Rightarrow\; b_z = 0$

$\vec{a} \cdot \vec{c} = 0 \;\Rightarrow\; -3 - 2 \cdot c_y + 15 = 0 \;\Rightarrow\; c_y = 6$

Then $\vec{b} \cdot \vec{c} = -6 + 6 + 0 = 0$ also holds.

Answer key:

The point for the core competencies is to be awarded if both components have been determined correctly and it has been shown that vectors $\vec{b}$ and $\vec{c}$ are also perpendicular. A correct method also needs to have been given.

Expected solution of the guiding question:

Possible solution:

$$g\colon X = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} + s \cdot \begin{pmatrix} 1 \\ -2 \\ 3 \end{pmatrix} \qquad h\colon X = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} + t \cdot \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix} \qquad i\colon X = \begin{pmatrix} 0 \\ 0 \\ 7 \end{pmatrix} + u \cdot \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix}$$

It needs to be shown that g and i do not cross:

$s = 2 \cdot u$
$-2 \cdot s = u$
$3 \cdot s = 7$

This system of equations in the variables s and u has no solution.

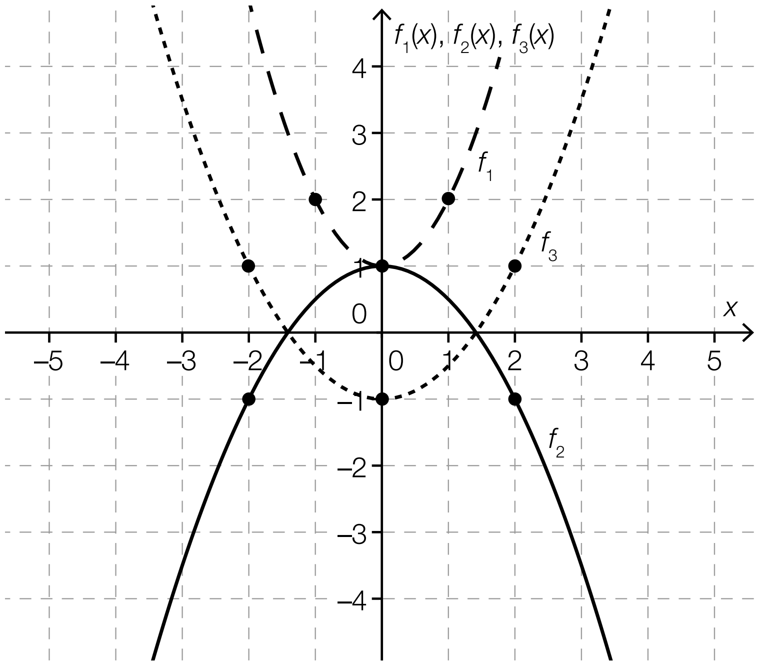
Answer key:

The point for the guiding question is to be awarded if three vector equations have been given that fulfil the conditions, a correct method has been used, and it has been shown that the lines g and i are skew.

Task 2

Functions

In the coordinate system below, three graphs of functions of the form $x \mapsto a \cdot x^2 + b$ are shown. The points marked in bold have integer coordinates.



Task:

Determine an equation of the function f_2.

Guiding question:

Explain the influence of the parameters a and b on the behaviour of the graph of a general function f where $f(x) = a \cdot x^2 + b$ and $a \neq 0$.

By comparing the parameters of the three functions f_1, f_2 and f_3, show how your explanation applies to concrete examples.

Solution of Task 2

Functions

Expected solution of the statement of the task:

If $f_2(x) = a \cdot x^2 + b$ and $f_2(0) = 1$ and, for example, $f_2(2) = -1$, we have $f_2(x) = -0.5 \cdot x^2 + 1$.

Answer key:

The point for the core competencies is to be awarded if an equation of the function f_2 has been determined correctly.

Expected solution of the guiding question:

The parameter b determines the intercept with the vertical axis.
The parameter a determines the concavity of the parabola.
For $a > 0$: The parabola is concave up; the larger a is, the "steeper" the graph of f is.
For $a < 0$: The parabola is concave down; the smaller a is, the "steeper" the graph of f is.

For f_1 and f_2, the parameter b is the same and has the value $b = 1$.
The parameter a is positive for f_1 and negative for f_2.

For f_3, $b = -1$ and a is positive (and has the same absolute value as in f_2).

Answer key:

The point for the guiding question is to be awarded if the general influence of the parameters a and b has been explained correctly and the differences in the parameters of the functions f_1, f_2 and f_3 have been correctly described.

Task 3

Wild Pigs

According to a newspaper article, the population of wild pigs in Bavaria in the year 2013 increased sharply, even though so many wild pigs had never been shot before. In the hunting season 2012/13, 66 000 wild pigs were shot. In the hunting season 2011/12, only 42 300 wild pigs had been shot.

Task:

Determine the absolute and relative increase of wild pig shootings in Bavaria from the hunting season of 2011/12 to that of 2012/13.

Guiding question:

State the type of functional relationship between time and the number of wild pig shootings that describes a constant annual rate of increase in shootings and corresponds to the relative change in shooting numbers in Bavaria calculated above.

Determine an equation for the function W that describes the number of wild pig shootings in Bavaria as a function of time, t (measured in years), where $W(0)$ gives the number of shootings in the 2012/13 season.

Using this equation, determine the number of wild pig shootings in the hunting season 2022/23. Estimate whether or not it is realistic for the number of wild pig shootings to develop in accordance with this function over a long period of time.

Solution of Task 3

Wild Pigs

Expected solution of the statement of the task:

Absolute increase: 23 700 shootings

Relative increase: $\frac{66\,000 - 42\,300}{42\,300} \approx 0.56$

The number of shootings increased by around 56 %.

Answer key:

The point for the core competencies is to be awarded if both values for the increase have been given correctly.

Expected solution of the guiding question:

The situation can be modelled by an exponential function:

$W(t) = 66\,000 \cdot 1.56^t$
$W(10) \approx 5.6$ million

It is not realistic, as this exponential function is strictly monotonically increasing, but the number of wild pig shootings cannot become infinitely large.

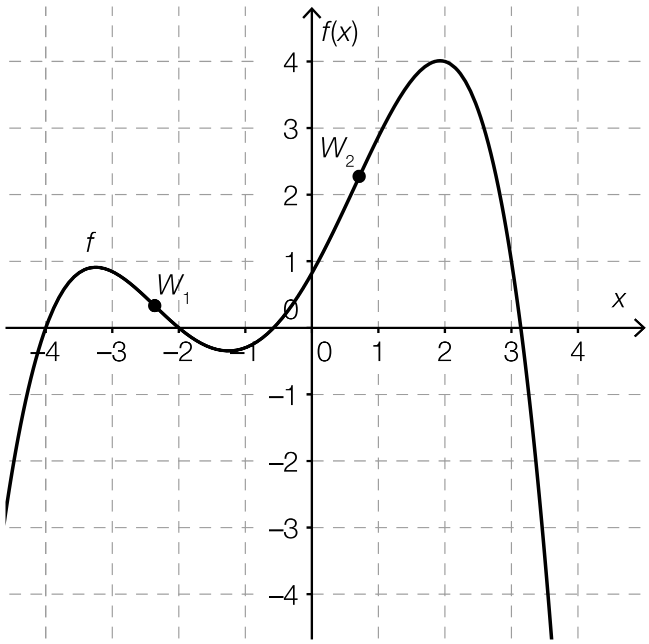
Answer key:

The point for the guiding question is to be awarded if a correct equation of the function has been given, the value for the 2022/23 season has been correctly determined, and a correct explanation has been provided.

Task 4

Derivatives and Antiderivatives

The diagram below shows a section of a graph of a fourth degree polynomial function, f, with the points of inflexion W_1 and W_2.



Task:

Determine whether the following statements are true or false. For each statement, justify your answer.

Statement 1: For all $x \in [-1, 1]$, $f'(x) > 0$.

Statement 2: There exists an $x \in [0, 1]$ for which $f'(x) = 0$.

Statement 3: For all $x \in [-4, -2]$, $f''(x) < 0$.

Statement 4: There exists an $x \in [1, 3]$ for which $f''(x) = 0$.

Guiding question:

Determine the intervals in the range $[-4, 3]$ for which an antiderivative of f is strictly monotonically increasing and explain your answer.

Solution of Task 4

Derivatives and Antiderivatives

Expected solution of the statement of the task:

Statement 1: The statement is true, as f is strictly monotonically increasing in this interval.
Statement 2: The statement is false, as f has no local maximum or minimum in this interval (and has no saddle point).
Statement 3: The statement is false, as the concavity of f changes in this interval.
Statement 4: The statement is false, as f has no point of inflexion in this interval.

Answer key:

The point for the core competencies is to be awarded if for each statement it has been correctly identified whether the statement is true or false and the decision has been justified correctly.

Expected solution of the guiding question:

An antiderivative of f is strictly monotonically increasing in the intervals $[-4, -2]$ and $[-0.5, 3]$ as the function f has non-negative values in these intervals.

Answer key:

The point for the guiding question is to be awarded if both intervals of an antiderivative of f have been given correctly. Different interval notations (open or half-open) as well as correct formal or verbal descriptions are all to be accepted as correct.
For the boundaries of the intervals, deviations of ± 0.2 are to be accepted.

Task 5

Medication

According to the information from a pharmaceutical company, only 2 % of people who consume a particular medication experience mild side effects.

The medication is consumed by 50 people.

As a simplification, it should be assumed in the following that the number of people who experience mild side effects is binomially distributed.

Task:

Determine how many people can be expected to experience mild side effects.

Determine the probability that more than two people experience mild side effects.

Guiding question:

Determine the lowest number n ($n \in \mathbb{N}$) of people that would have to take the medication such that there is a probability of at least 90 % that at least one person experiences mild side effects. Explain your method.

Solution of Task 5

Medication

Expected solution to the statement of the task:

Expectation value $E = n \cdot p = 1$

$P(X > 2) \approx 0.078 = 7.8\ \%$

Answer key:

The point for the core competencies is to be awarded if the expectation value and the probability have been given correctly.
Tolerance interval: [0.07, 0.08] or [7 %, 8 %]

Expected solution to the guiding question:

Possible method:

$P(X \geq 1) \geq 0.9 \quad \Rightarrow \quad 1 - P(X = 0) \geq 0.9$

$1 - 0.98^n \geq 0.9 \quad \Rightarrow \quad 0.98^n \leq 0.1 \quad \Rightarrow \quad n \geq 113.97...$

$n = 114$

Answer key:

The point for the guiding question is to be awarded if the lowest number of people has been given correctly and the method has been explained correctly.