

Name:	
Class:	



Standardised Competence-Oriented
Written School-Leaving Examination

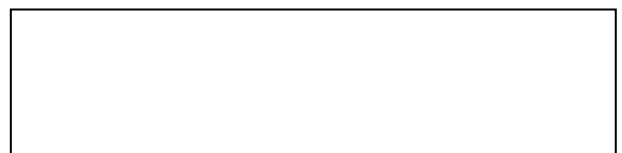
AHS

8th May 2019

Mathematics

Part 1 and Part 2 Tasks

Resit examination according to § 40 para. 3 SchUG
for students whose first examination was taken before May 2018



Advice for Completing the Tasks

Dear candidate!

The following booklet contains Part 1 tasks and Part 2 tasks (divided into sub-tasks). The tasks can be completed independently of one another. You have a total of *270 minutes* available in which to work through this booklet.

Please do all of your working out solely in this booklet and the paper provided to you. Write your name and that of your class on the cover page of the booklet in the spaces provided. Also, write your name and consecutive page numbers on each sheet of paper used. When answering each sub-task, indicate its name/number on your sheet.

In the assessment of your work, everything that is not crossed out will be considered. Your solutions must be marked clearly. If a solution is not clearly marked or if more than one solution is given, the task will be considered to be unsolved.

You may use the official formula booklet for this examination session as well as approved electronic device(s), provided there is no possibility to communicate via internet, Bluetooth, mobile networks, etc. and there is no access to your own data stored on the device.

An explanation of the task types is available in the examination room and can be viewed on request.

Please hand in the task booklet and all used sheets at the end of the examination.

Changing an answer for a task that requires a cross:

1. Fill in the box that contains the cross.
2. Put a cross in the box next to your new answer.

In this instance, the answer " $5 + 5 = 9$ " was originally chosen. The answer was later changed to be " $2 + 2 = 4$ ".

$1 + 1 = 3$	<input type="checkbox"/>
$2 + 2 = 4$	<input checked="" type="checkbox"/>
$3 + 3 = 5$	<input type="checkbox"/>
$4 + 4 = 4$	<input type="checkbox"/>
$5 + 5 = 9$	<input checked="" type="checkbox"/>

Selecting an item that has been filled in:

1. Fill in the box that contains the cross for the answer you do not wish to give.
2. Put a circle around the filled-in box you would like to select.

In this instance, the answer " $2 + 2 = 4$ " was filled in and then selected again.

$1 + 1 = 3$	<input type="checkbox"/>
$2 + 2 = 4$	<input checked="" type="checkbox"/>
$3 + 3 = 5$	<input type="checkbox"/>
$4 + 4 = 4$	<input checked="" type="checkbox"/>
$5 + 5 = 9$	<input type="checkbox"/>

Assessment

The tasks in Part 1 will be awarded either 0 points or 1 point or 0, $\frac{1}{2}$ or 1 point, respectively. The points that can be reached in each task are listed in the booklet for all Part 1 tasks. Every sub-task in Part 2 will be awarded 0, 1 or 2 points. The tasks marked with an **A** will be awarded either 0 points or 1 point.

Two assessment options

- 1) If you have reached **at least 16** of the 28 points (24 Part 1 points + 4 **A** points from Part 2), a grade will be awarded as follows:

Pass	16–23.5 points
Satisfactory	24–32.5 points
Good	33–40.5 points
Very Good	41–48 points

- 2) If you have reached **fewer than 16** of the 28 points (24 Part 1 points + 4 **A** points from Part 2), but have reached a **total of 24 points or more** (from Part 1 and Part 2 tasks), then a "Pass" or "Satisfactory" grade is possible as follows:

Pass	24–28.5 points
Satisfactory	29–35.5 points

From 36 points upward, the assessment key specified in 1) applies.

If you have reached fewer than 16 points in Part 1 (including the compensation tasks marked with an **A** from Part 2) and if the total is less than 24 points, you will not pass the examination.

Good luck!

Task 1

Basic Operations

For two integers a, b where $a < 0$ and $b < 0$, holds: $b = 2 \cdot a$.

Task:

Which of the following calculations always result in a natural number?
Put a cross next to each of the two correct calculations.

$a + b$	<input type="checkbox"/>
$b : a$	<input type="checkbox"/>
$a : b$	<input type="checkbox"/>
$a \cdot b$	<input type="checkbox"/>
$b - a$	<input type="checkbox"/>

[0/1 point]

Task 2

Stopping Distance

Student drivers learn in a driving school that the stopping distance s can be approximated by the following formula. In the formula, v is the speed of the vehicle (s in m, v in km/h).

$$s = \frac{v}{10} \cdot 3 + \left(\frac{v}{10}\right)^2$$

When driving “at sight”, the driver has to choose the speed of the vehicle in such a way that stopping within the viewable distance is possible at any moment. Viewable distance means the length of the road visible for the driver.

Task:

Determine the maximum permissible speed when the viewable distance is 25 m.

The maximum permissible speed is \approx _____ km/h.

[0/1 point]

Task 3

Solving Inequalities

Below, you will find two linear inequalities.

I: $7 \cdot x + 67 > -17$

II: $-25 - 4 \cdot x > 7$

Task:

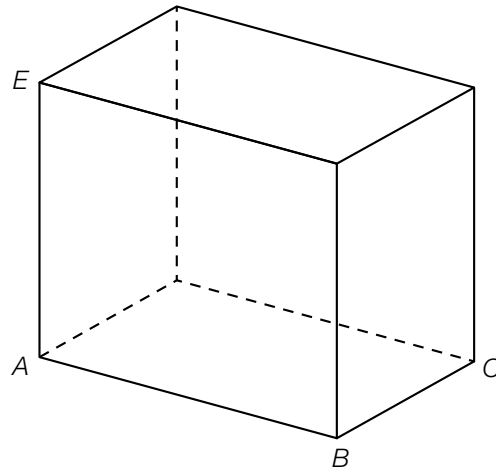
Find all real numbers x that solve both inequalities.
Write down the set of these numbers as an interval.

[0/1 point]

Task 4

Vertices of a Cuboid

The diagram below shows a cuboid. Its vertices A , B , C and E are shown in the diagram.



Task:

For the other vertices of the cuboid, R , S and T , the following relationships hold:

$$R = E + \overrightarrow{AB}$$

$$S = A + \overrightarrow{AE} + \overrightarrow{BC}$$

$$T = E + \overrightarrow{BC} - \overrightarrow{AE}$$

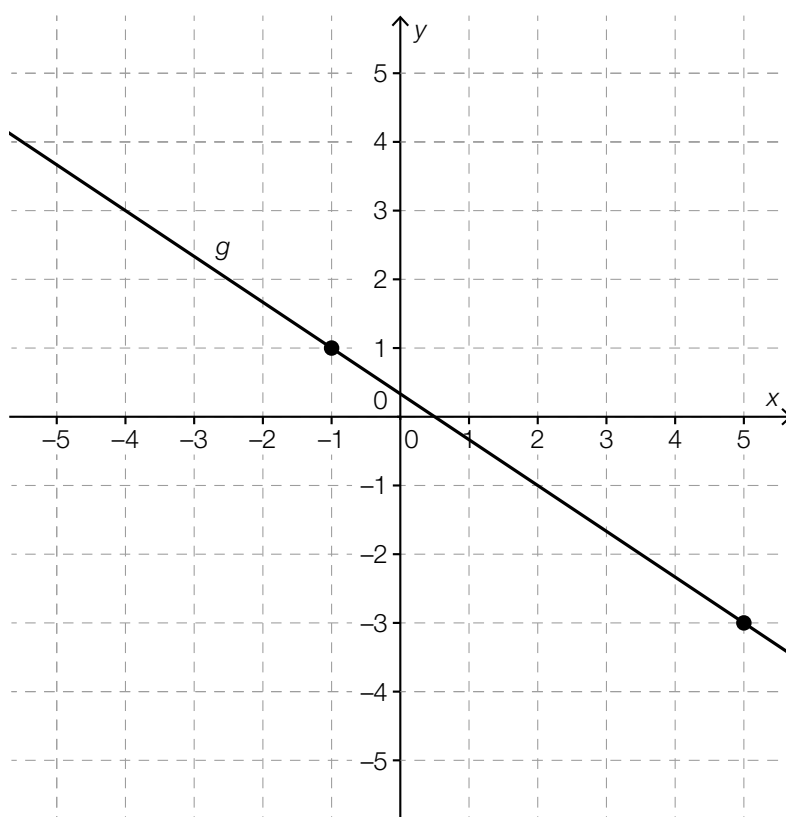
In the diagram above, label the vertices R , S and T in a clearly visible way.

[0/1 point]

Task 5

Vector Equation of a Line

The diagram below shows the line g . The points marked on the line g have integer coordinates.



Task:

Complete the following vector equation of the line g by writing down the values of a and b with $a, b \in \mathbb{R}$.

$$g: X = \begin{pmatrix} a \\ 3 \end{pmatrix} + t \cdot \begin{pmatrix} 3 \\ b \end{pmatrix} \text{ where } t \in \mathbb{R}$$

$a =$ _____

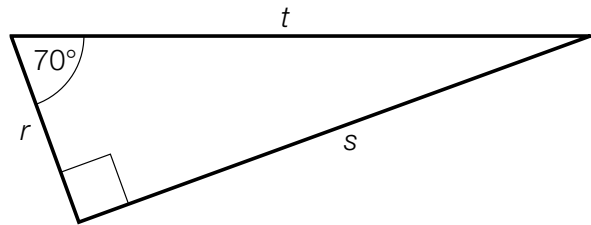
$b =$ _____

[0/1 point]

Task 6

Triangle

The following triangle with sides r , s and t is given.



Task:

Determine the ratio $\frac{r}{t}$ for this triangle.

[0/1 point]

Task 7

Matching Functions

The formula $F = \frac{a^2 \cdot b}{c^n} + d$, where $a, b, c, d \in \mathbb{R}$, $n \in \mathbb{N}$ and $c \neq 0$, $n \neq 0$, is given.

Assuming that only one of the quantities a, b, c, d or n is variable and all others are constant, F can be written as a function dependent of the respective variable.

Task:

Which of the relationships below describe a linear function (where domain and range are suitable)?

Put a cross next to both correct relationships.

$a \mapsto \frac{a^2 \cdot b}{c^n} + d$	<input type="checkbox"/>
$b \mapsto \frac{a^2 \cdot b}{c^n} + d$	<input type="checkbox"/>
$c \mapsto \frac{a^2 \cdot b}{c^n} + d$	<input type="checkbox"/>
$d \mapsto \frac{a^2 \cdot b}{c^n} + d$	<input type="checkbox"/>
$n \mapsto \frac{a^2 \cdot b}{c^n} + d$	<input type="checkbox"/>

[0/1 point]

Task 8

Unemployment Rate

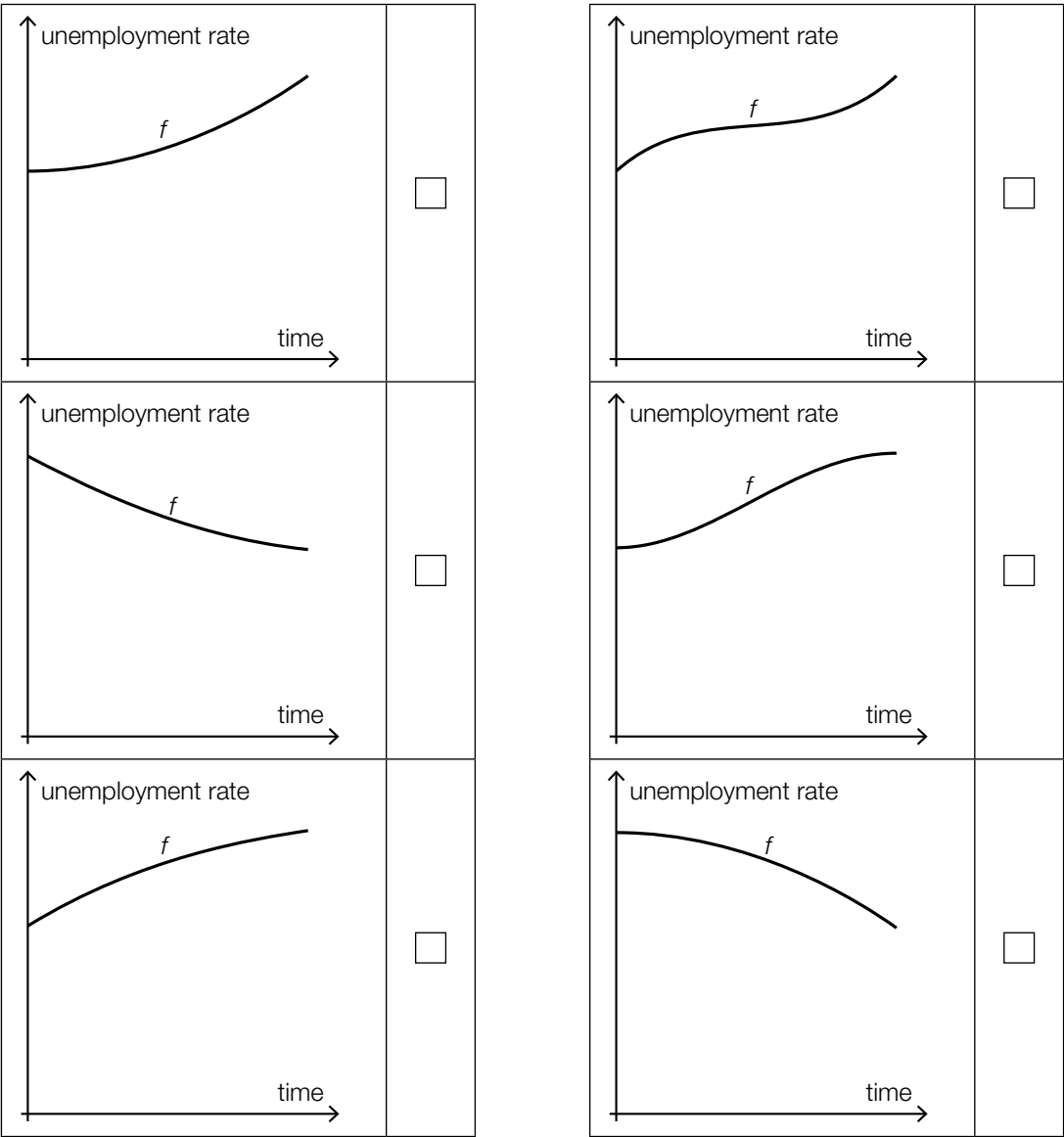
A politician, who wants to highlight the successful employment policies of one of the governing parties, says, “The growth of the unemployment rate has decreased throughout the year.”
An opposition politician replies, “The unemployment rate has increased throughout the year.”

Task:

The development of the unemployment rate throughout the year can be modelled by a function f in terms of the time.

Which of the following graphs shows the development of the unemployment rate throughout the year assuming that the statements of both politicians hold true?

Put a cross next to the correct graph.



[0/1 point]

Task 9

Water Container

The liquid in a cuboid water container stands at the height of 40 cm. The liquid fully drains 8 minutes from the opening of the drain.

A linear function h with equation $h(t) = k \cdot t + d$, where $t \in [0, 8]$, can be used to describe the height of the liquid in the container (in cm) t minutes from the opening of the drain.

Task:

Write down the values of k and d .

$k =$ _____

$d =$ _____

[0/1 point]

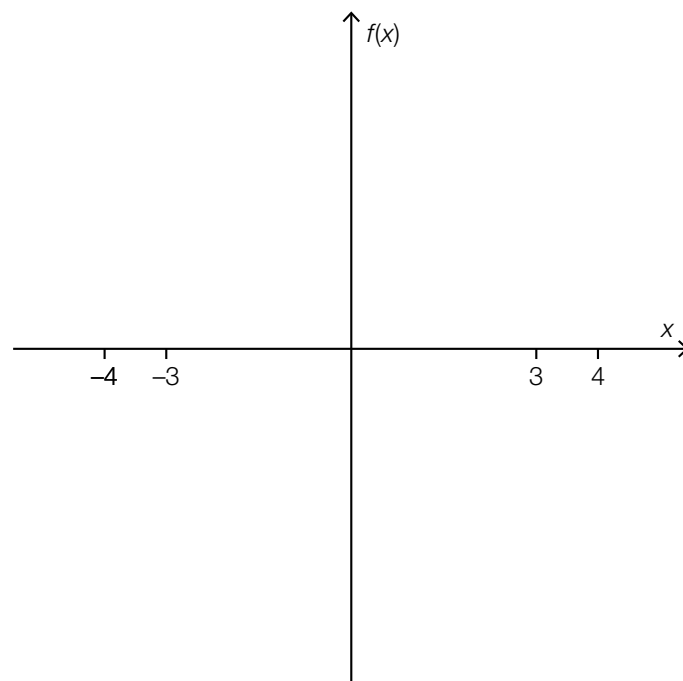
Task 10

Shape of a Fourth Degree Polynomial Function

There are fourth degree polynomial functions that have exactly three zeros at x_1 , x_2 and x_3 , where $x_1, x_2, x_3 \in \mathbb{R}$ and $x_1 < x_2 < x_3$.

Task:

In the interval $[-4, 4]$ in the coordinate system below, sketch the shape of such a function f that has all three zeros in the interval $[-3, 3]$.



[0/1 point]

Task 11

Active Ingredient

The decrease in the active ingredient of a medication in the bloodstream can be modelled by an exponential function.

After one hour, 10 % of the initial amount has been broken down.

Task:

Determine the percentage of the original amount of the ingredient that remains in the bloodstream after a total of four hours.

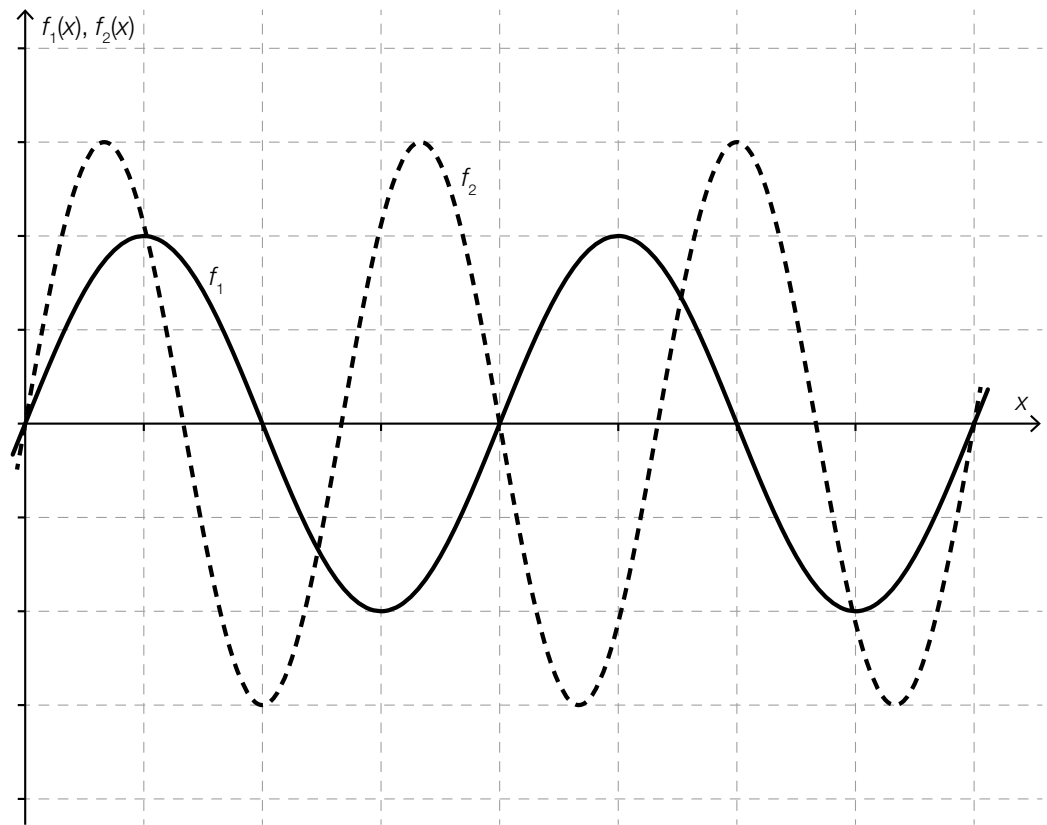
_____ % of the initial amount

[0/1 point]

Task 12

Graphs of Two Trigonometric Functions

The following diagram shows the graphs of the functions $f_1: \mathbb{R} \rightarrow \mathbb{R}$ and $f_2: \mathbb{R} \rightarrow \mathbb{R}$ with equations $f_1(x) = a_1 \cdot \sin(b_1 \cdot x)$ and $f_2(x) = a_2 \cdot \sin(b_2 \cdot x)$ where $a_1, a_2, b_1, b_2 > 0$.



Task:

Complete the following sentence by putting a cross next to one of the given possibilities for each gap so that the sentence becomes a correct statement.

For the values of the parameters, ① and ② hold.

①	
$a_2 < a_1$	<input type="checkbox"/>
$a_1 \leq a_2 \leq 2 \cdot a_1$	<input type="checkbox"/>
$a_2 > 2 \cdot a_1$	<input type="checkbox"/>

②	
$b_2 < b_1$	<input type="checkbox"/>
$b_1 \leq b_2 \leq 2 \cdot b_1$	<input type="checkbox"/>
$b_2 > 2 \cdot b_1$	<input type="checkbox"/>

[0/½/1 point]

Task 13

Crime Statistics 2010–2011

The following table describes how many criminal cases were reported in the years 2010 and 2011 in each Austrian province.

Province	Reported crimes 2010	Reported crimes 2011
Burgenland (Burgenland)	9 306	10 391
Carinthia (Kärnten)	30 192	29 710
Lower Austria (Niederösterreich)	73 146	78 634
Upper Austria (Oberösterreich)	66 141	67 477
Salzburg (Salzburg)	29 382	30 948
Styria (Steiermark)	55 167	55 472
Tyrol (Tirol)	44 185	45 944
Vorarlberg (Vorarlberg)	20 662	20 611
Vienna (Wien)	207 564	200 820

Source: http://www.bmi.gv.at/cms/BK/publikationen/krim_statistik/files/2011/KrimStat_Entwicklung_2011.pdf [24.10.2016].

Task:

Determine the relative change in crime cases of Burgenland reported in the year 2011 compared to the year 2010.

[0/1 point]

Task 14

Financial Growth

An amount of € 100,000 is invested at a fixed annual interest rate. The table below provides information about the growth of the investment. In this table, x_n gives the value of the investment after n years ($n \in \mathbb{N}$).

n in years	x_n in euros
0	100 000
1	103 000
2	106 090
3	109 272.7

Task:

Suggest an equation that can be used to determine the value of the investment x_{n+1} based on the value of the investment x_n .

$x_{n+1} =$ _____

[0/1 point]

Task 15

Values of a Derivative Function

Let $f: \mathbb{R} \rightarrow \mathbb{R}$ be a function with equation $f(x) = 3 \cdot e^x$.

Task:

The statements given below refer to the properties of the function f or its derivative function f' . Put a cross next to each of the two true statements.

There is one point $x \in \mathbb{R}$ for which $f'(x) = 2$.	<input type="checkbox"/>
For all $x \in \mathbb{R}$, $f'(x) > f'(x + 1)$ holds.	<input type="checkbox"/>
For all $x \in \mathbb{R}$, $f'(x) = 3 \cdot f(x)$ holds.	<input type="checkbox"/>
There is one point $x \in \mathbb{R}$ for which $f'(x) = 0$.	<input type="checkbox"/>
For all $x \in \mathbb{R}$, $f'(x) \geq 0$ holds.	<input type="checkbox"/>

[0/1 point]

Task 16

Antiderivative

Let $f: \mathbb{R} \rightarrow \mathbb{R}$ be a function with equation $f(x) = a \cdot x^3$ with $a \in \mathbb{R}$.

Task:

Determine a such that the function $F: \mathbb{R} \rightarrow \mathbb{R}$ with equation $F(x) = 5 \cdot x^4 - 2$ is an antiderivative of f .

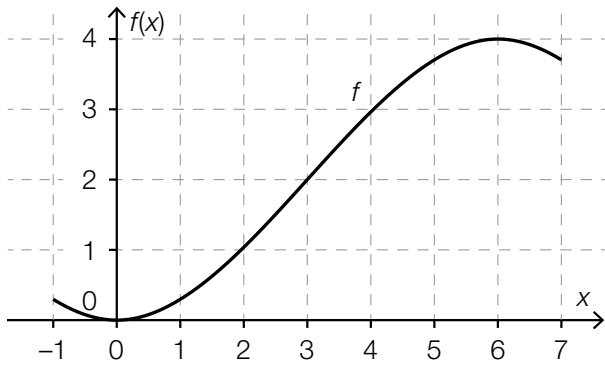
$a =$ _____

[0/1 point]

Task 17

Polynomial Function

The following diagram depicts the graph of a third degree polynomial function $f: \mathbb{R} \rightarrow \mathbb{R}$ in the interval $[-1, 7]$. The x-coordinates of all local maxima and minima and the point of inflexion of f in the interval $[-1, 7]$ are integers and can be read from the diagram.



Task:

Put a cross next to each of the two correct statements about the function f .

$f''(3) = 0$	<input type="checkbox"/>
$f'(1) > f'(3)$	<input type="checkbox"/>
$f''(1) = f''(5)$	<input type="checkbox"/>
$f''(1) > f''(4)$	<input type="checkbox"/>
$f'(3) = 0$	<input type="checkbox"/>

[0/1 point]

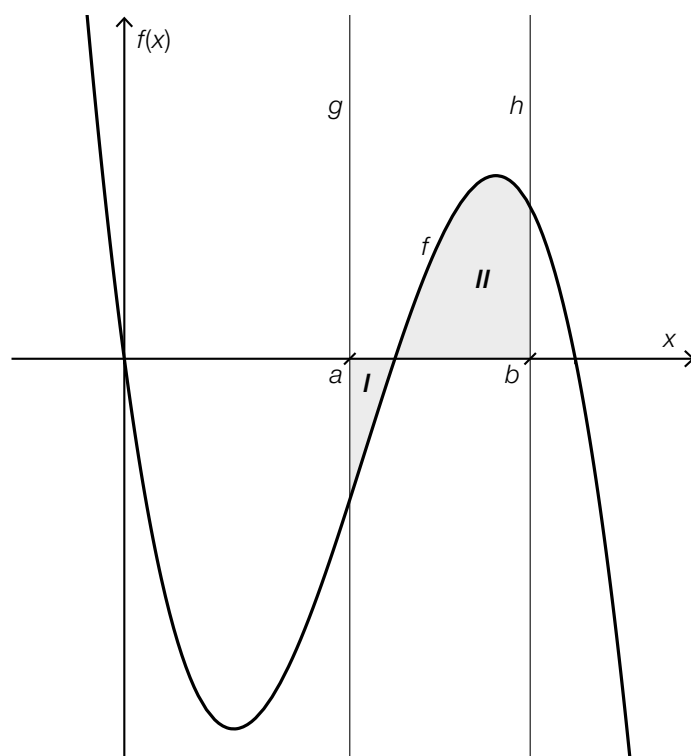
Task 18

Areas of Regions

The diagram below shows the graph of the function $f: \mathbb{R} \rightarrow \mathbb{R}$ and two shaded regions.

The graph of the function f , the x -axis and the line g with equation $x = a$ enclose region **I** with area A_1 .

The graph of the function f , the x -axis and the line h with equation $x = b$ enclose region **II** with area A_2 .



Task:

Write down the definite integral $\int_a^b f(x) dx$ using the areas A_1 and A_2 .

$$\int_a^b f(x) dx = \underline{\hspace{10cm}}$$

[0/1 point]

Task 19

Leisure Behaviour of Teenagers

400 teenagers were asked about their leisure behaviour. Among all respondents, 330 stated that they belong to a sports club, 146 stated that they play an instrument and 98 stated that they belong to a sports club and play an instrument as well.

The results of this survey have been documented in the following table.

	plays an instrument	does not play an instrument	total
belongs to a sports club	98		330
does not belong to a sports club			
total	146		400

Task:

Determine the relative frequency h of the teenagers that were asked who neither belong to a sports club nor play an instrument.

$h =$ _____

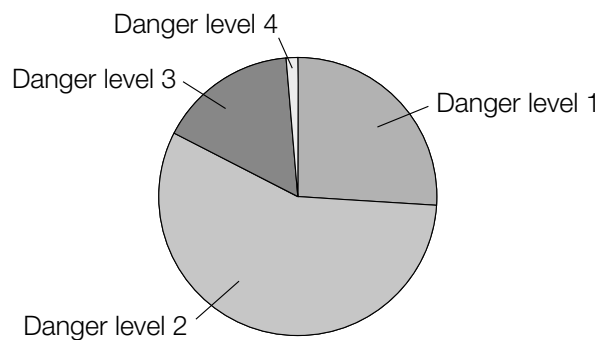
[0/1 point]

Task 20

Danger of Avalanches

During the winter months, the avalanche warning service publishes a daily *avalanche report*. This includes an assessment of the risk of avalanches according to five danger levels.

In a particular region, records of the danger levels were made during the winter of 2013/14. These records give a list of all the days with the according danger level 1 to 4. (There is no entry for danger level 5 in this list, as there was no day with danger level 5 during this period.) The following diagram shows the relative proportion of days with the respective danger level.



Task:

Justify why danger level 2 has to be the median of the data set that the diagram above is based on.

[0/1 point]

Task 21

Dice

A dice, where the sides are numbered 1, 2, 3, 4, 5 and 6, is used in a game. The dice is thrown three times. The following statement holds for each throw: the probability of any number being rolled is the same as for any other number.

Task:

Determine the probability p that a number that is divisible by 3 is thrown on the third throw.

$p =$ _____

[0/1 point]

Task 22

Frequency of Side Effects

Pharmaceutical companies are obliged to list all known side effects of a medication on the package leaflet. The information on the frequency of side effects are based on the following categories:

Frequency	Occurrence of Side Effects
Very common	More than 1 in 10 people under treatment experience side effects.
Common	Between 1 to 10 in 100 people under treatment experience side effects.
Uncommon	Between 1 to 10 in 1 000 people under treatment experience side effects.
Rare	Between 1 to 10 in 10 000 people under treatment experience side effects.
Very rare	Fewer than 1 in 10 000 people under treatment experience side effects.
Unknown	The frequency of side effects cannot be estimated based on the information available.

In the package leaflet of a medication, one particular side effect is categorised as “rare”. 50 000 people are treated with this medication independently from each other. A certain number of people experience this side effect.

Task:

Use the information given on the frequency of side effects above as probabilities and determine the minimum number of people expected to experience this side effect.

[0/1 point]

Task 23

Strike Probability

In a training session, a basketball player throws the ball at the basket six times in a row. If the ball falls into the basket, it is called strike. The probability of a strike for this player is 0.85 for each throw (irrespective of other throws).

Task:

Match each of the four events with the term (from among A to F) which describes the probability of this event occurring.

The player strikes exactly once.	
The player strikes not more than once.	
The player strikes at least once.	
The player strikes exactly twice.	

A	$1 - 0.85^6$
B	$0.15^6 + \binom{6}{1} \cdot 0.85^1 \cdot 0.15^5$
C	$1 - 0.15^6$
D	$0.85^6 + \binom{6}{1} \cdot 0.85^5 \cdot 0.15^1$
E	$6 \cdot 0.85 \cdot 0.15^5$
F	$\binom{6}{2} \cdot 0.85^2 \cdot 0.15^4$

[0/½/1 point]

Task 24

Confidence Interval

Someone wants to determine the unknown proportion p of voters who are going to vote for candidate A in an election and hires an opinion research institute to give an estimate of the proportion p . In the course of this estimation, 200 samples of the same size are drawn. For each of these samples, the respective 95 % confidence interval is determined.

Task:

Determine the expected number of intervals that contain the unknown proportion p .

[0/1 point]

Continue overleaf

Task 25 (Part 2)

System Reliability

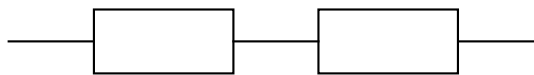
A system is defined as a machine that consists of more than one component. There is a certain probability that each component of a system may function correctly or that it might fail. If individual components fail to work, it depends on the design of the system whether the system as a whole continues to work or whether it fails as well.

The *reliability of a component* is the probability that the component functions correctly, i.e. does not break down. This holds true for a certain period of time and under certain conditions.

The *reliability of a system* is the probability that the system functions correctly. (It is assumed that breakdowns of components happen independently of one another.) The probability of the complementary event is called probability of failure.

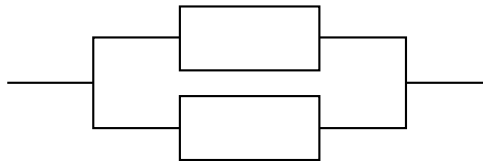
We distinguish between two simple types of systems:

- series systems:



A series system functions only when all its components function.

- parallel systems:



A parallel system functions when at least one of its components functions.

Task:

- a) The following system A is given:



Let p_1 be the probability of component T_1 and p_2 be the probability of component T_2 .

Consider the reliability of system A as a function z_A in terms of p_1 and p_2 .

☐ Write down $z_A(p_1, p_2)$.

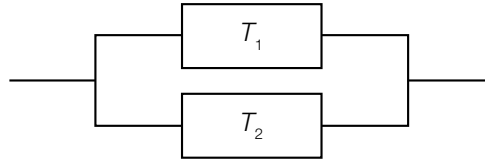
$z_A(p_1, p_2) =$ _____

The components of a different system with the same design have the same reliabilities $p_1 = p_2 = 0.7$. The probability of failure is to be reduced to a quarter of the current probability of failure.

Write down which value the reliability p_{new} (for both components) needs to take.

$p_{\text{new}} =$ _____

b) System B is given:



Both components T_1 and T_2 have the same reliability p .

Consider the reliability of system B as a function z_B of p .

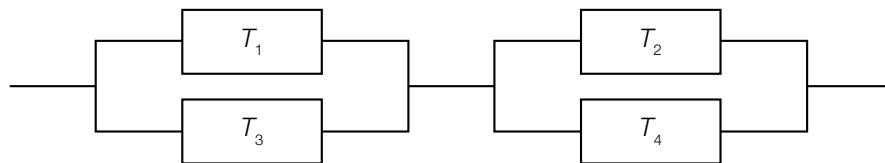
Write down $z_B(p)$.

$z_B(p) =$ _____

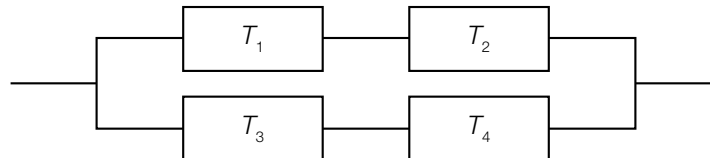
Show by calculation that the function z_B is strictly monotonically increasing in the interval $(0, 1)$.

c) Given are two systems C and D :

System C :



System D :



Every component T_1 , T_2 , T_3 and T_4 has the same reliability p .

The reliability z_C of system C is a function of p and can be described by the equation

$$z_C(p) = p^4 - 4 \cdot p^3 + 4 \cdot p^2.$$

Determine the quotient $\frac{1 - z_C(0.9)}{1 - z_C(0.8)}$ and interpret this value for system C .

The reliability z_D of system D is a function of p .

Justify why $z_C(p) > z_D(p)$ holds true for all $p \in (0, 1)$.

Use either an equation of z_D or provide justification on basis of the design of systems C and D .

Task 26 (Part 2)

Exponential Function and Linear Function

Let $f: \mathbb{R} \rightarrow \mathbb{R}$ be a function with equation $f(x) = e^x$.

Task:

- a) Let $g_1: \mathbb{R} \rightarrow \mathbb{R}$ be a linear function where $g_1(x) = k \cdot x + 2$ and $k \in \mathbb{R}$.

Write down all values of $k \in \mathbb{R}$ for which the graphs of the functions f and g_1 intersect at exactly two points.

For one particular value of k , the function $h: \mathbb{R} \rightarrow \mathbb{R}$ where $h(x) = g_1(x) - f(x)$ gives the difference between g_1 and f . For one value of x , x_0 , between the two points of intersection of the graphs, the relationship $h'(x_0) = 0$.

Determine x_0 in terms of k .

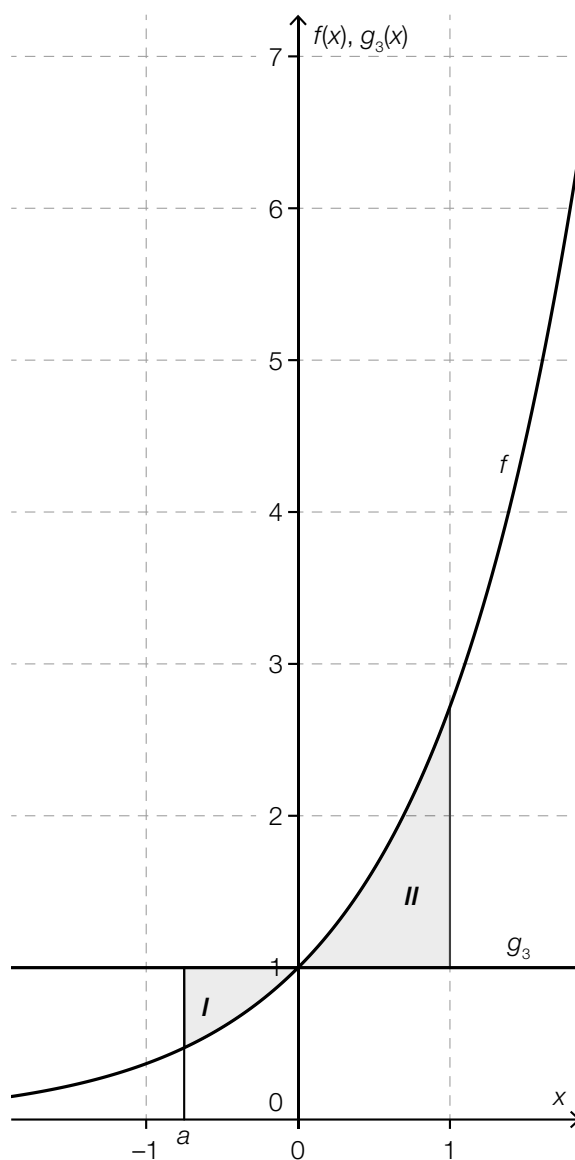
- b) The graph of a linear function $g_2: \mathbb{R} \rightarrow \mathbb{R}$ where $g_2(x) = 4 \cdot x + d$ and $d \in \mathbb{R}$ is a tangent to the graph of f .

Write down the coordinates of the point of contact B between the two graphs.

Write down the value of d .

c) Let $g_3: \mathbb{R} \rightarrow \mathbb{R}$ be a function where $g_3(x) = 1$.

The region **I**, which is bounded by the graphs of g_3 and f in the interval $[a, 0]$ (where $a \in \mathbb{R}$ and $a < 0$), has an area of A_1 . The region **II**, which is bounded by the graphs of f and g_3 in the interval $[0, 1]$, has an area of A_2 .



A Determine the size of the area A_2 .

Write the definite integral $\int_a^1 (f(x) - g_3(x)) dx$ in terms of the areas A_1 and A_2 .

Task 27 (Part 2)

First Names in Austria

Over decades, Statistik Austria, the statistical office of the Republic of Austria, has been collecting the first names that parents give their children. Here, the office only looks at the very first name of a child (if a child is given more than one first name). In addition, certain identical names or names that share the same background, like *Sophie*, *Sofie* and *Sofia*, are grouped together under one name.

For many years now, *Anna* and *Lukas* have been among the most popular names. Out of the children born in the year 2015 (40 777 girls, 43 604 boys), 2 144 girls were named *Anna* and 1 511 boys were named *Lukas*.

Task:

- a) For a statistical survey, 30 girls and 30 boys born in the year 2015 are selected randomly.

☐ A Determine the probability that at least one of the girls in this sample is called Anna.

Determine the probability that in this sample at least one of the girls is called Anna and one of the boys is called Lukas.

- b) In the year 1995, the relative proportion of the ten most popular first names for boys was 37.07 %. In 2005, it was 24.28 %. In the year 2015, it was 20.91 %.

This development of the relative proportion of the ten most popular names is described by a quadratic function f with equation $f(t) = 0.000471 \cdot t^2 + b \cdot t + 0.3707$ where $b \in \mathbb{R}$. Here, t stands for the number of years from 1995.

Determine b and write down an equation of f .

In which year did the relative proportion of the ten most popular names for boys fall below one third for the first time?

Write down the corresponding year.

- c) The random variable X models the number of girls born in Upper Austria in the year 2015, who were called *Anna*. This variable is taken to be binomially distributed with parameters $n = 7\,041$ and $p = 0.0526$.

Determine the expectation value μ and the standard deviation σ of the random variable X .

$\mu \approx$ _____

$\sigma \approx$ _____

In fact, in the year 2015, the name *Anna* was chosen most frequently in all nine provinces, whereby the percentage was highest in Upper Austria. In Upper Austria, 7 041 girls were born in 2015. Out of these, 494 were given the first name *Anna*.

$494 - \mu = c \cdot \sigma$ holds for $c \in \mathbb{R}^+$.

Determine c and interpret the value of c in the given context.

Task 28 (Part 2)

Solid with Rectangular Cross-Sections

Diagrams 1 and 2 below show a solid with flat faces from an oblique perspective as well as one cross-section.

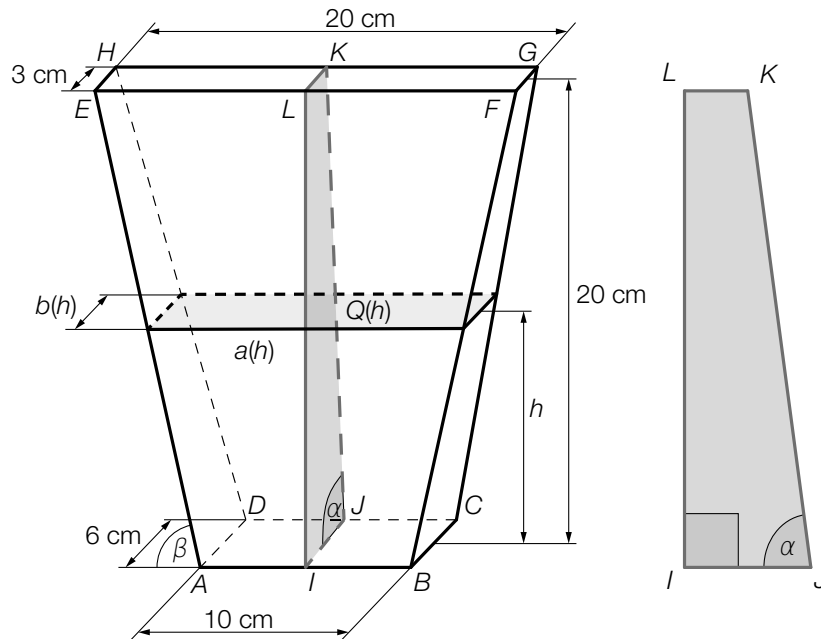


Diagram 1: Oblique perspective of the solid

Diagram 2: Cross-section $IJKL$

The front face $ABFE$ is perpendicular to the horizontal base $ABCD$ and to the horizontal lid $EFGH$. However, the rear face $DCGH$ meets the base at an angle α ($0^\circ < \alpha < 90^\circ$).

The side faces $ADHE$ and $BCGF$ both meet the base at the same angle β (with $\beta \approx 76^\circ$).

The horizontal cross-sections of the solid are rectangular at every height. The lengths $a(h)$ and the widths $b(h)$ of these rectangles change in a linear fashion with respect to the height h . The base has a length of 10 cm and a width of 6 cm; the lid has a length of 20 cm and a width of 3 cm. The height of the solid is 20 cm.

Task:

- a) The function $Q: [0, 20] \rightarrow \mathbb{R}$ gives the size of the area of the cross-section $Q(h)$ in terms of the height h (with $Q(h)$ in cm^2 , h in cm).

The equation $Q(h) = s \cdot h^2 + 1.5 \cdot h + t$ with $s, t \in \mathbb{R}$ holds.

Determine the values of s and t .

Determine the volume of the solid and give your answer using a correct unit.

- b) The instantaneous rate of change of the width $b(h)$ has a constant value of $c \in \mathbb{R}$ for every $h \in [0, 20]$.

Determine the value of c .

Write down an equation that describes the relationship between c and α .

- c) The function a gives the length $a(h)$ at height h where $a(h)$ and h are in cm .

☐ Write down the equation of the function of a .

If the angle β is changed to be 45° and the length of the base AB and the height remain unchanged, a new front face ABF_1E_1 is obtained. The function a_1 where $a_1(h) = 2 \cdot h + 10$ for this new front face gives the length $a_1(h)$ in terms of the height h ($a_1(h)$ and h in cm).

Determine the ratio $\int_0^{20} a_1(h) dh : \int_0^{20} a(h) dh$ and interpret the result in relation to the area of the new front face ABF_1E_1 and the area of the original front face $ABFE$.

