Exemplar für Prüfer/innen

Supplementary Examination for the
Standardised Competence-Oriented
Written School-Leaving Examination

AHS

May 2019

Mathematics

Supplementary Examination 6
Examiner’s Version

Bundesministerium
Bildung, Wissenschaft
und Forschung

Instructions for the supplementary examination

The following supplementary examination is comprised of five tasks that can be completed independently of one another.

Each task contains two parts: The statement of the task requires the candidate to demonstrate core competencies, and the guiding question that follows it requires the candidate to show their ability to communicate their ideas.

In the following document, the examiner will find the tasks as well as the expected solutions and the answer key.

The preparation time is to be at least 30 minutes and the examination time is to be at most 25 minutes.

Assessment

Each task can be awarded zero, one or two points. There is one point available for each demonstration of core competencies as well as for each guiding question. A maximum of 10 points can be achieved.

For the grading of the examination the following scale should be used:

Grade	Number of points
Pass	4 points for the core competencies + 0 points for the guiding questions 3 points for the core competencies + 1 point for the guiding questions
Satisfactory	5 points for the core competencies + 0 points for the guiding questions 4 points for the core competencies + 1 point for the guiding questions 3 points for the core competencies + 2 points for the guiding questions
Good	5 points for the core competencies + 1 point for the guiding questions 4 points for the core competencies + 2 points for the guiding questions 3 points for the core competencies + 3 points for the guiding questions
Very good	5 points for the core competencies + 2 (or more) points for the guiding questions 4 points for the core competencies + 3 (or more) points for the guiding questions

The examination board will decide on the final grade based on the candidate's performance in the supplementary examination as well as the result of the written examination.

Evaluation grid for the supplementary examination

This evaluation grid may be used to assist the examiner in assessing the candidate’s performance.

	Point for core competencies reached	Point for the guiding question reached
Task 1		
Task 2		
Task 3		
Task 4		
Task 5		

Task 1

Lines in $\mathbb{R}^2$

Let A and B be points where $A = (5,1)$ and $B = (1,2)$.

Task:

The lines p, n and s each go through the point A and can be described as follows:

- The line p is parallel to the x-axis.
- The line n is perpendicular to the x-axis.
- The line s has a gradient of 1.

Write down an equation for each of the three lines.

Guiding question:

A line g: $y = k \cdot x + d$ goes through the point B and makes an angle of α to the horizontal.

Write down expressions for k and d in terms of the angle α.

k = ____

d = ____

Write down the size of the angle $\alpha \in [0°, 90°)$ for which the lines g and p do not intersect.

The line g_1 goes through the point B and has the same gradient as the line s. Determine the point of intersection, S, of the line n and the line g_1.

Solution to Task 1

Lines in $\mathbb{R}^2$

Expected solution to the statement of the task:

Possible equations:

p: $y = 1$
n: $x = 5$
s: $y = x - 4$

Answer key:

The point for the core competencies is to be awarded if correct equations or vector equations for the lines p, n, and s have been given. Equivalent equations or vector equations are also to be marked as correct.

Expected solution to the guiding question:

$k = \tan(\alpha)$
$d = 2 - \tan(\alpha)$

The slope of both lines is zero, therefore $\alpha = 0°$.

Thus, g_1: $y = x + 1 \quad \Rightarrow \quad S = (5, 6)$.

Answer key:

The point for the guiding question is to be awarded if k and d have been given correctly and the angle α as well as the point of intersection of the lines g_1 and n have been given correctly.

Task 2

Barometric Altitude Formula

The relationship between the height h above sea level and the air pressure $p(h)$ at that height can be approximated by the barometric altitude formula:

$p(h) = p_0 \cdot e^{-\frac{h}{7991}}$

h ... height above sea level in metres (m)
$p(h)$ ... air pressure at a height of h in hectopascals (hPa)
p_0 ... air pressure at sea level (at $h = 0$); $p_0 > 0$

Task:

Determine the height h_1 at which the air pressure is only 80 % of p_0.

Guiding question:

The relationship between the height above sea level and the air pressure in the interval [0 m, 3500 m] can be approximated by a linear function f (in terms of h).

On a particular day the following values were obtained:

Height above sea level in m	Air pressure in hPa
1500	840
2000	790

Determine an equation of this function f so that it reflects the measured values.

For the height h_1 found above, determine the difference (in hPa) between the value of $p(h_1)$ calculated using the barometric altitude formula and the value calculated using the linear function f. In your calculations, assume that the value of p_0 in the function p is 1013 hPa.

Solution to Task 2

Barometric Altitude Formula

Expected solution to the statement of the task:

$0.8 \cdot p_0 = p_0 \cdot e^{-\frac{h_1}{7991}}$

$h_1 \approx 1\,783$ m

Answer key:

The point for the core competencies is to be awarded if the correct height h_1 has been determined.
Tolerance interval: [1 780 m, 1 790 m]

Expected solution to the guiding question:

Possible method:

$f(h) = -0.1 \cdot h + 990$

$p(h) = 1\,013 \cdot e^{-\frac{h}{7991}}$

$f(1\,783) = 811.7$ hPa

$p(1\,783) \approx 810.4$ hPa

The difference is around 1.3 hPa.

Answer key:

The point for the guiding question is to be awarded if a correct function of *f* has been given and the difference between the two values has been calculated correctly.
Tolerance interval: [1 hPa, 2 hPa]

Task 3

Rates of Change

Let $f\colon \mathbb{R} \to \mathbb{R}$ be a function where $f(x) = x^2 - 2 \cdot x - 1$.

Task:

Determine the differential quotient of f when $x = 1$ and write down the meaning of this value in terms of the behaviour of the graph.

Guiding question:

Write down an expression in terms of the parameter a ($a \in \mathbb{R}$ and $a < 3$) that can be used to calculate the difference quotient of f in the interval $[a, 3]$.

Determine the value of the parameter a such that this difference quotient is equal to the differential quotient of f when $x = 1$.

Solution to Task 3

Rates of Change

Expected solution to the statement of the task:

$f'(x) = 2 \cdot x - 2 \quad \Rightarrow \quad f'(1) = 0$

Possible meaning:
The graph of the function f has a horizontal tangent when $x = 1$.

Answer key:

The point for the core competencies is to be awarded if the differential quotient has been determined correctly and the meaning of the value for the shape of a graph has been explained correctly.

Expected solution to the guiding question:

Possible expression:

$$\frac{f(3) - f(a)}{3 - a} = \frac{2 - (a^2 - 2 \cdot a - 1)}{3 - a} = \frac{-a^2 + 2 \cdot a + 3}{3 - a} \ (= a + 1)$$

$-a^2 + 2 \cdot a + 3 = 0 \quad \Rightarrow \quad a = -1$

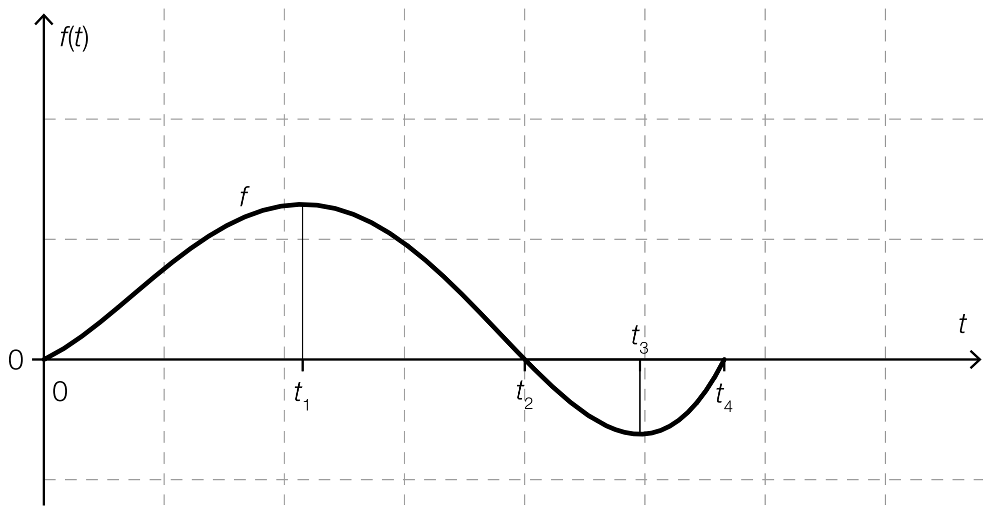
Answer key:

The point for the guiding question is to be awarded if a correct expression and the correct value for a have been given.

Task 4

Rain Water

There are 20 litres of water in a water butt.
From the time $t = 0$ the amount of water in the butt changes. The function f describes the instantaneous rate of change of the amount of water in the butt in terms of time ($f(t)$ is measured in litres per hour, t is measured in hours).
The diagram below shows the graph of the function f.



Task:

Write down at which point in time marked in the diagram above (from t_1 to t_4) the amount of water in the butt is highest and justify your decision.

Guiding question:

Write down a formula for the amount of water M in the butt at time t_4.

In the diagram above in the interval $[0, t_4)$, label the approximate time t^* at which there is exactly the same amount of water in the butt as at t_4. Explain your method.

Solution to Task 4

Rain Water

Expected solution to the statement of the task:

At time t_2 the amount of water in the butt is the highest because the value of f is positive until this time, which means that the amount of water in the butt increases until this time (afterwards the value of f is negative, which means that the amount of water in the butt decreases after this point).

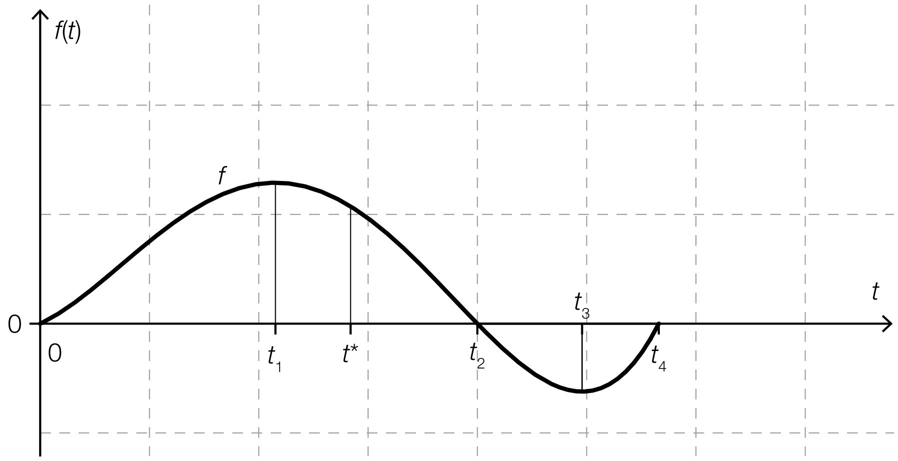
Answer key:

The point for the core competencies is to be awarded if the time t_2 has been stated and this decision has been justified correctly.

Expected solution to the guiding question:

Possible formula:

$M = 20 + \int_0^{t_4} f(t)\,\mathrm{d}t$



The areas bounded by the graph of f and the x-axis in the intervals $[t^*, t_2]$ and $[t_2, t_4]$ must be the same size because the amount of water in the butt in the interval $[t^*, t_2]$ has to increase by the same amount as it decreases in the interval $[t_2, t_4]$.

Answer key:

The point for the guiding question is to be awarded if a correct formula for M has been given, the time t^* has been labelled in approximately the correct place, and the method has been explained correctly.

Task 5

Multi-Step Random Experiment

In a game three urns, each containing six balls, are used. Urn *A* contains five white balls and one black ball, urn *B* contains four white and two black balls, and urn *C* contains three white and three black balls. A player wins the game if a white ball is selected.

Victoria participates in this game and thus has to complete the following steps:
First, she chooses one urn and takes a ball out of this urn. In both cases (the choice of the urn and the choice of the ball) the selections are made at random.

Task:

Determine the probability that Victoria wins the game.

Guiding question:

In one option of the game, twelve white and six black balls are being distributed anew, whereby there are six balls again in each of the three urns. In the first urn there are x white balls now, in the second urn there are y white balls, and in the third urn there are z white balls.
Show by calculation that the probability of Victoria winning does not change.

Solution to Task 5

Multi-Step Random Experiment

Expected solution to the statement of the task:

$P(\text{“Victoria wins”}) = \frac{1}{3} \cdot \frac{5}{6} + \frac{1}{3} \cdot \frac{4}{6} + \frac{1}{3} \cdot \frac{3}{6} = \frac{2}{3}$

Answer key:

The point for the core competencies is to be awarded if the probability has been calculated correctly.

Expected solution to the guiding question:

$P(\text{“Victoria wins”}) = \frac{1}{3} \cdot \frac{x}{6} + \frac{1}{3} \cdot \frac{y}{6} + \frac{1}{3} \cdot \frac{z}{6} = \frac{1}{3} \cdot \frac{x + y + z}{6} = \frac{1}{3} \cdot \frac{12}{6} = \frac{2}{3}$

Answer key:

The point for the guiding question is to be awarded if it has been correctly shown by calculation that the probability of Victoria winning is $\frac{2}{3}$ again.