Supplementary Examination for the Standardised Competence-Oriented Written School-Leaving Examination

AHS

## October 2019

## Mathematics

Supplementary Examination 2
Candidate's Version
= Bundesministerium
Bildung, Wissenschaft
und Forschung

## Instructions for the supplementary examination

Dear candidate,
The following supplementary examination is comprised of five tasks that can be completed independently of one another.

Each task contains two parts: The statement of the task requires you to demonstrate core competencies, and the guiding question that follows it requires you to demonstrate your ability to communicate your ideas.

You will be given preparation time of at least 30 minutes, and the examination will last at the most 25 minutes.

## Assessment

Each task can be awarded zero, one or two points. There is one point available for each demonstration of core competencies as well as for each guiding question. A maximum of 10 points can be achieved.

For the grading of the examination the following scale will be used:

| Grade | Number of points |
| :--- | :--- |
| Pass | 4 points for the core competencies + 0 points for the guiding questions <br> 3 points for the core competencies + 1 point for the guiding questions |
| Satisfactory | 5 points for the core competencies + 0 points for the guiding questions <br> 4 points for the core competencies + 1 point for the guiding questions <br> 3 points for the core competencies + 2 points for the guiding questions |
| Good | 5 points for the core competencies + 1 point for the guiding questions <br> 4 points for the core competencies + 2 points for the guiding questions <br> 3 points for the core competencies + 3 points for the guiding questions |
| Very good | 5 points for the core competencies + 2 (or more) points for the guiding questions <br> 4 points for the core competencies + 3 (or more) points for the guiding questions |

The examination board will decide on the final grade based on your performance in the supplementary examination as well as the result of the written examination.

## Good Luck!

## Task 1

## Sets of Numbers

Numbers are contained in one or more sets of numbers.

## Task:

- For each of the numbers shown below, put a cross next to each set of numbers that the number is an element of.

|  | $\mathbb{Z}^{-}$ | $\mathbb{Q}$ | $\mathbb{R}^{+}$ |
| :--- | :--- | :--- | :--- |
| $\frac{\pi}{2}$ |  |  |  |
| $3 \cdot \sqrt{3}$ |  |  |  |
| $-\frac{16}{8}$ |  |  |  |
| $1.23 \cdot 10^{-3}$ |  |  |  |

## Guiding question:

If the result of an operation on any two numbers that are members of a particular set is also a member of this set, then this set is said to be closed under this operation.
For example: for any $a, b \in \mathbb{N}$ holds: $a \cdot b \in \mathbb{N}$. Therefore, the set of natural numbers is closed under multiplication.

Consider the operations subtraction, multiplication and taking a square root.

- Write down whether the set of numbers $\mathbb{Q}^{-}$is closed under the operations given above and justify your answers.


## Task 2

## Solutions to Quadratic Equations

Let $x^{2}-2 \cdot x=p$ with $p \in \mathbb{R}$ be a quadratic equation.

## Task:

- Write down all values of $p$ for which the equation shown above has solutions in the set $\mathbb{R}$.


## Guiding question:

- Write down the possible solution scenarios for a quadratic equation of the form
$a \cdot x^{2}+b \cdot x+c=0(a, b, c \in \mathbb{R} ; a \neq 0)$ and show these scenarios graphically by sketching an appropriate graph of a quadratic function for each scenario.


## Task 3

## Movement of an Object

An object moves along a straight path. The distance (in metres) of the object from its starting point is modelled in terms of time $t$ (in seconds) by the third degree polynomial function $s$. The graph of this function $s$ is shown in the diagram below; the point of inflexion $W$, the maximum $H$, and the zero $N$ have integer coordinates.


## Task:

- Describe the movement of the object in words, and explain the significance of the coordinates of the points $W, H$ and $N$.


## Guiding question:

The function $v$ describes the velocity of the object in the time interval $[0,12]$.

- Write down the value of the area enclosed by the graph of the function $v$ and the time axis in the time interval $[0,8]$.
- Based on the diagram shown above, explain why the maximum speed is greater than $4 \mathrm{~m} / \mathrm{s}$.


## Task 4

## Pellet Consumption

In 2016 in Germany, 8.1 \% more pellets were used than in 2015.
In 2017, 5 \% more were used than in 2016.
In 2018, the consumption was 4.8 \% higher than in 2017.
In 2017, 2.1 million tonnes of pellets were used.

## Task:

- Write down the absolute and relative change in pellet consumption from 2015 to 2018.


## Guiding question:

- Determine the annual percentage rate of change $p$ in pellet consumption from 2015 to 2018 if a constant increase is assumed.
- Using the consumption value for 2017 and the annual percentage rate of change $p$ determined above, write down the number of years after which the pellet consumption first reaches 2.5 million tonnes.


## Task 5

## Gladiolas

Gladiolas are popular cut flowers that grow from gladiola bulbs. By looking at the gladiola bulb, it is impossible to say which colour the gladiola's flowers will be. It is assumed that for a particular type of gladiola, $12 \%$ of all gladiolas have red flowers.

## Task:

A hobby gardener plants $n$ randomly selected gladiola bulbs in the ground.

- Determine the value of $n$ if 6 gladiola plants with red flowers are expected.
- Write down the probability that there are at least 5 gladiola plants with red flowers that grow from the $n$ gladiola bulbs that have been planted.


## Guiding question:

A wholesaler sells gladiola bulbs in sacks that each contain 200 bulbs. He would like to guarantee that the number of gladiolas with red flowers in a sack does not differ from the expected value by more than a particular number $c$. He would like to be able to keep the promise of this guarantee with a probability of at least $95 \%$.

- Write down the smallest value that the deviation c must be

