Exemplar für Prüfer/innen

Supplementary Examination for the
Standardised Competence-Oriented
Written School-Leaving Examination

AHS

October 2019

Mathematics

Supplementary Examination 2
Examiner's Version

Bundesministerium
Bildung, Wissenschaft
und Forschung

Instructions for the supplementary examination

The following supplementary examination is comprised of five tasks that can be completed independently of one another.

Each task contains two parts: The statement of the task requires the candidate to demonstrate core competencies, and the guiding question that follows it requires the candidate to show their ability to communicate their ideas.

In the following document, the examiner will find the tasks as well as the expected solutions and the answer key.

The preparation time shall be at least 30 minutes and the examination time shall be at most 25 minutes.

Assessment

Each task can be awarded zero, one or two points. There is one point available for each demonstration of core competencies as well as for each guiding question. A maximum of 10 points can be achieved.

The following scale will be used for the grading of the examination:

Grade	Number of points
Pass	4 points for the core competencies + 0 points for the guiding questions 3 points for the core competencies + 1 point for the guiding questions
Satisfactory	5 points for the core competencies + 0 points for the guiding questions 4 points for the core competencies + 1 point for the guiding questions 3 points for the core competencies + 2 points for the guiding questions
Good	5 points for the core competencies + 1 point for the guiding questions 4 points for the core competencies + 2 points for the guiding questions 3 points for the core competencies + 3 points for the guiding questions
Very good	5 points for the core competencies + 2 (or more) points for the guiding questions 4 points for the core competencies + 3 (or more) points for the guiding questions

The examination board will decide on the final grade based on the candidate's performance in the supplementary examination as well as the result of the written examination.

Evaluation grid for the supplementary examination

This evaluation grid may be used to assist the examiner in assessing the candidate's performance.

	Point for core competencies reached	Point for the guiding question reached
Task 1		
Task 2		
Task 3		
Task 4		
Task 5		

Task 1

Sets of Numbers

Numbers are contained in one or more sets of numbers.

Task:

– For each of the numbers shown below, put a cross next to each set of numbers that the number is an element of.

	$\mathbb{Z}^-$	$\mathbb{Q}$	$\mathbb{R}^+$
$\frac{\pi}{2}$			
$3 \cdot \sqrt{3}$			
$-\frac{16}{8}$			
$1.23 \cdot 10^{-3}$			

Guiding question:

If the result of an operation on any two numbers that are members of a particular set is also a member of this set, then this set is said to be closed under this operation.
For example: for any $a, b \in \mathbb{N}$ holds: $a \cdot b \in \mathbb{N}$. Therefore, the set of natural numbers is closed under multiplication.

Consider the operations subtraction, multiplication and taking a square root.

– Write down whether the set of numbers $\mathbb{Q}^-$ is closed under the operations given above and justify your answers.

Solution to Task 1

Sets of Numbers

Expected solution to the statement of the task:

	$\mathbb{Z}^-$	$\mathbb{Q}$	$\mathbb{R}^+$
$\frac{\pi}{2}$			×
$3 \cdot \sqrt{3}$			×
$-\frac{16}{8}$	×	×	
$1.23 \cdot 10^{-3}$		×	×

Answer key:

The point for the core competencies shall be awarded if the correct set(s) of numbers has/have been crossed for each of the numbers.

Expected solution to the guiding question:

A set of numbers is not closed under an operation if at least one counter-example exists.

The set of numbers $\mathbb{Q}^-$ is not closed under any of the operations listed because, for example:

- Subtraction: $-\frac{1}{3} - \left(-\frac{1}{2}\right) = \frac{1}{6} \notin \mathbb{Q}^-$
- Multiplication: $-\frac{1}{3} \cdot \left(-\frac{1}{2}\right) = \frac{1}{6} \notin \mathbb{Q}^-$
- Taking a square root: $\sqrt{-\frac{1}{6}} \notin \mathbb{Q}^-$

Answer key:

The point for the guiding question shall be awarded if it has been correctly determined for each of the operations that the set of numbers is not closed and this has been justified correctly in each case.

Task 2

Solutions to Quadratic Equations

Let $x^2 - 2 \cdot x = p$ with $p \in \mathbb{R}$ be a quadratic equation.

Task:

– Write down all values of p for which the equation shown above has solutions in the set $\mathbb{R}$.

Guiding question:

– Write down the possible solution scenarios for a quadratic equation of the form $a \cdot x^2 + b \cdot x + c = 0$ $(a, b, c \in \mathbb{R}; a \neq 0)$ and show these scenarios graphically by sketching an appropriate graph of a quadratic function for each scenario.

Solution to Task 2

Solutions to Quadratic Equations

Expected solution to the statement of the task:

$x^2 - 2 \cdot x - p = 0$
$x_{1,2} = 1 \pm \sqrt{1 + p}$
$1 + p \geq 0 \quad \Rightarrow \quad p \geq -1$

Answer key:

The point for the core competencies shall be awarded if all correct values of p have been given.

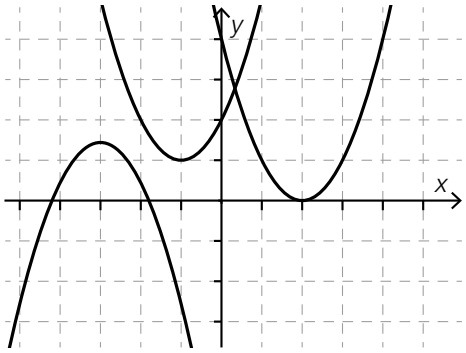
Expected solution to the guiding question:

A quadratic equation has either no real solutions, one real solution or two real solutions.

Possible graphical representation:
Graphs of quadratic functions are parabolas, which either do not cross the x-axis, touch it, or cross the x-axis twice.

The quadratic equation has no real solutions if the corresponding parabola does not cross the x-axis.
The quadratic equation has exactly one real solution if the corresponding parabola touches the x-axis.
The quadratic equation has two real solutions if the corresponding parabola crosses the x-axis twice.

Possible sketch:



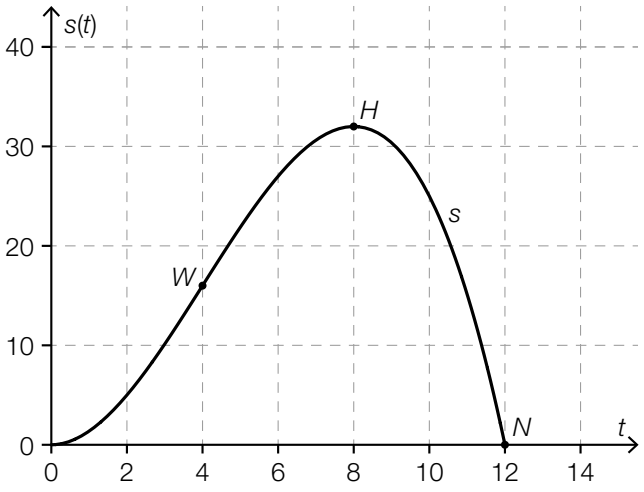
Answer key:

The point for the guiding question shall be awarded if the three solution scenarios have been given correctly and a correct graphical representation has been shown.

Task 3

Movement of an Object

An object moves along a straight path. The distance (in metres) of the object from its starting point is modelled in terms of time t (in seconds) by the third degree polynomial function s. The graph of this function s is shown in the diagram below; the point of inflexion W, the maximum H, and the zero N have integer coordinates.



Task:

– Describe the movement of the object in words, and explain the significance of the coordinates of the points W, H and N.

Guiding question:

The function v describes the velocity of the object in the time interval [0, 12].

– Write down the value of the area enclosed by the graph of the function v and the time axis in the time interval [0, 8].
– Based on the diagram shown above, explain why the maximum speed is greater than 4 m/s.

Solution to Task 3

Movement of an Object

Expected solution to the statement of the task:

Possible description:
The object accelerates for 4 s. After 4 s, when it is 16 m away from its starting point, the object begins to move more slowly. After 8 s, when it is 32 m away from its starting point, the object changes its direction of movement, and after a total of 12 s it is back at its starting point.

Answer key:

The point for the core competencies shall be awarded if a correct description of the movement of the object, including the coordinates of the points, has been given.

Expected solution to the guiding question:

The size of the area between the graph of v and the time axis in the interval [0, 8] has the value 32.

Possible justification:
The maximum of v is at $t = 4$.
The gradient of the tangent at point W on the graph of s is greater than 4 as this tangent is steeper than, for example, the straight line connecting the origin and H, whose gradient is exactly 4.

Answer key:

The point for the guiding question shall be awarded if the correct value for the size of the area has been determined and if a correct explanation of why the maximum must have a value greater than 4 has been given.

Task 4

Pellet Consumption

In 2016 in Germany, 8.1 % more pellets were used than in 2015.
In 2017, 5 % more were used than in 2016.
In 2018, the consumption was 4.8 % higher than in 2017.

In 2017, 2.1 million tonnes of pellets were used.

Task:

– Write down the absolute and relative change in pellet consumption from 2015 to 2018.

Guiding question:

– Determine the annual percentage rate of change p in pellet consumption from 2015 to 2018 if a constant increase is assumed.
– Using the consumption value for 2017 and the annual percentage rate of change p determined above, write down the number of years after which the pellet consumption first reaches 2.5 million tonnes.

Solution to Task 4

Pellet Consumption

Expected solution to the statement of the task:

Consumption in 2015: $\frac{2.1}{1.05 \cdot 1.081} = 1.850...$
Consumption in 2018: $2.1 \cdot 1.048 = 2.2...$
$\Rightarrow$ The absolute change is approximately 0.35 million tonnes.

$1.081 \cdot 1.05 \cdot 1.048 = 1.189...$
$\Rightarrow$ The pellet consumption increased by around 19 % in this time period.

Answer key:

The point for the core competencies shall be awarded if the correct absolute change and the correct percentage change have been given.

Expected solution to the guiding question:

$1.85 \cdot \left(1 + \frac{p}{100}\right)^3 = 2.2 \quad \Rightarrow \quad p = 5.94... \quad \Rightarrow \quad p \approx 6\ \%$

The pellet consumption increases by an average of 6 % per year.

$2.1 \cdot 1.06^t = 2.5 \quad \Rightarrow \quad t \approx 3$
The pellet consumption will reach 2.5 million tonnes for the first time after around 3 years.

Answer key:

The point for the guiding question shall be awarded if the correct value for the rate of change and the correct time period have been given.

Task 5

Gladiolas

Gladiolas are popular cut flowers that grow from gladiola bulbs. By looking at the gladiola bulb, it is impossible to say which colour the gladiola's flowers will be. It is assumed that for a particular type of gladiola, 12 % of all gladiolas have red flowers.

Task:

A hobby gardener plants n randomly selected gladiola bulbs in the ground.

– Determine the value of n if 6 gladiola plants with red flowers are expected.
– Write down the probability that there are at least 5 gladiola plants with red flowers that grow from the n gladiola bulbs that have been planted.

Guiding question:

A wholesaler sells gladiola bulbs in sacks that each contain 200 bulbs. He would like to guarantee that the number of gladiolas with red flowers in a sack does not differ from the expected value by more than a particular number c. He would like to be able to keep the promise of this guarantee with a probability of at least 95 %.

– Write down the smallest value that the deviation c must be.

Solution to Task 5

Gladiolas

Expected solution to the statement of the task:

$n \cdot 0.12 = 6 \quad \Rightarrow \quad n = 50$

The random variable X gives the number of gladiola plants that grow to have red flowers.

$P(X \geq 5) = 0.732...$

The probability is around 73 %.

Answer key:

The point for the core competencies shall be awarded if the correct sample size n and the correct probability have been given.

Expected solution to the guiding question:

$$\left.\begin{array}{l} n = 200 \\ p = 12\ \% \end{array}\right\} \quad E(X) = 24$$

$P(24 - c \leq X \leq 24 + c) \geq 0.95$

$c \geq 9 \quad \Rightarrow \quad c$ must be at least 9

Answer key:

The point for the guiding question shall be awarded if the correct deviation c has been given.