Bundesministerium
Bildung, Wissenschaft
und Forschung

Mathematics Core Competencies for the SRP in Mathematics (AHS)

Fundamental content areas for the core competencies in mathematics

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Content Area *Algebra and Geometrie (AG)*

Algebra is the language of mathematics, through which two central mathematical ideas become particularly visible: generalisation and operational flexibility. Variables shift the attention from specific numbers to a defined set of numbers (or other mathematical objects); defined operations make it possible to combine variables and thus to represent relationships between them; and, finally, algebra provides a system of rules for the formal operational rearranging of relationships of this type, through which yet more relationships become visible.

In order to use mathematics and for communicating about and reflecting on mathematics, a proper handling of basic terms and concepts in algebra is essential. In particular, this includes various sets of numbers, variables, expressions, equations (formulae), and inequalities as well as systems of equations. A proper handling includes an appropriate interpretation of these terms and concepts in context as well as suitable usage of these terms and concepts to represent abstract situations and their rearrangement following certain rules. Reflection on possible solutions and cases as well as the (limits and exploration of the) applicability of the respective concepts are also significant for certain communicative purposes.

The extension of the concept of number to vectors and the determination of suitable rules to combine these mathematical objects operationally leads to an important generalisation of the concepts of number and variable.

By introducing coordinates, it is possible to place points in two- and three-dimensional space so that geometric objects can be described algebraically using vectors and can therefore be solved through purely geometric (using angles, lengths or volumes) means and geometric problems can be considered through the lens of algebra.

This connection between algebra and geometry does not only allow geometric situations to be represented by algebraic means (e.g. vectors as algebraic representations of arrows or points) and to be treated as such, but also algebraic situations can be considered geometrically (e.g. lists of three numbers as points or arrows in three dimensions) and to gain new insights from this shift in perspectives. These interpretations of algebraic objects in geometry as well as geometric objects in algebra and a flexible exchange between these representations and perspectives are of great significance for various communicative purposes.

In trigonometry, relationships in right-angled triangles are of interest, as well as the expansion to general triangles. Elementary relationships of this type should be understood, and more complex geometric relationships should be able to be represented using these elementary relationships.

Core Competencies

Fundamental Concepts in Algebra

AG 1.1 to be able to apply knowledge of the sets of numbers $\mathbb{N}$, $\mathbb{Z}$, $\mathbb{Q}$, $\mathbb{R}$, $\mathbb{C}$

AG 1.2 to be able to apply knowledge of algebraic concepts appropriately: variables, expressions, formulae, inequalities, equations, systems of equations, equivalency, rearrangements, solvability

Comments:

For the sets of numbers, the names of the sets and the subset relationships should be familiar, elements of the sets should be able to be given and assigned, and the set of real numbers should be understood as the basis for continuous models. Knowledge of the real numbers also includes knowing that sets of numbers exist that go beyond $\mathbb{R}$.

The algebraic concepts should be able to be described/explained using simple examples and to be applied appropriately.

Inequalities, Equations and Systems of Equations

AG 2.1 to be able to construct and rearrange simple expressions and formulae and interpret them in context

AG 2.2 to be able to construct, interpret and rearrange/solve linear equations and to interpret the solution in context

AG 2.3 to be able to rearrange/solve quadratic equations in one variable, understand solution cases, interpret solutions and solution cases (also geometrically)

AG 2.4 to be able to construct, interpret and rearrange/solve inequalities, understand solution cases, interpret solutions and solution cases (also geometrically)

AG 2.5 to be able to construct, interpret and rearrange/solve systems of linear equations, understand solution cases, interpret solutions and solution cases (also geometrically)

Comments:

Simple expressions can also include exponents, roots, logarithms, sine etc.

Using electronic aids, more complex rearrangements of expressions, formulae, equations, inequalities and systems of equations can also be conducted.

Vectors

AG 3.1 to be able to appropriately use vectors as an ordered list of numbers and interpret them in context

AG 3.2 to be able to interpret and appropriately use vectors geometrically (as points or arrows)

AG 3.3 to be able to understand the definitions of operations with vectors (addition, multiplication with a scalar, scalar product), appropriately apply these operations and interpret them (also geometrically)

AG 3.4 to be able to write lines in $\mathbb{R}^2$ as vector equations and equations and in $\mathbb{R}^3$ as vector equations and interpret these representations; analyse positional relationships (between lines and between a point and a line), determine points of intersection

AG 3.5 to be able to write normal vectors in $\mathbb{R}^2$, apply them appropriately and interpret them

Comments:

Vectors are to be understood as ordered lists of numbers i.e. as algebraic objects and be applied in appropriate contexts. Points and arrows in two- and three-dimensional space must be able to be interpreted as geometric visualisations of these algebraic objects.

The geometric interpretation of the scalar (in $\mathbb{R}^2$ and $\mathbb{R}^3$) refers here only to the special case $\vec{a} \cdot \vec{b} = 0$. Lines should be able to be written and interpreted as vector equations and as equations (without parameters) in $\mathbb{R}^2$.

Trigonometry

AG 4.1 to be able to understand the definitions of *sine, cosine* and *tangent* in right-angled triangles and apply them to solve right-angled triangles

AG 4.2 to be able to understand and apply the definitions of *sine* and *cosine* for angles greater than 90°

Comments:

The contexts are restricted to simple cases in two- and three-dimensional space, more complex (measurement) tasks are not referred to; the sine and cosine rules are not required.

Content Area *Functional Relationships (FA)*

When experts use mathematics, they often employ the tool of functions. Therefore, it is necessary for comprehensible communication to be familiar and competent with the specific perspective of functions. This means being able to focus the attention on the relationship between two (or more) amounts in various contexts as well as being able to work flexibly with the common forms of representation.

The understanding of the use of the most important function types is central to the fundamental knowledge of mathematics: knowing their names and equations, recognising and knowing the typical behaviours of their graphs, changing between the forms of representation, knowing their characteristic properties and being able to interpret these.

In general, communicative purposes (representing, interpreting, justifying) are of significance; sometimes also constructive purposes (building models) may be helpful. Mathematical-operational purposes are of lesser importance in communicative situations.

Furthermore, (reflective) knowledge of the advantages and disadvantages of functions is very important. In this respect, knowledge about the different types of models (constructive, explanatory, descriptive) as well as their significance and usage is helpful.

If an overview of the most important types of functions has been gained and important characteristics needed to describe functions are known (monotonicity, change in monotonicity, points of inflexion, periodicity, zeros, poles), communication can be extended to unknown functions as well as compositions of functions.

Core Competencies

Concept of a function, real functions, representations and properties

FA 1.1 to be able to decide whether given relations can be considered as functions

FA 1.2 to be able to interpret formulae as representations of functions and classify them as a type of function

FA 1.3 to be able to convert between tabular and graphical representations of functional relationships

FA 1.4 to be able to determine (pairs of) values of functions from tables, graphs[1] and equations and interpret them in context

FA 1.5 to be able to recognise properties of functions, name them, interpret them in context and apply them in constructing graphs of functions: (change of) monotonicity, local maxima and minima, points of inflexion, periodicity, symmetry about the axes, asymptotic behaviour, points of intersection with the axes

FA 1.6 to be able to determine points of intersection between two graphs of functions graphically and by calculation and interpret them in context

FA 1.7 to be able to understand functions as mathematical models and thus work with them appropriately

[1] The graph of a function is defined as the set of its pairs of values. Following a widely used linguistic convention, we also call the graphical representation of the graph in the Cartesian coordinate system a "graph".

FA 1.8 to be able to interpret functions in multiple variables given as equations (formulae) in context and determine values of these functions

FA 1.9 to be able to give an overview of the most important types of mathematical functions (listed below) and compare their characteristics

Comments:

Value is placed on the ability to consistently distinguish between functional and non-functional relationships; theoretical properties (e.g. injectivity, survectivity, invertibility) are not focused on. The role of functions as models and the appropriate use of fundamental types of functions and their characteristics as well as the various forms of representation of functions (including $f\colon A \to B, x \mapsto f(x)$) are prioritised.

Working with functions in multiple variables is restricted to the interpretation of the equation of the function in context as well as determining values of these functions.

The behaviour of functions should not only be described mathematically but also interpreted in the given context.

Linear Functions $[f(x) = m \cdot x + c]$

FA 2.1 to be able to recognise and work with linear relationships in verbal, tabular or graphical form or represented as an equation (formula) and to convert between these forms of representation

FA 2.2 to be able to determine (pairs of) values as well as the parameters m and c from tables, graphs and equations and interpret these in context

FA 2.3 to be able to recognise the effect of the parameters m and c and interpret these parameters in various contexts

FA 2.4 to be able to understand important properties and interpret them in context:
$f(x + 1) = f(x) + m; \frac{f(x_2) - f(x_1)}{x_2 - x_1} = m = [f'(x)]$

FA 2.5 to be able to evaluate the appropriacy of using a linear function to describe a situation

FA 2.6 to be able to describe directly proportional relationships as linear functions of the type $f(x) = m \cdot x$

Comments:

The parameters *m* and *c* should be able to be interpreted for concrete values as well as in general in a given context. This also applies to the effect of the parameters and the effect of varying them.

Power functions with $f(x) = a \cdot x^z$ **and functions of the type** $f(x) = a \cdot x^z + b$ **with** $z \in \mathbb{Z}\backslash\{0\}$ **or** $z = \frac{1}{2}$

FA 3.1 to be able to recognise and work with these types of relationships in verbal, tabular or graphical form or represented as an equation (formula) and to convert between these forms of representation

FA 3.2 to be able to determine (pairs of) values as well as the parameters *a* and *b* from tables, graphs and equations and interpret these in context

FA 3.3 to be able to recognise the effect of the parameters *a* and *b* and interpret these parameters in context

FA 3.4 to be able to describe indirectly proportional relationships as power functions of the type
$f(x) = \frac{a}{x}$ (bzw. $f(x) = a \cdot x^{-1}$)

Polynomial functions $[f(x) = \sum_{i=0}^{n} a_i \cdot x^i$ **with** $n \in \mathbb{N}]$

FA 4.1 to be able to recognise and understand typical behaviours of graphs based on the degree of the polynomial function

FA 4.2 to be able to convert between tabular and graphical representations of relationships of this type

FA 4.3 to be able to determine values of the function from tables, graphs and equations and to determine argument values from tables, graphs and the equations of quadratic functions

FA 4.4 to be able to understand the relationship between the degree of a polynomial function and the number of (possible) zeros, maxima and minima and points of inflexion

Comments:

The relationship between the degree of a polynomial function and the number of (possible) zeros, maxima and minima and points of inflexion should be known for general *n*, concrete tasks are restricted to polynomial functions with $n \leq 4$.

Using technology, argument values for higher degree polynomial functions can be determined.

Exponential functions [$f(x) = a \cdot b^x$ **or** $f(x) = a \cdot e^{\lambda \cdot x}$ **with** $a, b \in \mathbb{R}^+, \lambda \in \mathbb{R} \setminus \{0\}$**]**

FA 5.1 to be able to recognise and work with exponential relationships in verbal, tabular or graphical form or represented as an equation (formula) and to convert between these forms of representation

FA 5.2 to be able to determine (pairs of) values from tables, graphs and equations of exponential functions and interpret these in context

FA 5.3 to be able to recognise the effect of the parameters *a* and *b* as well as λ and interpret these parameters in various contexts

FA 5.4 to be able to understand important properties ($f(x + 1) = b \cdot f(x)$; $[e^x]' = e^x$) and interpret them in context

FA 5.5 to be able to understand the terms *half-life* and *doubling time*, calculate the corresponding values and interpret these in context

FA 5.6 to be able to evaluate the appropriacy of using an exponential function to describe a situation

Comments:

The parameters *a* and *b* as well as λ should be able to be interpreted for concrete values as well as in general in a given context. This also applies to the effect of the parameters and the effect of varying them.

Sine functions, cosine functions

FA 6.1 to be able to recognise given relationships of the type $f(x) = a \cdot \sin(b \cdot x)$ as a general sine function given a graph or an equation (formula) and work with these types of function; to be able to convert between these forms of representation

FA 6.2 to be able to determine (pairs of) values from tables, graphs and equations of sine functions and interpret these in context

FA 6.3 to be able to recognise the effect of the parameters a and b and interpret these parameters in context

FA 6.4 to be able to understand periodicity as a characteristic property and interpret this in context

FA 6.5 to be able to understand that $\cos(x) = \sin\left(x + \frac{\pi}{2}\right)$

FA 6.6 to be able to understand that $[\sin(x)]' = \cos(x)$ and $[\cos(x)]' = -\sin(x)$ hold

Comments:

While sine, cosine and tangent are used to solve right-angled triangles, the functional perspective is (mainly) restricted to the general sine function. The interpretation of the parameters (in a graph as well as in appropriate contexts), the behaviour of the graph of the function and the periodicity are essential.

Content Area *Calculus (AN)*

Calculus provides concepts for formal, analytical descriptions of discrete and continuous change behaviour, which are of fundamental importance in mathematics and also in many areas of application. The terms difference quotient and differential quotient are general mathematical means of quantitatively describing this change behaviour of variables in various contexts, which is also used in many subject areas to create new terms.

In terms of communication skills, it is therefore important to be able to interpret these mathematical terms in various applications and also to recognise any connections between technical terms on the basis of the mathematical concepts mentioned here (e.g. the relationship between charge and current in physics or, in general, the relationship between stock and flow variables). Some of the terms mentioned here are also used colloquially (e.g. instantaneous velocity, acceleration, decay velocity, progressive growth). In terms of communication with the general public, it is therefore also important for a generally educated person to be able to draw on the mathematical core of these technical terms when explaining them.

With regard to the ability to communicate, the central concept of integral calculus is the definite integral. It is important to know what this concept enables in mathematics and concretely in various application situations. One the one hand, the view of the definite integral as the limit of a sum of products in different contexts results from this; on the other hand, the fact that the typical applications of the definite integral can be generally described and the term itself can be used in various contexts to represent corresponding relationships (e.g. physical work as a path integral of force or the distance covered as a time integral of speed) follow from this too.

The mathematical representation of each term is generally a symbolic one with the symbols having a particular meaning within calculus. A knowledgeable understanding of this formalism is necessary for the accessibility of elementary specialist literature i.e. the various symbolic representations of the differential quotient, the derivative function as well as the definite integral should be recognised as such and they should be able to be interpreted in context and used independently as a means of representation. It is important to know that these symbols can also be calculated with and what in each concrete case is being calculated; the execution of the calculation itself can largely be omitted. It is sufficient to focus on the simplest rules of differentiation, especially as the symbolic representation of the concepts is available alongside the graphical representation of the corresponding functions, with which the relevant properties and relationships can be recognised and also quantitatively estimated.

Core Competencies

Measures of change

AN 1.1 to be able to distinguish between absolute and relative (percentage) measures of change and use them appropriately

AN 1.2 to be able to understand the relationship between the *difference quotient* (average rate of change) and the *differential quotient* ("instantaneous" or local rate of change) on the basis of an intuitive concept of limits and apply these concepts (verbally as well as in formal notation) in context

AN 1.3 to be able to interpret the difference and differential quotients in various contexts and be able to describe corresponding facts using the difference and differential quotients

Comments:

The focus is on the representation of changes through the differences between the values of functions, through percentage changes, through difference quotients and through differential quotients, in particular though also on the interpretation of these measures of change in context.

Using technology, the calculation of difference and differential quotients for arbitrary (differentiable) functions is possible.

Differentiation rules

AN 2.1 to be able to understand and apply simple differentiation rules: the power rule, the $[k \cdot f(x)]'$ and $[f(k \cdot x)]'$ (see also the content area *Functional Relationships*)

Derivative function / antiderivative

AN 3.1 to be able to understand the terms *derivative function* and *antiderivative* and apply them in the description of functions

AN 3.2 to be able to recognise, understand and describe the relationship between a function and its derivative (and a function and its antiderivative) in graphical form

AN 3.3 to be able to describe properties of functions using the derivative (function): montonicity, local maxima and minima, concavity, points of inflexion

Comments:

The term *derivative (function)* should be used sensibly and purposefully to describe functions.

Using technology, differentiating functions that are not included in the differentiation rules in the core competencies is possible.

Summation and integral

AN 4.1 to be able to interpret and describe the concept of the definite integral as the limit of a sum of products

AN 4.2 to be able to understand and apply simple integration rules: power rule, sum rule, $\int k \cdot f(x)\,\mathrm{d}x$, $\int f(k \cdot x)\,\mathrm{d}x$ (see also the content area *Functional Relationships*), and determine definite integrals of polynomial functions

AN 4.3 to be able to interpret the definite integral in various contexts and describe situations using integrals

Comments:

Similarly to differential quotients, the focus of the definite integral is on the description of situations through definite integrals as well as, above all, the interpretation of the definite integral in context.

Using technology, integrating functions that are not included in the differentiation rules in the core competencies is possible.

Content Area *Probability and Statistics (WS)*

Mathematicians as well as users of mathematics often employ the concepts, the forms of representation and the (basic) procedures of descriptive statistics and probability theory. For general laypeople, it is important for their communication skills to be able to use the stochastic terms and representations appropriately in context and assess and evaluate their significance or appropriateness.

The independent creation of statistical tables and diagrams is limited to situations of low complexity (e.g. when communicating with the general public); a similar approach also applies to the determination of statistical parameters (measures of central tendency and dispersion).

In the case of probability, too, the content is limited to the fundamental interpretations of probability, the basic terms (random variable, (random) sample, probability distribution, expectation value and variance/standard deviation) and concepts (the binomial distribution) as well as the simplest probability calculations; on the other hand, it is important to understand probability as a way of modelling and quantifying random chance (from a given level of information).

Core Competencies

Descriptive statistics

WS 1.1 to be able to read values from tables and simple graphical representations (and determine composite values i.e. to be able to use data that can be read from diagrams to calculate further parameters) and interpret them appropriately in context

Comments:

(un)ordered lists, tally charts, pictograms, column graphs, bar charts, line graphs, stem-and-leaf diagrams, scatter plots, histograms (as a special case of a column graph), percentage strips, box plots

WS 1.2 to be able to create tables and simple statistical diagrams and convert between forms of representation

WS 1.3 to be able to interpret statistical parameters (absolute and relative frequencies, mean, median, mode, quartiles, range, empirical variance/standard deviation) in context and determine the parameters listed above for simple data sets

WS 1.4 to be able to state and use the definitions and important properties of the mean and the median, determine and interpret quartiles and justify the choice of a particular parameter for a certain purpose

Comments:

Although statistical parameters (for simple data sets) should be determined and basic statistical diagrams should be created, the focus is on the sensible interpretation of diagrams (taking manipulation into consideration) and parameters. Especially for the mean and the median (as well as quartiles), the most important properties (defining properties, data type compatibility, outlier sensitivity) should be known, properly applied and taken into account. For the mean, any necessary weightings must be observed ("weighted mean") and used (calculation of the mean from the means of subsets).

Probability calculations

WS 2.1 to be able to state the sample space (set of possible outcomes of an experiment) and events for situations verbally as well as formally

WS 2.2 to be able to use and apply the relative frequency as an estimate of probability

WS 2.3 to be able to calculate and interpret probabilities using the Laplace assumption (Laplace probability) and apply and interpret the addition and multiplication rules

Comments:

The multiplication rule should also be able to be determined using combinatorial principles and the application of Laplace's rule should (also) be able to be avoided.

WS 2.4 to be able to calculate and interpret binomial coefficients

Probability Distribution(s)

WS 3.1 to be able to understand and apply the terms *random variable, (probability) distribution, expectation value* and *standard deviation*

WS 3.2 to be able to understand the binomial distribution as a model of a discrete distribution, determine the expectation value as well as the variance/standard deviation of binomially distributed random variables, state the probability distribution of binomially distributed random variables and apply the binomial distribution in appropriate situations

WS 3.3 to be able to recognise and describe situations that can be modelled using the binomial distribution

Contexts

A central purpose of school is to impart fundamental knowledge; in particular, pupils should be empowered and encouraged to acquire knowledge independently and actively. Furthermore, they should be encouraged to critically examine the available knowledge again and again during their years at school.

In this way, pupils learn to define and tackle problems appropriate for their age and to monitor their own success. The “interdisciplinary” competencies developed in various areas of education can and should be included in the standardised written school-leaving examination in mathematics. It is particularly important to note that pupils must be given the opportunity to connect prior experiences and knowledge to relevant contexts.

The list of contexts given below represents a selection of available areas of application of mathematics. This list is intended as an aid to prepare for the standardised written school-leaving examination. In any case, the contexts listed below can arise without detailed explanation in the standardised written school-leaving examination. For all other contexts, the necessary and sufficient explanations for each task will be given in the introductory text.

When applying mathematics in everyday situations, one cannot avoid dealing with proportions, (physical) quantities in general and units in particular. In any case, the correct way of working with (proportions of) amounts and units in unavoidable in communication situations and demonstrates a deeper understanding of connections.

1 percent = 10^{-2} = 10 000 ppm = parts per hundred = 1 %
1 tenth of a percent = 10^{-3} = 1 000 ppm = parts per thousand = 0.1 % = 1 ‰
1 ppm (parts per million) = 10^{-6} = parts per million = 0.0001 %

Prefixes

Prefix	Meaning	Symbol	
tera-	trillion	T	10^{12} = 1 000 000 000 000
giga-	billion	G	10^{9} = 1 000 000 000
mega-	million	M	10^{6} = 1 000 000
kilo-	thousand	k	10^{3} = 1 000
hecto-	hundred	h	10^{2} = 100
deca-	ten	da	10^{1} = 10
deci-	tenth	d	10^{-1} = 0.1
centi-	hundredth	c	10^{-2} = 0.01
milli-	thousandth	m	10^{-3} = 0.001
micro-	millionth	µ	10^{-6} = 0.000 001
nano-	billionth	n	10^{-9} = 0.000 000 001
pico-	trillionth	p	10^{-12} = 0.000 000 000 001

Quantities and their units

Quantity	Unit	Symbol	Relationship
temperature	degrees Celsius or Kelvin	°C, K	$\Delta t = \Delta T$
frequency	hertz	Hz	$1\ \text{Hz} = 1\ \text{s}^{-1}$
energy, work, amount of heat	joule	J	$1\ \text{J} = 1\ \text{kg} \cdot \text{m}^2 \cdot \text{s}^{-2}$
force	newton	N	$1\ \text{N} = 1\ \text{kg} \cdot \text{m} \cdot \text{s}^{-2}$
torque	newton metre	N · m	$1\ \text{N} \cdot \text{m} = 1\ \text{kg} \cdot \text{m}^2 \cdot \text{s}^{-2}$
electrical resistance	ohm	Ω	$1\ \Omega = 1\ \text{V} \cdot \text{A}^{-1} = 1\ \text{kg} \cdot \text{m}^2 \cdot \text{A}^{-2} \cdot \text{s}^{-3}$
pressure	pascal	Pa	$1\ \text{Pa} = 1\ \text{N} \cdot \text{m}^{-2} = 1\ \text{kg} \cdot \text{m}^{-1} \cdot \text{s}^{-2}$
electrical current	ampere	A	$1\ \text{A} = 1\ \text{C} \cdot \text{s}^{-1}$
voltage	volt	V	$1\ \text{V} = 1\ \text{J} \cdot \text{C}^{-1} =$ $1\ \text{kg} \cdot \text{m}^2 \cdot \text{A}^{-1} \cdot \text{s}^{-3}$
power	watt	W	$1\ \text{W} = 1\ \text{J} \cdot \text{s}^{-1} = 1\ \text{kg} \cdot \text{m}^2 \cdot \text{s}^{-3}$

Technical-scientific basics

density	$\varrho = \frac{m}{V}$		
power	$P = \frac{\Delta E}{\Delta t} = \frac{\Delta W}{\Delta t}$	$P = \frac{dW}{dt}$	
force	$F = m \cdot a$		
work	$W = F \cdot s$		
	$W = \int F(s)\,ds$	$F = \frac{dW}{ds}$	
kinetic energy	$E_{kin} = \frac{1}{2} \cdot m \cdot v^2$		
potential energy	$E_{pot} = m \cdot g \cdot h$		
uniform linear motion	$v = \frac{s}{t}$	$v = \frac{ds}{dt}$	$v(t) = s'(t)$
uniformly accelerated linear motion	$v = a \cdot t + v_0$	$a = \frac{dv}{dt} = \frac{d^2s}{dt^2}$	$a(t) = v'(t) = s''(t)$

Financial mathematics

The curriculum for mathematics does not only draw on scientific and technical contexts but also thematises economic affairs. Therefore, basic concepts in this field are also necessary.

Compound interest calculations

K_0 ... initial investment

K_n ... final capital after n years

i ... annual percentage rate of interest

$K_n = K_0 \cdot (1 + i)^n$

Cost-price theory

x ... amount produced, offered, required or sold ($x \geq 0$)

cost function K	$K(x)$
fixed costs F	$K(0)$
variable cost function K_v	$K_v(x) = K(x) - F$
marginal cost function K'	$K'(x)$
price function p	$p(x)$
revenue function E	$E(x) = p(x) \cdot x$
marginal revenue function E'	$E'(x)$
profit function G	$G(x) = E(x) - K(x)$
marginal profit function G'	$G'(x)$
break-even point	$E(x) = K(x)$ ... at the (first) zero x of the profit function