Name:	
Class:	

Standardised Competence-Oriented
Written School-Leaving Examination

AHS

18th September 2024

Mathematics

Bundesministerium
Bildung, Wissenschaft
und Forschung

Advice for Completing the Tasks

Dear candidate,

The following booklet contains Part 1 and Part 2 tasks (divided into sub-tasks). The tasks can be completed independently of one another. You have a total of *270 minutes* available in which to work through this booklet.

Please do all of your working out solely in this booklet and on the paper provided to you. Write your name and that of your class on the cover page of the booklet in the spaces provided. Please also write your name on any separate sheets of paper used and number these pages consecutively. When responding to the instructions of each task, write the task reference (e. g. 25a1) on your sheet.

In the assessment of your work, everything that is not crossed out will be considered.

The use of the official formula booklet for this examination that has been approved by the relevant government authority is permitted. Furthermore, the use of electronic device(s) (e. g. graphic display calculators or other appropriate technology) is allowed provided there is no possibility of communicating via the internet, Bluetooth, mobile networks etc. and there is no access to your own data stored on the device.

An explanation of the task types is displayed in the examination room.

Instructions for Completing the Tasks

– Solutions must be unambiguous and clearly recognisable.
– Solutions must be given alongside their corresponding units if this has been explicitly required in the task instructions.

For tasks with open answer formats, evidence of the targeted core competency is required for the award of the point. When completing tasks with open answer formats, it is recommended that you:

– document how the solution was reached, even if electronic devices were used,
– explain any variables you have chosen yourself and give their corresponding units,
– avoid rounding prematurely,
– label diagrams or sketches.

Changing an answer for a task that requires a cross:

1. Fill in the box that contains the cross.
2. Put a cross in the box next to your new answer.

In this instance, the answer “5 + 5 = 9” was originally chosen. The answer was later changed to be “2 + 2 = 4”.

1 + 1 = 3	☐
2 + 2 = 4	☒
3 + 3 = 5	☐
4 + 4 = 4	☐
5 + 5 = 9	■
6 + 6 = 10	☐

Selecting an item that has been filled in:

1. Fill in the box that contains the cross for the answer you do not wish to give.
2. Put a circle around the filled-in box you would like to select.

In this instance, the answer “2 + 2 = 4” was filled in and then selected again.

1 + 1 = 3	☐
2 + 2 = 4	◉
3 + 3 = 5	☐
4 + 4 = 4	■
5 + 5 = 9	☐
6 + 6 = 10	☐

Grading System

points awarded	grade
32 – 36 points	very good
27 – 31.5 points	good
22 – 26.5 points	satisfactory
17 – 21.5 points	pass
0 – 16.5 points	fail

Best-of Assessment: A best-of assessment approach will be applied to tasks 26, 27 and 28. Of these three Part 2 tasks, the task with the lowest point score will not be included in the total point score.

Good luck!

Task 1

Knowledge about Sets of Numbers

Let a and b be two natural numbers with $b > a$.

Task:

Complete the following sentence by putting a cross next to the correct option for each gap so that the sentence becomes a true statement.

$a - b$ is definitely _____①_____ and $b - a$ is definitely _____②_____.

①	
a natural number	☐
a rational number but not a natural number	☐
a rational number but not an integer	☐

②	
a natural number	☐
an integer but not a natural number	☐
a rational number but not a natural number	☐

[0 / ½ / 1 p.]

Task 2

Volume of a Sphere

A particular sphere has radius r and volume V, where $r, V \in \mathbb{R}^+$.
Another sphere has radius $2 \cdot r$ and volume $k \cdot V$.

Task:

Determine k.

$k =$ _____

[0/1 p.]

Task 3

Language Trip

A language trip is attended by U lower-secondary school pupils, O upper-secondary school pupils and B chaperones. The total number of pupils (lower- and upper-secondary) is at least as great as 5 times the number of chaperones.

Task:

Write down an inequality that describes the relationship between U, O and B given above.

[0 / 1 p.]

Task 4

Vektors in $\mathbb{R}^3$

Let be $\vec{a} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$, $\vec{b} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$ and $\vec{n} = \begin{pmatrix} n_1 \\ n_2 \\ n_3 \end{pmatrix}$ be three non-zero vectors in $\mathbb{R}^3$.

The following statements hold:

The vector $\vec{n}$ is perpendicular to both vector $\vec{a}$ and vector $\vec{b}$.
The vectors $\vec{a}$ and $\vec{b}$ are not perpendicular to each other.
The vectors $\vec{a}$ and $\vec{b}$ are not parallel to each other.

Task:

Put a cross next to each of the two statements that are definitely true. *[2 out of 5]*

$\vec{a} \cdot \vec{b} = \vec{b} \cdot \vec{n}$	☐
$(\vec{a} + \vec{b}) \cdot \vec{n} = 0$	☐
$a_1 \cdot n_1 + a_2 \cdot n_2 + a_3 \cdot n_3 = 0$	☐
There exists a number $k \in \mathbb{R}$ such that $\vec{a} + \vec{b} = k \cdot \vec{n}$ holds	☐
There exists a number $k \in \mathbb{R}$ such that $\vec{a} = k \cdot \vec{b}$ holds	☐

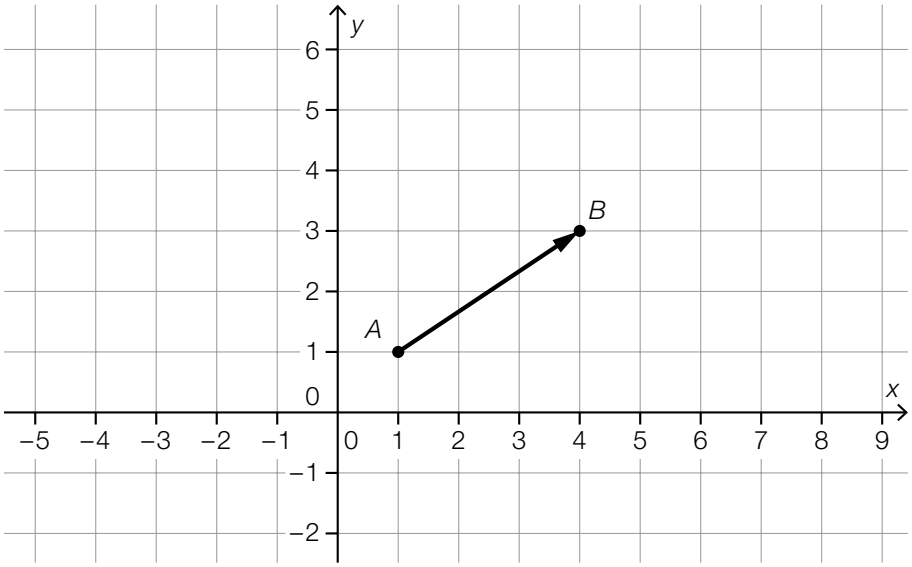
[0/1 p.]

Task 5

Directions

A path goes from point *A* via point *B* to point *C*. At point *B*, the path branches off to the right at a right angle. The path is straight between points *B* and *C*.

The path *AB* is modelled in the coordinate system below.



Task:

Write down a vector $\vec{a}$ that gives the direction of the path *BC*.

$\vec{a} = \begin{pmatrix} \square \\ \square \end{pmatrix}$

[0 / 1 p.]

Task 6

Sine and Cosine

For a particular angle $\alpha \in [0°, 360°)$, the relationship $\sin(\alpha) \geq \cos(\alpha)$ holds.

Task:

Write down the largest possible interval for α such that this relationship holds.

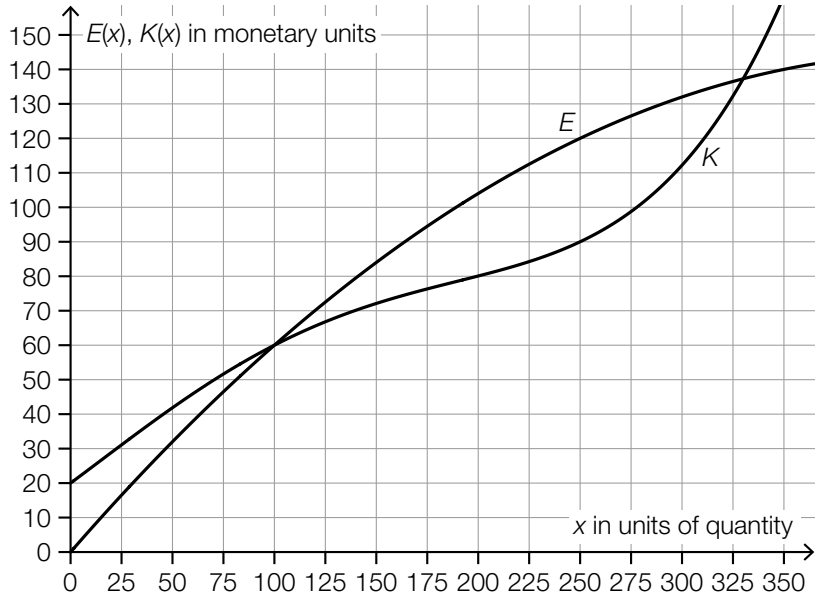
$\alpha \in [$ _____ °, _____ °$]$

[0 / 1 p.]

Task 7

Profit

The diagram below shows the graph of the cost function $K: x \mapsto K(x)$ and the graph of the revenue function $E: x \mapsto E(x)$ for a particular product (x in units of quantity, $E(x)$, $K(x)$ in monetary units).



It can be assumed for this product that all of the units of quantity produced are also sold.

A positive profit for this product is first reached at more than x_1 produced and sold units of quantity.

The profit for this product reaches a maximum at x_2 produced and sold units of quantity.

Task:

Determine x_1 and x_2 using the diagram above.

x_1 = _____ units of quantity

x_2 = _____ units of quantity

[0 / ½ / 1 p.]

Task 8

Parameters of a Linear Function

Let $f\colon \mathbb{R} \to \mathbb{R}$ be a linear function with $f(x) = k \cdot x + d$ with $k, d \in \mathbb{R}$. The graph of this linear function goes through the points $A = (a, a)$ and $B = (3 \cdot a, 2 \cdot a)$, where $a \in \mathbb{R}\backslash\{0\}$.

Task:

Complete the following sentence by putting a cross next to the correct option for each gap so that the sentence becomes a true statement.

_____① _____ holds for the parameter k, and _____② _____ holds for the parameter d.

①	
$k = \frac{a}{2}$	☐
$k = \frac{1}{2}$	☐
$k = 2 \cdot a$	☐

②	
$d = \frac{a}{2}$	☐
$d = \frac{1}{2}$	☐
$d = 2 \cdot a$	☐

[0 / 1 p.]

Task 9

Amount of Water in a Swimming Pool

The linear function V: $[0, 10] \to \mathbb{R}_0^+$ models the amount of water in a swimming pool in terms of the time t (t in min, $V(t)$ in l).

The following statement holds for all $t \in [0, 9]$:
$V(t + 1) - V(t) = -10$

Task:

Interpret the equation above in the given context; write down the corresponding unit.

[0/1 p.]

Task 10

Oxygen

The function S assigns the temperature T of water to its maximum absorption capacity $S(T)$ of pure oxygen (T in °C, $S(T)$ in mg/l). A table of values for S is shown below.

temperature T (in °C)	maximum absorption capacity $S(T)$ (in mg/l)
0	14.6
20	9.1

It can be assumed that S is an exponential function.

Task:

Determine the temperature T_1 at which $S(T_1)$ is only half the value of $S(0)$.

[0 / 1 p.]

Task 11

Gamma Radiation

In an experiment with a radioactive compound, the intensity of the gamma radiation after penetration of three lead plates of various thicknesses (2 cm, 5 cm and 7 cm) is measured. The results are shown in the table below.

plate thickness (in cm)	2	5	7
intensity (in %)	38.94	9.46	3.69

Julian claims: “The data suggest that the intensity decreases approximately exponentially with increasing plate thickness.”

Task:

Show by calculation that Julian’s claim is correct.

[0 / 1 p.]

Task 12

Period Length

Let $f\colon \mathbb{R} \to \mathbb{R}$ with $f(x) = \sin(a \cdot x)$ and $g\colon \mathbb{R} \to \mathbb{R}$ with $g(x) = \sin\left(\frac{1}{a} \cdot x\right)$ be functions.
For these functions, $a \in \mathbb{R}$ and $a > 1$.
The (smallest) period length of f is given by p_f; the (smallest) period length of g is given by p_g.

Task:

Complete the following sentence by putting a cross next to the correct option for each gap so that the sentence becomes a true statement.

_____ ① _____ holds for p_g; _____ ② _____ holds for $\frac{p_f}{p_g}$.

①	
$p_g = 2 \cdot \pi$	☐
$p_g = \frac{2 \cdot \pi}{a}$	☐
$p_g = 2 \cdot \pi \cdot a$	☐

②	
$\frac{p_f}{p_g} = a^2$	☐
$\frac{p_f}{p_g} = \frac{1}{a^2}$	☐
$\frac{p_f}{p_g} = 2 \cdot \pi \cdot a^2$	☐

[0/1 p.]

Task 13

Price Difference

A particular product costs x euros in a local shop; from an online retailer it costs y euros.
The following statement holds: $x > y > 0$

The relative proportion by which the product is more expensive in the local shop than from the online retailer is given by h.

Task:

Write down a formula in terms of x and y that could be used to calculate h.

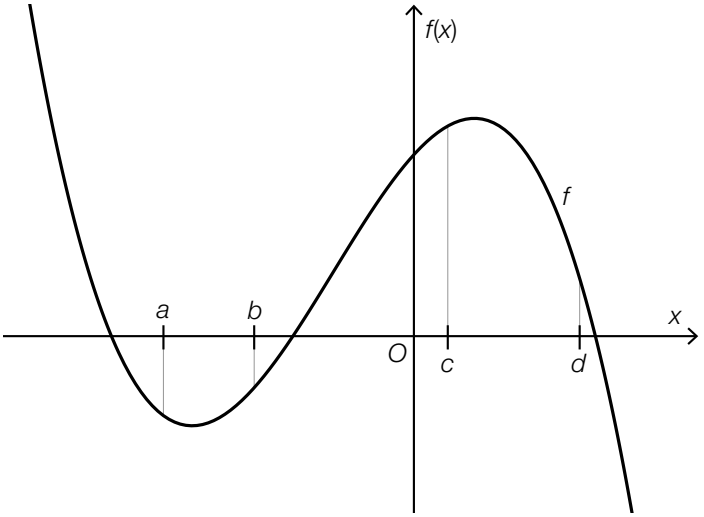
$h =$ _____

[0 / 1 p.]

Task 14

Difference Quotient and Differential Quotient

The diagram below shows the graph of a 3rd degree polynomial function f.



Task:

Put a cross next to each of the two statements that are true for the function f. *[2 out of 5]*

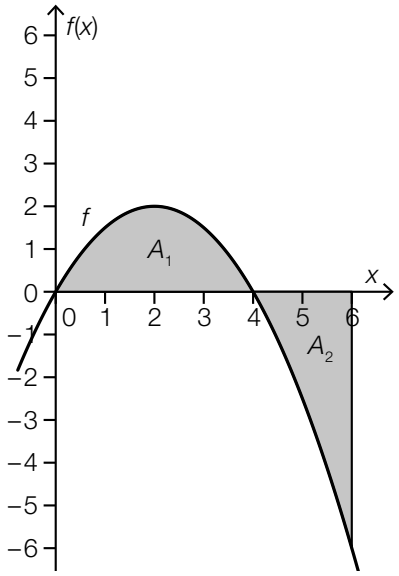
$\frac{f(b)-f(a)}{b-a} > 0$	☐
$\frac{f(d)-f(c)}{d-c} > f'(c)$	☐
$f'(b) < 0$	☐
$\frac{f(c)-f(b)}{c-b} < 0$	☐
$f'(d) < f'(c)$	☐

[0 / 1 p.]

Task 15

Values of an Antiderivative

The diagram below shows the graph of the polynomial function f.



A_1 ... area of the region between the graph of f and the x-axis in the interval [0, 4]
A_2 ... area of the region between the graph of f and the x-axis in the interval [4, 6]

The following statement holds: $A_1 = A_2 = \frac{16}{3}$

For an antiderivative F of f, $F(0) = 0$ holds.

Task:

Write down the values of $F(4)$ and $F(6)$.

$F(4)$ = _____

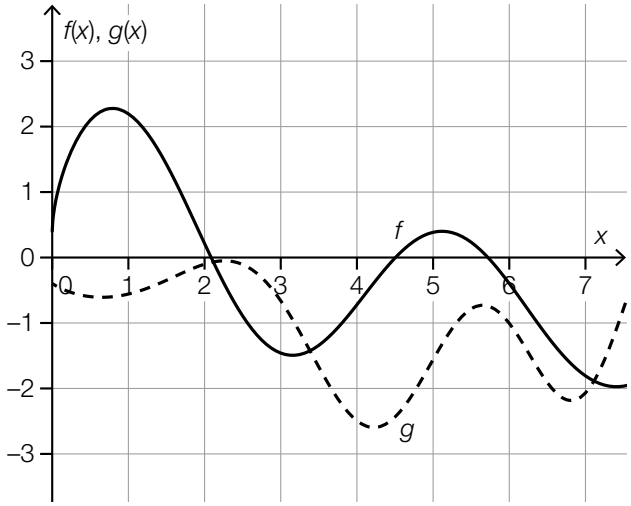
$F(6)$ = _____

[0 / ½ / 1 p.]

Task 16

Derivatives of two Functions

The graphs of the two twice differentiable real functions f and g are shown below.



Task:

Put a cross next to each of the two correct statements. *[2 out of 5]*

$g'(1) > 1$	☐
$f'(3) > g'(3)$	☐
$f'(5) > g'(5)$	☐
$f''(1) > g''(1)$	☐
$f''(3) > g''(3)$	☐

[0/1 p.]

Task 17

Definite Integral

Let f be a linear function with $f(x) = -0.5 \cdot x + a$ with $a > 0$.

$\int_0^{x_1} f(x)\,dx = 0$ with $x_1 > 0$ holds.

Task:

Write down the missing number in the equation below in the box provided.

$x_1 = \square \cdot a$

[0/1 p.]

Task 18

Geometric Interpretation of the Sum Rule

Let f and g be polynomial functions with $g(x) = f(x) + 2$ with $x \in [a, b]$.

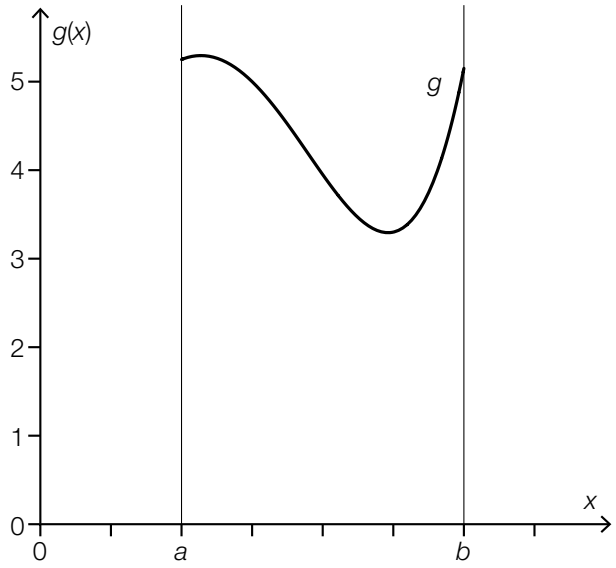
The region between the graph of g and the x-axis in the interval $[a, b]$ can be divided into a subregion with area A and a subregion with area B.

The following statements hold:

$A = \int_a^b f(x)\,dx$ and $B = \int_a^b 2\,dx$

Task:

Draw the subregion with area A on the diagram below.



[0 / 1 p.]

Task 19

Email Addresses

As part of a survey, teenagers and adults were asked how many email addresses they use. The responses are summarised in the table below.

	at most 2 email addresses	more than 2 email addresses
teenagers	205	295
adults	935	565

The relative proportion of all people surveyed (teenagers and adults) who use more than 2 email addresses is given by p.

The relative proportion of the teenagers surveyed who use more than 2 email addresses is given by q.

Task:

Determine p and q.

$p =$ _____

$q =$ _____

[0/½/1 p.]

Task 20

Statistical Parameters

The list of data $x_1, x_2, \ldots, x_{10}$ comprises 10 different numbers and is in ascending order.

Task:

Put a cross next to each of the two correct statements. *[2 out of 5]*

The number x_7 is the 3rd quartile q_3 of the list of data.	☐
The number x_3 is the 1st quartile q_1 of the list of data.	☐
The sum of the numbers $x_1, \ldots, x_{10}$ is 10-times as big as the mean of the list of data.	☐
The mean of the list of data is definitely less than x_9.	☐
The number x_5 is the median of the list of data.	☐

[0 / 1 p.]

Task 21

Football Team

The heights of the 11 players of a school football team are measured.
The collected data are listed in order of size.

- The shortest player is 1.40 m tall.
- Exactly 2 players are 1.45 m tall.
- The rest of the players are taller than 1.70 m.
- The tallest player is 1.80 m tall.

Task:

Put a cross next to each of the two correct statements. *[2 out of 5]*

The range of the collected data is 0.5 m.	☐
The median of the collected data is greater than 1.70 m.	☐
The mean of the collected data is greater than 1.75 m.	☐
More than 60 % of the players are taller than 1.70 m.	☐
Fewer than 20 % of the players are shorter than 1.50 m.	☐

[0 / 1 p.]

Task 22

Balls in a Jar

There are 12 red and 15 white balls in a jar. 3 balls are removed from this jar at random without replacement.

Task:

Put a cross next to the event E whose probability can be calculated with: $P(E) = 1 - \frac{15}{27} \cdot \frac{14}{26} \cdot \frac{13}{25}$.
[1 out of 6]

At most 1 white ball is removed.	☐
At least 1 red ball is removed.	☐
At least 1 white ball is removed.	☐
No white balls are removed.	☐
At most 1 red ball is removed.	☐
At least 2 white balls are removed.	☐

[0/1 p.]

Task 23

Expected Values and Standard Deviations

The tables below show the respective probability distributions for the random variables X and Y with $a \in \mathbb{R}$.

random variable X:

k	$a - 2$	a	$a + 2$
$P(X = k)$	0.1	0.8	0.1

random variable Y:

k	a	$a + 2$	$a + 4$
$P(Y = k)$	0.4	0.2	0.4

Task:

Complete the sentence below by putting a cross next to one of the options for each gap so that the sentence becomes a correct statement.

_____① _____ holds for the expected values, and _____② _____ holds for the standard deviations.

①	
$E(X) < E(Y)$	☐
$E(X) = E(Y)$	☐
$E(X) > E(Y)$	☐

②	
$\sigma(X) < \sigma(Y)$	☐
$\sigma(X) = \sigma(Y)$	☐
$\sigma(X) > \sigma(Y)$	☐

[0 / ½ / 1 p.]

Task 24

Experiment

An experiment is made up of n independent trials ($n \in \mathbb{N}$, $n \geq 2$). Each trial occurs under the same conditions and can only result in the two possible outcomes A and B with probabilities $P(A) = a$ and $P(B) = b$.

The probability P_1 corresponds to the event that outcome A occurs at most 2 times.

Task:

Write down an expression that could be used to calculate P_1.

$P_1 =$ _____

[0 / 1 p.]

Task 25 (Part 2)

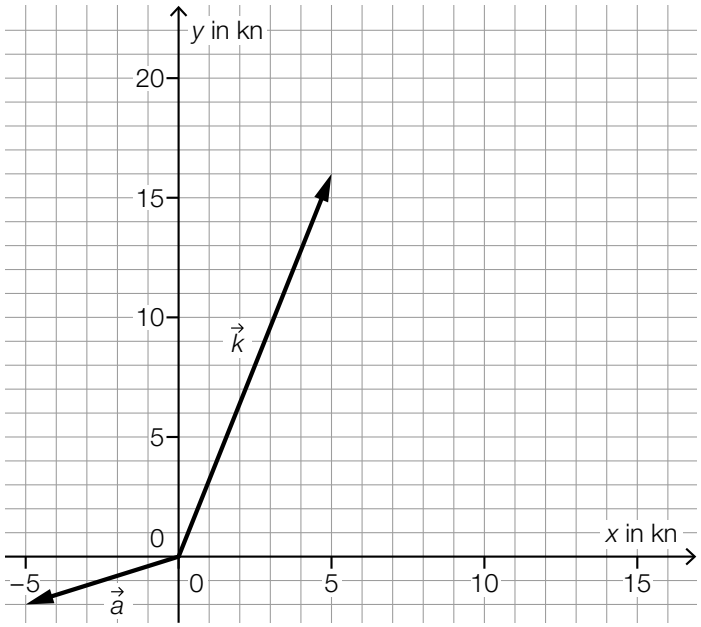
Container Ships

Container ships make low-cost transport of large quantities of a wide variety of goods at sea possible.

In shipping, distances are given in nautical miles (NM), and speeds are given in knots (kn).

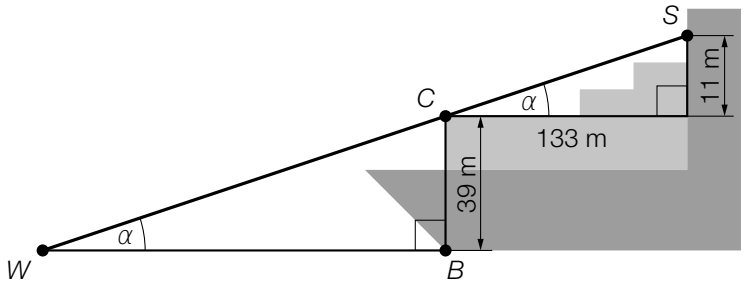
Task:

a) The course of a container ship is modelled by the velocity vectors $\vec{k}$, $\vec{a}$ and $\vec{s}$.
The target course $\vec{k}$ is the result of the intended course $\vec{s}$ and the so-called drift $\vec{a}$.
$\vec{k} = \vec{s} + \vec{a}$ holds.
For a particular point in time, the velocity vector $\vec{k}$ and $\vec{a}$ are shown in the diagram below.



1) Draw the vector $\vec{s}$ as an arrow starting from the origin in the diagram above. *[0/1 p.]*

b) A model of a particular container ship as seen from the side is shown in the diagram below. The view from point *S* to the surface of the water is obstructed by containers.



1) Determine the length of the line segment *BW*. *[0 / 1 p.]*

c) For speeds from 10 kn to 30 kn, the fuel consumption of a particular container ship can be modelled in terms of the speed by the function *f*.

$f(v) = a \cdot v^3 + b$

v ... speed in kn
f(*v*) ... fuel consumption at speed *v* in kg/NM
a, *b* ... real parameters

The following statements hold:

- At a speed of 10 kn, the fuel consumption is 90 kg/NM.
- At a speed of 25 kn, the fuel consumption is 260 kg/NM.

1) Determine the coefficients *a* and *b*. *[0 / 1 p.]*

d) For every journey of a particular container ship, both filled as well as empty containers are transported.

The corresponding number of filled and empty containers for 10 particular journeys is shown in the table below.

journey	1	2	3	4	5	6	7	8	9	10
filled containers	6 800	7 100	7 600	6 900	7 000	6 800	6 600	7 800	8 000	*c*
empty containers	1 200	1 000	500	1 200	1 500	1 300	1 100	300	200	*d*

The mean of the number of filled containers for all of these 10 journeys is 7 200.

The median of the number of empty containers for these 10 journeys is equal to the median of the number of empty containers for the first 9 of these 10 journeys.

1) Determine *c* and *d*.

c = _____

d = _____ *[0 / ½ / 1 p.]*

Task 26 (Part 2, Best-of Assessment)

Slackline

Slacklining is a sport in which people balance on a strap.

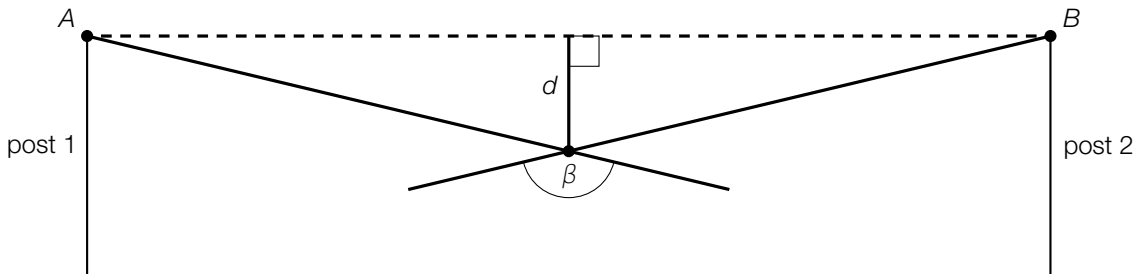
Two vertical posts suitable for this type of sport are positioned 12 m apart from each other. The strap is attached to two anchor points, *A* and *B*, that are at the same height on these posts (see diagram below).

Task:

a) Slacklining involves an elastic strap that is secured horizontally above the ground.

Theo is standing exactly in the middle of the strap so that a particular sag *d* occurs.

The situation is represented in the diagram below.



The following statements hold for the resulting force *F* (in N) at an anchor point:

$F = \frac{10 \cdot e \cdot m}{4 \cdot d}$
e ... horizontal distance between the anchor points *A* and *B* (in m)
m ... body mass (in kg)
d ... sag (in m)

The function $d: \mathbb{R}^+ \to \mathbb{R}^+, m \mapsto d(m)$ where *F* and *e* are held constant as well as the function $F: \mathbb{R}^+ \to \mathbb{R}^+, d \mapsto F(d)$ where *m* and *e* are held constant each describe a particular relationship.

1) Complete the following sentence by putting a cross next to the correct option for each gap so that the sentence becomes a true statement. *[0/½/1 p.]*

The function d describes _____①_____ relationship; the function F describes a _____②_____ relationship.

①	
a directly proportional	☐
an indirectly proportional	☐
a non-proportional	☐

②	
a directly proportional	☐
an indirectly proportional	☐
a non-proportional	☐

Theo has a body mass m of $m = 80$ kg.

2) For $\beta = 160°$, determine the size of the resulting force F (in N) at an anchor point. *[0/1 p.]*

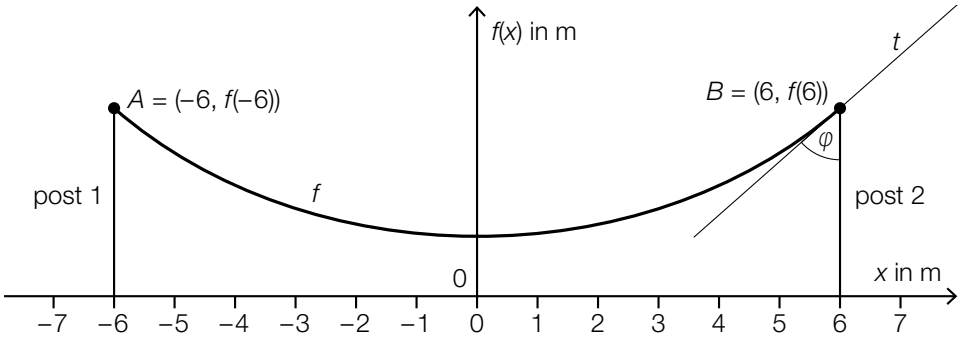
Theo tries many times to walk across the strap. He manages to cross the strap each time with probability p.
It can be assumed that the attempts are independent of each other and the probability of success p is the same for each attempt.

The probability that Theo successfully crosses the strap at least 2 times in 10 attempts is 99 %.

3) Determine p. *[0/1 p.]*

b) A variant of slacklining is *rodeolining* in which the strap is not under tension.
The function $f\colon [-6, 6] \to \mathbb{R}^+,\ x \mapsto f(x)$ models the height of the strap above the ground at point x (x in m, $f(x)$ in m).

The graph of f is shown in the diagram below.



Post 2 makes an angle φ with the tangent t of the graph of f at anchor point B.

1) Write down an equation that describes the relationship between $f'(6)$ and φ. *[0 / 1 p.]*

Task 27 (Part 2, Best-of Assessment)

Diving in Lake Grundl

Mia and Laurin are on holiday at Lake Grundl so that they can dive in the lake.

Task:

a) The total pressure exerted on a diver is the sum of the air pressure at the water's surface and the pressure of the water.

Mia dives. The resulting total pressure in terms of the diving depth d is modelled by the linear function p.

d ... diving depth in m
$p(d)$ ... total pressure at the diving depth d in millibars (mbar)

The following statements hold:

- The total resulting pressure on Mia increases by 98 mbar per metre.
- The air pressure at the water's surface at Lake Grundl is 930 mbar.

1) Write down an equation of the function p.

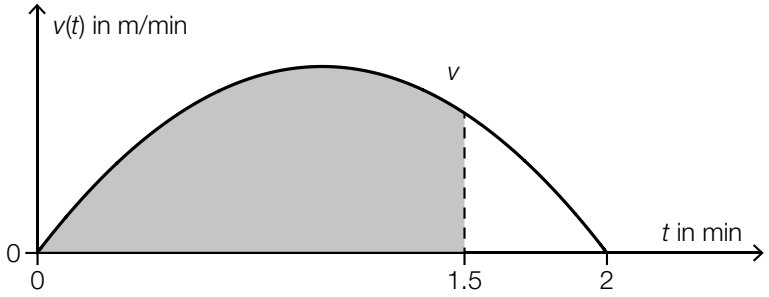
$p(d) =$ _____ *[0 / 1 p.]*

b) Laurin dives diagonally downwards. His vertical velocity whilst diving can be modelled by the quadratic function v: $[0, 2] \to \mathbb{R}$.

$v(t) = c \cdot t \cdot (t - 2)$ with $c \in \mathbb{R}\backslash\{0\}$ holds.

t ... time from the beginning of the dive in min
$v(t)$ ... vertical velocity at time t in m/min

The graph of v is shown in the diagram below.



1) Interpret the area of the region shaded grey in the diagram above in the given context. *[0/1 p.]*

2 min after the start of the dive, Laurin reaches a diving depth of 16 m.

2) Determine c. *[0/1 p.]*

3) Determine the time interval for which Laurin's vertical velocity is at least 9 m/min. *[0/1 p.]*

Task 28 (Part 2, Best-of Assessment)

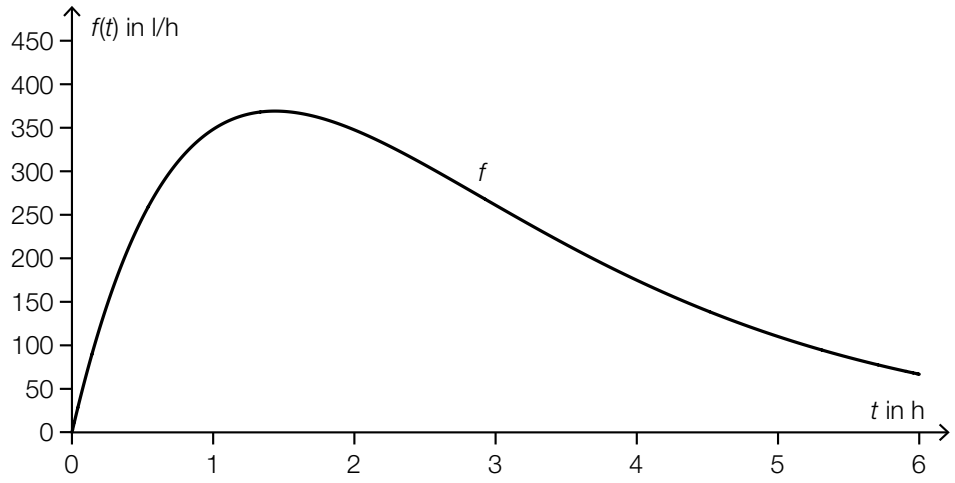
Rooftop

Task:

a) A rainwater butt catches the rainwater that flows from a rooftop. At the start of a rain shower that lasts many hours, the instantaneous rate of change of the amount of water in the rainwater butt can be modelled by the function f: $[0, 6] \to \mathbb{R}_0^+$, $t \mapsto f(t)$ (t since the start of the rain shower in h, $f(t)$ in l/h). At the start of the rain shower, there are already 400 l of water in the rainwater butt.

1) Interpret $400 + \int_0^6 f(t)\,dt$ in the given context. Write down the corresponding unit. *[0 / 1 p.]*

The graph of the function f is shown in the diagram below. The function f has a local maximum at $t_1 = 1.4$ and a point of inflexion at $t_2 = 2.9$.

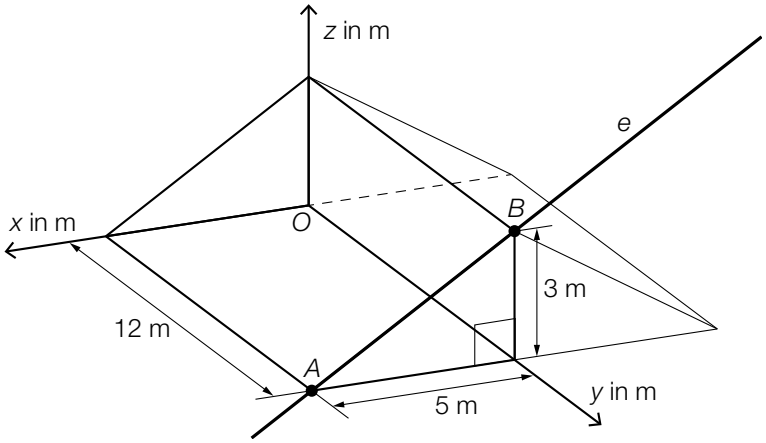


The function F: $[0, 6] \to \mathbb{R}_0^+$, $t \mapsto F(t)$ with $F(0) = 400$ is an antiderivative of f (t in h, $F(t)$ in l).

2) Put a cross next to each of the two correct statements. *[2 out of 5]* *[0 / 1 p.]*

F is strictly monotonically increasing.	☐
F has the value 370 at $t_1 = 1.4$.	☐
The graph of F changes concavity at $t_1 = 1.4$.	☐
The graph of F has a horizontal tangent at $t_1 = 1.4$.	☐
F has a local maximum at $t_2 = 2.9$.	☐

b) A rooftop is modelled in a Cartesian coordinate system in the diagram below. The roof is bordered by roof edges.



The roof edge AB lies on a straight line e.
The point A lies in the xy-plane; the point B lies in the yz-plane.

1) Write down a vector equation of e. *[0/1 p.]*

Two other roof edges lie on the lines g and h. The following statements hold:

$g: X = \begin{pmatrix}5\\0\\0\end{pmatrix} + s \cdot \begin{pmatrix}-5\\0\\3\end{pmatrix}$ and $h: X = \begin{pmatrix}-5\\12\\0\end{pmatrix} + t \cdot \begin{pmatrix}0\\1\\0\end{pmatrix}$ with $s, t \in \mathbb{R}$

2) Complete the following sentence by putting a cross next to the correct option for each gap so that the sentence becomes a true statement. *[0/½/1 p.]*

The two lines _____①_____ and _____②_____.

①	
are parallel to each other	☐
intersect	☐
are skew	☐

②	
g is parallel to the x-axis	☐
h is parallel to the y-axis	☐
h goes through the point (0, 0, 3)	☐