Exemplar für Prüfer/innen

Supplementary Examination for the
Standardised Competence-Oriented
Written School-Leaving Examination

AHS

June 2016

Mathematics

Supplementary Examination 9
Examiner's Version

BM|BF Bundesministerium für Bildung und Frauen

Bundesinstitut bifie

Instructions for the supplementary examination

The following supplementary examination is comprised of five tasks that can be completed independently of one another.

Each task contains two parts: The statement of the task requires the candidate to demonstrate core competencies, and the guiding question that follows it requires the candidate to show their ability to communicate their ideas.

In the following document, the examiner will find the tasks as well as the expected solutions and the answer key.

The preparation time is to be at least 30 minutes and the examination time is to be at most 25 minutes.

Assessment

Each task can be awarded zero, one or two points. There is one point available for each demonstration of core competencies as well as for each guiding question. A maximum of 10 points can be achieved.
For the grading of the examination the following scale should be used:

Grade	Minimum number of points
Pass	4 points for the core competencies + 0 points for the guiding questions 3 points for the core competencies + 1 point for the guiding questions
Satisfactory	5 points for the core competencies + 0 points for the guiding questions 4 points for the core competencies + 1 point for the guiding questions 3 points for the core competencies + 2 points for the guiding questions
Good	5 points for the core competencies + 1 point for the guiding questions 4 points for the core competencies + 2 points for the guiding questions 3 points for the core competencies + 3 points for the guiding questions
Very good	5 points for the core competencies + 2 points for the guiding questions 4 points for the core competencies + 3 points for the guiding questions

The examination board will decide on the final grade based on the candidate’s performance in the supplementary examination as well as the result of the written examination.

Evaluation grid for the supplementary examination

This evaluation grid may be used to assist the examiner in assessing the candidate's performance.

	Point for core competencies reached	Point for the guiding question reached
Task 1		
Task 2		
Task 3		
Task 4		
Task 5		

Task 1

Lines in $\mathbb{R}^3$

Let g and h be two lines in $\mathbb{R}^3$.

The line g goes through the point $P = (3,1,5)$ and is parallel to the y-axis.

Task:

Determine a vector equation of the line g.

Explain why it is not possible to find a point Q, where $Q = (1,y_Q,z_Q)$, such that the point Q lies on the line g.

Guiding question:

Describe all possible relative positions of two lines in $\mathbb{R}^3$.

The line h is described by the vector equation $X = \begin{pmatrix} x_h \\ 1 \\ 3 \end{pmatrix} + s \cdot \begin{pmatrix} 2 \\ y_h \\ 1 \end{pmatrix}$ where $s, x_h, y_h \in \mathbb{R}$.

Is it possible to determine values for x_h and y_h such that the two lines g and h are perpendicular to one another and meet at point P?
If not, justify by means of calculation why this is not possible.
If so, determine the relevant values of x_h and y_h.

Solution of Task 1

Lines in $\mathbb{R}^3$

Expected solution of the statement of the task:

Possible vector equation:

$$g\colon X = \begin{pmatrix} 3 \\ 1 \\ 5 \end{pmatrix} + t \cdot \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \text{ with } t \in \mathbb{R}$$

Possible justification:
For all points on the line g, $x = 3$; $y = 1 + t$, and $z = 5$ (with $t \in \mathbb{R}$) hold.
⇒ Therefore, a point with $x = 1$ cannot lie on the line g.

Answer key:

The point for the core competencies is to be awarded if a correct vector equation of the line g has been given and a correct justification has been provided.

Expected solution of the guiding question:

Two lines in $\mathbb{R}^3$ can be identical, parallel, intersecting or skew.

It is possible to determine values for x_h and y_h such that the two conditions are fulfilled:

$x_h = -1$ (from the value of the parameter $s = 2$)

$y_h = 0$ (from $\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \cdot \begin{pmatrix} 2 \\ 0 \\ 1 \end{pmatrix} = 0$)

Answer key:

The point for the guiding question is to be awarded if all the different possible relative positions have been given and the values of x_h and y_h have been given correctly.

Task 2

Quadratic Function

Let f be a function with $f(x) = r \cdot x^2 + s$ where $r, s \in \mathbb{R}, r \neq 0$.

Task:

Explain the effects that changes of the values of the parameters r and s have on the behaviour of the graph of f.

Guiding question:

The graph of the function f goes through the two points $B = (a,b)$ and $E = (0,e)$, where $a \neq 0$.

Determine expressions for r and s in terms of the coordinate values a, b, and e.

State the value of b for which the function f is not a quadratic function.

Solution of Task 2

Quadratic Function

Expected solution of the statement of the task:

For $r > 0$, the graph of the function f is a parabola that opens upwards; for $r < 0$, the graph of the function f is a parabola that opens downwards. The bigger the absolute value of r is, the "steeper" the graph of f is.

The change ofvalue of the parameter s determines the extent of the vertical translation of the parabola along the y-axis.

or:

The vertex of the parabola is at the point $(0,s)$.

or:

$(0,s)$ is the point where the graph crosses the vertical axis.

Answer key:

The point for the core competencies is to be awarded if the effects the changes of the values of the parameters r and s have on the behaviour of the graph of f have been explained comprehensibly and correctly.

Expected solution of the guiding question:

As B and E lie on the graph of f and E is the vertex of the parabola, we have:

$s = e$

$b = r \cdot a^2 + e \;\Rightarrow\; r = \dfrac{b - e}{a^2}$

For $b = e$, the function f is not a quadratic function.

Answer key:

The point for the guiding question is to be awarded if the values of the parameters r and s have been given correctly and it has been justified comprehensibly and correctly that $b = e$, so that f is not a quadratic function.

Task 3

Spring Force

If a spring is stretched, then the force required to stretch the spring is directly proportional to the amount by which the spring is stretched. The function F describes the force required in terms of the amount by which the spring is stretched, x.

Therefore: $F(x) = k \cdot x$.

The distance x is measured in metres (m) and $F(x)$ is measured in Newtons (N). The constant k is known as the spring constant and describes the “strength” of a spring.

Task:

Sketch a possible graph of F and label k on your sketch.

Guiding question:

Write down an expression in terms of k that could be used to calculate the work required to stretch the spring by a distance of x_0.

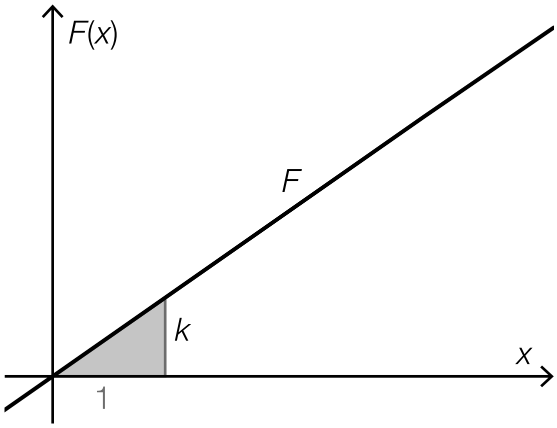
Explain how the work done changes if the spring is stretched by the length $2 \cdot x_0$.

Solution of Task 3

Spring Force

Expected solution of the statement of the task:

Possible sketch:



Answer key:

The point for the core competencies is to be awarded if an appropriate sketch of a homogenous linear function has been drawn and *k* has been labelled correctly.

Expected solution of the guiding question:

$$W = \int_0^{x_0} k \cdot x \, dx = \frac{k \cdot x_0^{\,2}}{2}$$

For the case where the spring is stretched by a length that is twice this amount, the following calculation holds:

$$W = \int_0^{2x_0} k \cdot x \, dx = \frac{k \cdot (2x_0)^2}{2}$$

The work done increases fourfold.

Answer key:

The point for the guiding question is to be awarded if an expression for calculating the work done has been given correctly and the change in work done has been described correctly.
Equivalent expressions are to be marked as correct.

Task 4

Marginal Costs

For a business, the cost function, K, where $K(x) = 4 \cdot x^3 - 60 \cdot x^2 + 400 \cdot x + 1\,000$, gives the cost of manufacturing x units of a product. The production costs, $K(x)$, are given in monetary units (GE), and the number of units manufactured is given in production units (ME).

The marginal costs (in GE/ME) are the additional costs accrued from increasing the number of units manufactured by 1 ME.

Task:

The approximate calculation of the marginal costs for a particular production amount, x_0, can be approximated by the first derivative $K'(x_0)$.

Using the derivative of the function K, determine the marginal costs for a production volume of 15 ME.

Guiding question:

Determine the difference in GE between the approximated marginal costs for a production volume of 15 ME and the actual increase in cost when the production volume is increased from 15 ME to 16 ME.

The first derivative K' is strictly monotonically increasing from $x = 5$ ME. Explain the implications of this statement for the production costs if the production volume increases.

Solution of Task 4

Marginal Costs

Expected solution of the statement of the task:

$K'(x) = 12 \cdot x^2 - 120 \cdot x + 400$
$K'(15) = 1\,300$

Answer key:

The point for the core competencies is to be awarded if $K'(15)$ has been calculated correctly.

Expected solution of the guiding question:

$K'(15) = 1\,300$

$K(16) - K(15) = 8\,424 - 7\,000 = 1\,424$

The value of the approximated marginal costs differs for $x_0 = 15$ ME from the actual cost increase for an extra 1 ME by 124 GE.

This means that the increase in costs upwards from the production volume $x = 5$ is progressive (i. e. the costs increase more and more the higher the production volume is).

Answer key:

The point for the guiding question is to be awarded if the difference has been calculated correctly and the meaning has been interpreted comprehensibly and correctly.

Task 5

Discrete Random Variable

For a discrete random variable, X, the following table shows all of its possible values as well as their corresponding probabilities. The parameter n is a natural number with $n \neq 0$.

k	1	4	7	10	15
$P(X = k)$	0.2	$\frac{2}{n}$	$\frac{6}{n}$	0.1	0.3

Task:

Determine the value of the parameter n and the expectation value of the random variable X. Explain your method.

Guiding question:

The standard deviation σ for the random variable described above has a value of $\sigma = 5.2$.

In the table, change the given probabilities $P(X = k)$ for at least two values of k such that the standard deviation becomes smaller and the table still shows a valid probability distribution. The values of the random variable (in the first row of the table) should remain unchanged. Explain your method.

Solution of Task 5

Discrete Random Variable

Expected solution of the statement of the task:

As the sum of all probabilities is 1, this means that $\frac{8}{n} = 0.4$. Therefore, $n = 20$.

$E(X) = 1 \cdot 0.2 + 4 \cdot 0.1 + 7 \cdot 0.3 + 10 \cdot 0.1 + 15 \cdot 0.3 = 8.2$

Answer key:

The point for the core competencies is to be awarded if both n and $E(X)$ have been calculated correctly and the method has been explained correctly.

Expected solution of the guiding question:

Possible method:
The table must be changed in such a way that both demands – a smaller standard deviation and a valid probability distribution i. e. the sum of the probabilities adds up to 1 – are fulfilled.
A sensible strategy is to reduce the probabilities of the events at the edges of the distribution and to increase the probabilities of the events in the middle.

Possible example:

k	1	4	7	10	15
$P(X = k)$	0.1	0.2	0.3	0.3	0.1

Answer key:

The point for the guiding question is to be awarded if a correct strategy for reducing the standard deviation has been given and an acceptable probability distribution remains. A calculation of the new standard deviation does not need to be given.